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25.

iii) $\sum_{k=0}^{144} k \cdot \binom{144}{k}$

Desarrollo:

$$\sum_{k=1}^{144} k \cdot \binom{144}{k} = \sum_{k=1}^{144} k \cdot \frac{144!}{k! \cdot (144-k)}$$
$$= \sum_{k=1}^{144} \frac{144!}{(k-1)! \cdot (144-k)}$$
$$= \sum_{k=1}^{144} \frac{144!}{(k-1)! \cdot (144-k+1-1)}$$
$$= \sum_{k=1}^{144} \frac{144!}{(k-1)! \cdot (143-k+1)}$$
$$= \sum_{k=1}^{144} \frac{144!}{(k-1)! \cdot (143-(k-1))}$$
$$= \sum_{k=1}^{144} \frac{143! \cdot 144}{(k-1)! \cdot (143-(k-1))}$$
$$= 144 \sum_{k=1}^{144} \frac{143!}{(k-1)! \cdot (143-(k-1))}$$
$$= 144 \sum_{k=1}^{144} \binom{143}{k-1}$$
$$= 144 \cdot \left(\binom{143}{0} + \binom{143}{1} + \binom{143}{2} + \ldots + \binom{143}{142} + \binom{143}{143} \right)$$
$$= 144 \sum_{k=0}^{143} \binom{143}{k}$$
$$= 144 \sum_{k=0}^{143} \binom{143}{k} \cdot 1^{k} \cdot 1^{143-k}$$
$$= 144 \cdot (1+1)^{143}$$
$$= 144 \cdot 2^{143}$$

iv) $\sum_{k=0}^{1998} \frac{1}{(k+1)(k+2)} \cdot \binom{1998}{k}$

Desarrollo:

Multiplicaremos por 1, para reordenar la combinatoria.

$$\sum_{k=0}^{1998} \frac{1}{(k+1)(k+2)} \cdot \binom{1998}{k} = \sum_{k=0}^{1998} \frac{1}{(k+1)(k+2)} \cdot \binom{1998}{k} \frac{1999 \cdot 2000}{1999 \cdot 2000}$$
$$= \sum_{k=0}^{1998} \frac{1}{(k+1)(k+2)} \cdot \frac{1998!}{k! \cdot (1998-k)!} \frac{1999 \cdot 2000}{1999 \cdot 2000}$$
$$= \sum_{k=0}^{1998} \frac{2000!}{(k+2)! \cdot (1998-k)!} \frac{1}{1999 \cdot 2000}$$
$$= \frac{1}{1999 \cdot 2000} \sum_{k=0}^{1998} \frac{2000!}{(k+2)! \cdot (1998-k-2+2)!}$$
$$= \frac{1}{1999 \cdot 2000} \sum_{k=0}^{1998} \frac{2000!}{(k+2)! \cdot (2000-(k+2))!}$$
$$= \frac{1}{1999 \cdot 2000} \sum_{k=0}^{1998} \binom{2000}{k+2}$$
$$= \frac{1}{1999 \cdot 2000} \left[\binom{2000}{2} + \binom{2000}{3} + \binom{2000}{4} + \binom{2000}{5} + \ldots + \binom{2000}{1999} + \binom{2000}{2000} \right]$$

Ahora, sumemos cero dentro del paréntesis.

$$= \frac{1}{1999 \cdot 2000} \left[\binom{2000}{2} + \binom{2000}{3} + \ldots + \binom{2000}{2000} + \overbrace{\binom{2000}{0} - \binom{2000}{0}}^{=0} + \underbrace{\binom{2000}{1} - \binom{2000}{1}}_{=0} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[\binom{2000}{0} + \binom{2000}{1} + \binom{2000}{2} + \ldots + \binom{2000}{1999} + \binom{2000}{2000} - \binom{2000}{0} - \binom{2000}{1} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[\sum_{k=0}^{2000} \binom{2000}{k} - \binom{2000}{0} - \binom{2000}{1} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[\sum_{k=0}^{2000} \binom{2000}{k} 1^k \cdot 1^{2000-k} - \binom{2000}{0} - \binom{2000}{1} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[(1+1)^{2000} - \binom{2000}{0} - \binom{2000}{1} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[2^{2000} - \binom{2000}{0} - \binom{2000}{1} \right]$$
$$= \frac{1}{1999 \cdot 2000} \left[2^{2000} - 2001 \right]$$