$F = C_1 r v + C_2 r^2 v^2 \qquad F = \dfrac{mMG}{r^2} \qquad \boldsymbol{F} = \dfrac{\boldsymbol{dp}}{dt} = m\boldsymbol{a} \qquad dW = \boldsymbol{F} \cdot \boldsymbol{dr} \qquad a_{cent} = \dfrac{v^2}{r} = \omega^2 r$

$U = \dfrac{-mMG}{r} \qquad U = mgh \qquad U = \dfrac{1}{2}kx^2 \qquad K = \dfrac{1}{2}mv^2 \qquad K = \dfrac{1}{2}I\omega^2$

$E_{\text{tot}} = K + U = \dfrac{1}{2}mv^2 - \dfrac{mMG}{r} = \dfrac{-mMG}{2a} \qquad \boldsymbol{L = r \times p} \qquad I = \sum_i m_i r_i^2$

$m_1 r_1 = m_2 r_2 \qquad v = \omega r \qquad T^2 = \dfrac{4\pi^2 (r_1 + r_2)^3}{G(m_1 + m_2)} \qquad \omega = \dfrac{d\theta}{dt} \qquad \omega = \sqrt{k/m}$

$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F} = I\boldsymbol{\alpha} = \dfrac{\boldsymbol{dL}}{dt} \qquad \alpha = \dfrac{d\omega}{dt} \qquad T = \dfrac{2\pi}{\omega} \qquad L = I\omega \qquad \boldsymbol{I} = \int_0^{\Delta t} \boldsymbol{F}\, dt = \boldsymbol{p_f} - \boldsymbol{p_i}$

$\omega = \sqrt{g/l} \qquad \omega_{\text{pr}} = \dfrac{\tau}{L_s} \qquad T^2 = \dfrac{4\pi^2 a^3}{GM}$

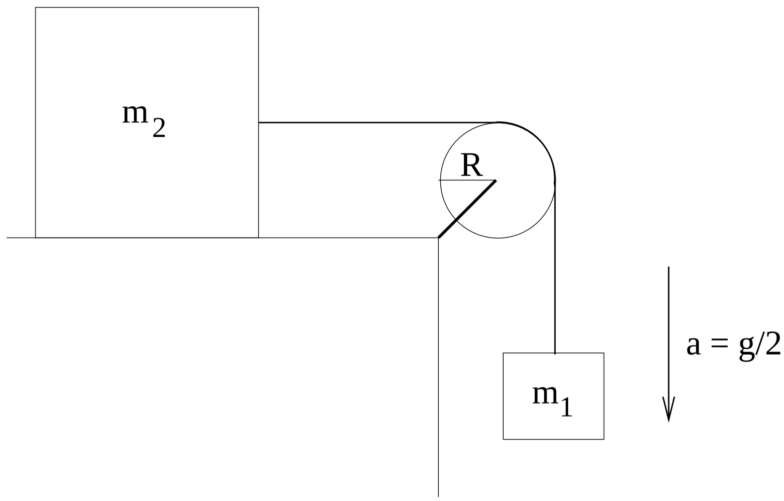
Solid disk of mass M and radius R rotating about its cylindrical axis: $I = \frac{1}{2}MR^2$

$v_f - v_i = -u \ln\left(\dfrac{m_f}{m_i}\right) - gt \qquad I = I_{\text{cm}} + Md^2 \qquad I_z = I_x + I_y$

$f' = f\left(1 + \dfrac{v}{c}\cos\theta\right) \qquad \lambda' = \lambda\left(1 - \dfrac{v}{c}\cos\theta\right)$

Problem 1 (35 points)

An unknown mass, m_1, hangs from a massless string and descends with an acceleration $g/2$. The other end is attached to a mass m_2 which slides on a frictionless horizontal table. The string goes over a uniform cylinder of mass $m_2/2$ and radius R (see figure). The cylinder rotates about a horizontal axis without friction and the string does not slip on the cylinder. Express your answers in parts b, c, and d in terms of g, m_2, and R.



a. (8) Draw free-body diagrams for the cylinder and the two masses.

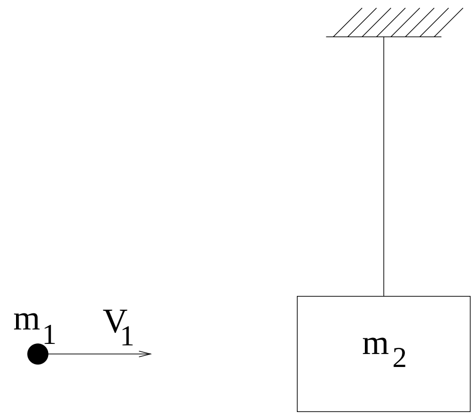
b. (9) What is the tension in the horizontal section of the string?

c. (9) What is the tension in the vertical section of the string?

d. (9) What is the value of the unknown mass m_1?

Problem 2 (30 points)

A bullet of mass m_1 is fired into a pendulum of mass m_2 and length L. The speed of the bullet as it enters the mass m_2 is V_1 (see figure).



First, assume that the collision is elastic, and that $m_1 \ll m_2$.

a. (6) If the pendulum is initially at rest, what is the speed of the bullet after the collision?

b. (8) Now suppose that when the collision occurs, the pendulum, at the bottom of its swing, is moving to the left with velocity V_2. What now is the speed of the bullet after the elastic collision?

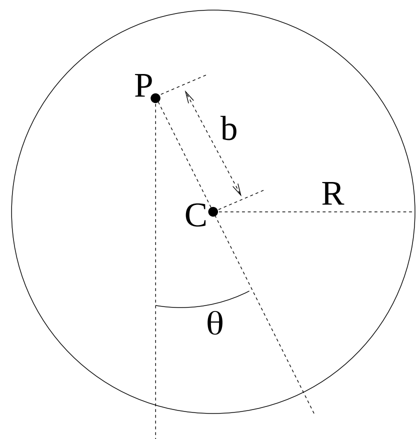
Now assume that the collision is completely inelastic. The pendulum is at rest before the collision, $m_1 < m_2$, but the speed V_1 of the bullet is unknown.

c. (8) After the collision the pendulum moves to the right and it comes to a halt when the string makes an angle $\theta_{\max}$ with the vertical. What was the speed of the bullet? Substitute in your answer $\theta_{\max} = 0$. Does your result make sense?

d. (8) Could $\theta_{\max}$ be $90°$? Explain your answer.

Problem 3 (35 points)

A solid, uniform disk of mass M and radius R is oscillating about an axis through P. The axis is perpendicular to the plane of the disk. Friction at P is negligibly small and can be ignored. The distance from P to the center, C, of the disk is b (see figure). The gravitational acceleration is g.



a. (7) When the displacement angle is θ, what then is the torque relative to point P?

b. (7) What is the moment of inertia for rotation about the axis through P?

c. (7) The torque causes an angular acceleration about the axis through P. Write down the equation of motion in terms of the angle θ and the angular acceleration.

As the disk oscillates, the maximum displacement angle, $\boldsymbol{\theta}_{\mathbf{max}}$, is very small, and the motion is a near perfect simple harmonic oscillation.

d. (7) What is the period of oscillation?

e. (7) As the disk oscillates, is there any force that the axis at P exerts on the disk? Explain your answer.