
Laplace Transform Pairs

Laplace Domain
Time Domain

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

 $f(t)$

1. $c_1 F(s) + c_2 G(s)$

$c_1 f(t) + c_2 g(t)$

2. $F\left(\frac{s}{a}\right)$

$f(a \cdot t) a$

3. $F(s)G(s)$

$$\int_0^t f(t - \tau) g(\tau) d\tau = \int_0^t f(\tau) g(t - \tau) d\tau$$

4. $s^n F(s) - \sum_{j=1}^n s^{n-j} \frac{d^{j-1} f}{dt^{j-1}}(0)$

$\frac{d^n f}{dt^n}(t)$

5. $\frac{1}{s^n} F(s)$

$$\underbrace{\int_0^t \cdots \int_0^t}_{n} f(\tau) d\tau \cdots d\tau$$

6. $F(s + a)$

$e^{-at} f(t)$

7. $\frac{n!}{s^{n+1}}$

$t^n, n = 1, 2, \dots, t$

8. $\frac{1}{s + a}$

e^{-at}

9. $\frac{1}{(s + a)^2}$

te^{-at}

10. $\frac{a}{s(s + a)}$

$1 - e^{-at}$

11. $\frac{s + a}{s^2}$

$1 + at$

12. $\frac{a^2}{s^2(s + a)}$

$at - (1 - e^{-at})$

13. $\frac{s + b}{s(s + a)}$

$\frac{b}{a} \left\{ 1 - \left(1 - \frac{a}{b} \right) e^{-at} \right\}$

14. $\frac{a}{s^2 + a^2}$

$\sin at$

15. $\frac{s}{s^2 + a^2}$

$\cos at$

16. $\frac{a^2}{s(s^2 + a^2)}$

$1 - \cos at$

17. $\frac{s}{s^2 - a^2}$

$\cosh at$

18. $\frac{a}{s^2 - a^2}$

$\sinh at$

19. $\frac{a(s^2 - a^2)}{(s^2 + a^2)^2}$

$at \cos at$

Laplace Transform Pairs

Laplace Domain

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Time Domain

$f(t)$

$$20. \frac{2sa^2}{(s^2 + a^2)^2}$$

$$at \sin at$$

$$21. \frac{s + a}{(s + a)^2 + b^2}$$

$$e^{-at} \cos bt$$

$$22. \frac{b}{(s + a)^2 + b^2}$$

$$e^{-at} \sin bt$$

$$23. \frac{1}{(s + a)(s + b)}$$

$$\frac{e^{-at} - e^{-bt}}{(b - a)}$$

$$24. \frac{s + w}{(s + a)(s + b)}$$

$$\frac{\{(w - a)e^{-at} - (w - b)e^{-bt}\}}{(b - a)}$$

$$25. \frac{1}{(s + a)(s + b)(s + c)}$$

$$\frac{e^{-at}}{(b - a)(c - a)} + \frac{e^{-bt}}{(a - b)(c - b)} + \frac{e^{-ct}}{(a - c)(b - c)}$$

$$26. \frac{s + w}{(s + a)(s + b)(s + c)}$$

$$\frac{(w - a)e^{-at}}{(b - a)(c - a)} + \frac{(w - b)e^{-bt}}{(a - b)(c - b)} + \frac{(w - c)e^{-ct}}{(a - c)(b - c)}$$

$$27.* \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$$

$$28. \frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$-\frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t - \phi_1)$$

$$29.* \frac{s + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi_1)$$

$$30.* \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$1 - \frac{\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t + \phi_1)$$

$$31.* \frac{s + \zeta\omega_n}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

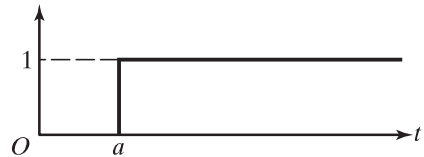
$$e^{-\zeta\omega_n t} \sin(\omega_d t + \phi_1)$$

$$32. 1$$



Unit impulse at $t = 0$

$$33. \frac{e^{-as}}{s}$$



Unit step function at $t = a$

$$*\omega_d = \omega_n \sqrt{1 - \zeta^2}; \zeta < 1$$

$$\phi_1 = \cos^{-1} \zeta; \zeta < 1$$