Massachusetts Institute of Technology - Physics Department

Physics - 8.01 **Exam #2** Fall 1999

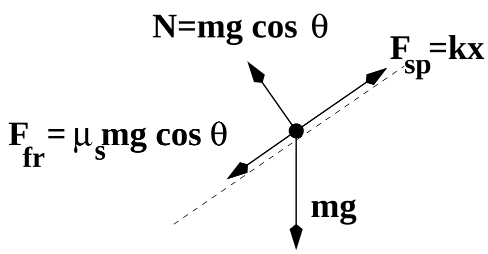
SOLUTIONS

Problem 1 42 points

6 pts **a)** max. extension: $mg\sin\theta + \mu_s mg\cos\theta = kx_{\max}$

$\Rightarrow \boldsymbol{x_{\max} = \frac{mg}{k}(\sin\theta + \mu_s\cos\theta)}$

6 pts **b)**



10 pts **c)** net force along slope once the block starts moving: $kx - mg\sin\theta - \mu_k mg\cos\theta = ma$

max speed when $a = 0$

$\Rightarrow \boldsymbol{x = \frac{mg}{k}(\sin\theta + \mu_k\cos\theta)}$

10 pts **d)** gravity: $\boldsymbol{-mg\sin\theta(x_{\max} - x)}$

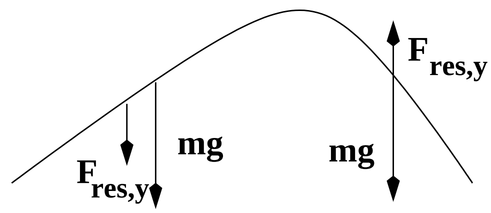
spring: $\boldsymbol{+\frac{1}{2}k(x_{\max}^2 - x^2)}$

fricion: $\boldsymbol{-\mu_k mg\cos\theta(x_{\max} - x)}$

10 pts **e)** Substitute in the results of d) for $x = 0$. Add up the total work done by the three forces. If this is zero or larger than zero, the spring will become as short as l. Thus, $\boldsymbol{\frac{1}{2}kx_{\max}^2 \geq mgx_{\max}(\sin\theta + \mu_k\cos\theta)}$.

Problem 2 32 points

6 pts **a)** As the object goes up, the component of the resistive force in the vertical (y) direction is in the same direction as the gravitational force mg (see figure).



When the object is on the way down, the y component of the resistive force is in the opposite direction. Thus, the component of the force in the y direction is smaller on the way down than on the way up. Thus, it will take **longer than 2 sec** to come down.

6 pts **b)** For zero acceleration, $T = 2\pi\sqrt{\frac{l}{g}}$. For an acceleration a downwards, $T' = 2\pi\sqrt{\frac{l}{g-a}}$. If a were 10 m/sec^2, we'd have free fall, and the pendulum would be weightless (no oscillations, $T \to \infty$). Since $a = 5$ m/sec^2, $\boldsymbol{T' = T\sqrt{2}}$.

6 pts **c)** The viscous term will dominate when $v_{\text{term}} \ll v_{\text{crit}}$ where $v_{\text{crit}} = \frac{C_1}{C_2 r}$. When the viscous term dominates, $v_{\text{term}} = \frac{mg}{C_1 r}$. The mass of the oil drop is given by $m = \frac{4}{3}\pi r^3 \rho$. Thus, $\frac{4\pi r^3 \rho g}{3 C_1 r} \ll \frac{C_1}{C_2 r}$.

$$\boldsymbol{\Rightarrow r \ll \left(\frac{3C_1^2}{4\pi\rho g C_2}\right)^{\frac{1}{3}}}$$

6 pts **d)** $\omega = 2$, $T = \frac{2\pi}{\omega}$, $f = \frac{1}{T} = \frac{\omega}{2\pi} = \boldsymbol{\frac{1}{\pi}}$ **Hz**

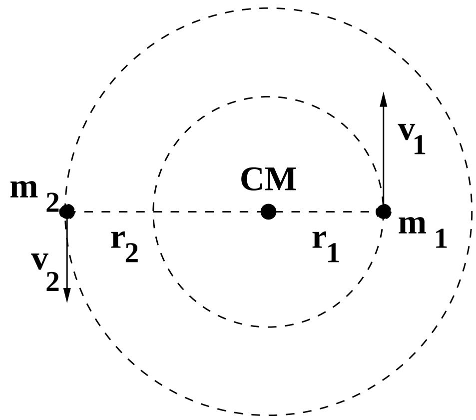
8 pts **e)** The speed is maximum when $x = 0$, i.e., when $\sin(2t + \frac{\pi}{4}) = 0$

This happens when $2t + \frac{\pi}{4} = n\pi$ where $n = 0, \pm1, \pm2, \pm3, \ldots$

$\boldsymbol{\Rightarrow t = \frac{\pi}{2}(n - \frac{1}{4})}$

Alternatively, $v = -0.6\cos(2t + \frac{\pi}{4})$, and the max speed is when $\cos(2t + \frac{\pi}{4}) = \pm1$. This leads to the same times.

Problem 3 26 points



6 pts **a)**

5 pts **b)** $\boldsymbol{F = \frac{m_1 m_2 G}{(r_1+r_2)^2}}$

5 pts **c)** $\boldsymbol{a_1 = \frac{m_2 G}{(r_1+r_2)^2}}$ and $\boldsymbol{a_2 = \frac{m_1 G}{(r_1+r_2)^2}}$

10 pts **d)** Let ω be the angular velocity, which must be the same for both stars. The centripetal acceleration must equal that of gravity.

star 1: $\omega^2 r_1 = \frac{m_2 G}{(r_1+r_2)^2}$

star 2: $\omega^2 r_2 = \frac{m_1 G}{(r_1+r_2)^2}$

Add these two equations and substitute $\omega = \frac{2\pi}{T}$ to find $\frac{4\pi^2}{T^2}(r_1 + r_2) = \frac{(m_1+m_2)G}{(r_1+r_2)^2}$.

Thus,

$$\boldsymbol{T = \frac{2\pi(r_1 + r_2)^{\frac{3}{2}}}{G^{\frac{1}{2}}(m_1 + m_2)^{\frac{1}{2}}}}$$