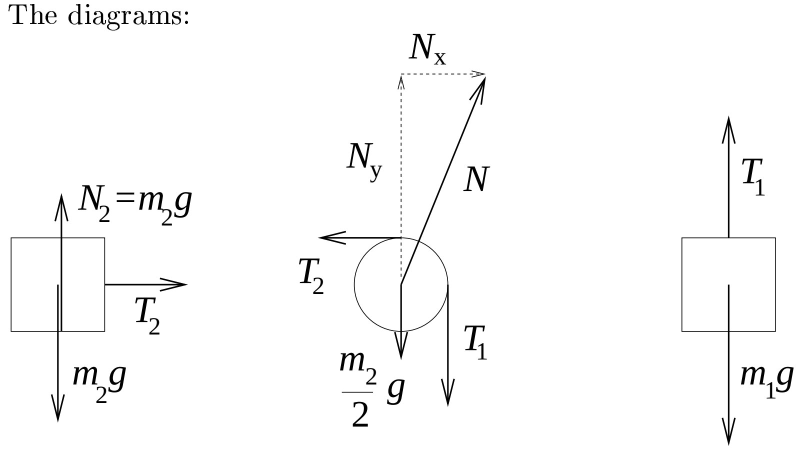
Massachusetts Institute of Technology - Physics Department

Physics - 8.01 **Exam #3** Fall 1999

SOLUTIONS

Problem 1 35 points

8 pts **a)** The diagrams:



The contact force N on the pulley must have components $N_{\mathrm{x}} = T_2$ and $N_{\mathrm{y}} = \frac{m_2}{2}g + T_1$ for the pulley to stay in place.

9 pts **b)** $T_2 = m_2 a = \boldsymbol{m_2\left(\frac{g}{2}\right)}$

9 pts **c)** $R(T_1 - T_2) = I\alpha \qquad I = \frac{1}{2}\frac{m_2}{2}R^2 = \frac{m_2 R^2}{4} \qquad \alpha = \frac{a}{R}$

$R(T_1 - m_2\frac{g}{2}) = I\frac{a}{R} = \frac{Ig}{2R} = \frac{m_2 g R}{8}$

$\Rightarrow T_1 = \frac{m_2 g}{8} + \frac{m_2 g}{2} = \boldsymbol{\frac{5}{8}m_2 g}$

9 pts **d)** $m_1 g - T_1 = m_1 a = m_1\frac{g}{2}$

$m_1\frac{g}{2} = T_1 = \frac{5}{8}m_2 g$

$\Rightarrow \boldsymbol{m_1 = \frac{5}{4}m_2}$

Problem 2 30 points

6 pts **a)** Since $m_2 \gg m_1$, the bullet will leave with the same relative speed with which it came in.
$\boldsymbol{V_1' = V_1}$

8 pts **b)** The speed of the bullet after the collision is now $V_1 + V_2$ *relative* to m_2. Therefore, the speed of m_1 is $\boldsymbol{V_1' = V_1 + 2V_2}$

8 pts **c)** Momentum conservation gives the speed of both masses after the collision, $V' = \frac{m_1 V_1}{m_1+m_2}$.
This kinetic energy brings the system to a height given by

$\frac{1}{2}(m_1 + m_2)V'^2 = (m_1 + m_2)gh$

$\frac{1}{2}\frac{m_1^2 V_1^2}{(m_1+m_2)^2} = gL(1 - \cos\theta_{\max})$

$\Rightarrow \boldsymbol{V_1 = \frac{m_1+m_2}{m_1}\sqrt{2gL(1-\cos\theta_{\max})}}$ Notice for $\theta_{\max} = 0$, $V_1 = 0$ as it should!

8 pts **d)** If $\theta_{\max} = 90°$, then $\cos\theta_{\max} = 0$ and $V_1 = \frac{m_1+m_2}{m_1}\sqrt{2gL}$, which **is possible**. For example, if $L = 1$ m and $\frac{m_2}{m_1} \approx 10^2$, then $V_1 \approx 450$ m/sec.

Problem 3 35 points

7 pts **a)** $\tau_p = |\vec{r_p} \times F| = \boldsymbol{-bMg\sin\theta}$

7 pts **b)** $I_p = I_c + Mb^2 = \boldsymbol{\frac{1}{2}MR^2 + Mb^2}$

7 pts **c)** $\sum \tau_p = I_p \alpha$

$-bMg\sin\theta = (\frac{1}{2}MR^2 + Mb^2)\ddot{\theta}$

$\Rightarrow \boldsymbol{\ddot{\theta} + \frac{bg}{\frac{1}{2}R^2+b^2}\sin\theta = 0}$

7 pts **d)** Under the small angle approximation, $\sin\theta \approx \theta$, and the equation of motion is given by $\ddot{\theta} + \frac{bg}{\frac{1}{2}R^2+b^2}\theta = 0$ which is simple harmonic with an angular frequency given by $\omega = \sqrt{\frac{bg}{\frac{1}{2}R^2+b^2}}$. The period is given by $T = \frac{2\pi}{\omega} = \boldsymbol{2\pi\sqrt{\frac{\frac{1}{2}R^2+b^2}{bg}}}$.

7 pts **e)** There must be a force at P or the CM would accelerate straight downwards.