2. Fixed Income Securities

1. True or False? Briefly explain (or qualify) your answers.
   (a) The duration of a coupon bond maturing at date T is always less than the duration of a zero-coupon bond maturing on the same date.
   (b) When investing in bonds, we should invest in bonds with higher yields to maturity (YTM) because they give higher expected returns.
   (c) The phrase "On the run" refers to junk bonds that have recently defaulted.

2. True or false? Briefly explain (or qualify) your answers.
   (a) Investors expect higher returns on long-term bonds than short-term bonds because they are riskier. Thus the term structure of interest rates is always upward sloping.
   (b) Bonds whose coupon rates fall when the general level of interest rates rise are called reverse floaters. Everything else the same, these bonds have a lower modified duration than their straight bond counterparts.

3. True, false or "it depends" (give a brief explanation):
   (a) Term structure of interest rates must be always upward sloping because longer maturity bonds are riskier.
   (b) Bonds with higher coupon rates have more interest rate risk.

4. True, false (give a brief explanation): The term structure of interest rates is always upward sloping because bonds with longer maturities are riskier and earn higher returns.

5. True or false (give a brief explanation): A flat term structure (identical spot rates for all maturities) indicates that investors do not expect interest rates to change in the future.

6. True or false (give a brief explanation): To reduce interest rate risk, an over-funded pension fund, i.e., a fund with more assets than liabilities, should invest in assets with longer duration than its liabilities.

7. Which security has a higher *effective* annual interest rate?
   (a) A three-month T-bill selling at $97, 645 with face value of $100, 000.
   (b) A coupon bond selling at par and paying a 10% coupon semi-annually.

8. The Wall Street Journal quotes 6.00% for the Treasury bill with a par value of $100,000 due two months from now. What is the effective annual yield on the bill?

9. Which security has a higher effective annual interest rate?
   (a) A six-month T-bill selling at $98,058 with face value of $100,000.
   (b) A coupon bond selling at par and paying a 4.2% coupon (2.1% every six months).

10. Spot rates.

You are given the following prices of US Treasury Strips (discount or zero coupon bonds):

Maturity	Price (per 100 FV)
1	96.2
2	91.6
3	86.1

(a) Compute the spot rates for years 1, 2 and 3.

(b) Now, suppose you are offered a project which returns the following cashflows:
$300m at the end of year 1
$210m at the end of year 2
$400m at the end of year 3

The project costs $600m today.
Calculate the NPV of the project using the spot rates computed above.

11. Assume that spot interest rates are as follows:

Maturity (year)	Spot Rate (%)
1	3.0
2	3.5
3	4.0
4	4.5

Compute the prices and YTMs of the following bonds:

(a) A zero-coupon bond with 3 years to maturity.

(b) A bond with coupon rate 5% and 2 years to maturity.

(c) A bond with coupon rate 6% and 4 years to maturity.

Assume that spot rates and YTMs are with annual compounding, coupon payments are annual, and par values are $100.

12. Treasury bonds paying an 8% coupon rate with *semiannual* payments currently sell at par value. What coupon rate would they have to pay in order to sell at par if they paid their coupons *annually*?

13. Yields on three Treasury notes are given as follows:

Bonds and Notes

Coupon Rate	Maturity	Bid	Asked	Asked yield
7.250	Aug. 04	110:00	110:00	2.07%
10.750	Aug. 05	123:12	123:13	2.55%
5.750	Aug. 10	112:00	112:01	3.97%
5.00	Aug. 11	106:24	106:25	4.09%
8.750	Aug. 20	143:26	143:27	5.02%
6.125	Aug. 29	114:11	114:12	5.13%

Strips

Maturity	Bid	Asked	Asked yield
Aug. 04	96:03	96:04	2.00%
Aug. 05	92:19	92:21	2.58%
Aug. 10	71:13	71:17	4.24%
Aug. 11	67:06	67:08	4.47%

Table 1: Treasury Prices and Yields, August 20, 2002

Maturity (yrs)	Coupon rate (%)	Yield to maturity (%)
1	0	5.25
2	5	5.50
3	6	6.00

Coupons are paid annually.

(a) What are the prices of the 1-year, 2-year, and 3-year notes?

(b) What are the spot interest rates for years 1, 2 and 3?

(c) What is the implied forward rate for year 2 to year 3?

14. Using the following data given in Table 1, answer these questions:

(a) What were the 2-, 3- and 8-year spot interest rates?

(b) What is the forward interest rate from August 2004 to August 2005? From August 2010 to August 2011?

(c) What does the slope of the term structure imply about future interest rates? Explain briefly.

Express your answers to (a) and (b) as effective annual interest rates.

15. Forward rates.

You are given the following spot rates:

Maturity	Spot rate
1	2.9%
2	3.2%
3	3.6%
4	4.2%

(a) Compute the forward rate between years 1 and 2.

(b) Compute the forward rate between years 1 and 3.

(c) Suppose one of your 401 TAs, offers to *commit* to borrowing money from you between years 3 and 4 at a rate of 6.3%. Is there any way you can profit from this? What sort of risk are you exposed to? Is this strategy an arbitrage?

16. You are a bond trader and see on your screen the following information on three bonds with annual coupon payments and par value of $100:

Bond	Coupon rate (%)	Maturity (year)	YTM(%)
A	0	1	5.00
B	5	2	5.50
C	6	3	6.00

Coupon payments are annual.

(a) What are the prices of the above bonds?

(b) Construct the current term-structure of spot interest rates.

(c) Explain how you would synthetically replicate a zero-coupon bond with a maturity of 3 years and a par value of $100.

(d) What should be the price of the bond so that there is no arbitrage?

17. You are given the following information:

Bond	Coupon Rate	Maturity	Price
A	10%	1	106.80
B	5%	2	101.93
C	10%	3	111.31

All coupon payments are annual and par values are 100.

(a) Determine the 1-, 2- and 3-year spot interest rates from the given prices.

(b) Compute the annual forward rate from year one to year two, i.e., f_2.

18. The Wall Street Journal gives the following prices for STRIPS (with a principal of 100):

Bond	Maturity Year	Price
A	1	95.92
B	2	92.01
C	3	87.00

(a) Determine the 1-, 2- and 3-year spot interest rates from the given prices.

(b) Compute the annual forward rate from year two to year three, i.e., f_3 (or $f_{2,3}$).

(c) Compute the yield to maturity of a 2-year coupon bond with a principal of 100 and a coupon rate of 4.25%. Assume annual coupon payments.

19. The Wall Street Journal gives the following prices for the STRIPS:

Maturity (years)	1	2	3
Price (% of par value)	97.56	95.18	92.86

Suppose that you have a short term liability of $10 million every year for the next three years.

(a) Calculate the present value of the liability.

(b) Calculate the duration of your liability.

(c) Suppose that you want to set aside $20 million to pay part of the liability and the fund will be invested in STRIPS. In order to avoid interest rate risk, what maturity for the STRIPS should you pick?

(d) If the interest rates increase by 0.10%, how much will be the remaining short fall for your liability?

20. The Wall Street Journal gives the following prices for the STRIPS:

Maturity (years)	1	2	3
Price (% of par value)	96.1538	92.4556	88.8996

You are holding an asset which yields a sure income of $20 million every year for the next two years.

(a) Calculate the present value of the asset.

(b) Calculate the modified duration of the asset.

(c) If the interest rates increase by 0.10%, how much will the asset's value change in dollars?

(d) Suppose that you want to use an interest rate future to hedge the interest rate risk. The futures has a contract value of $100,000 and a modified duration of 5. Assume a flat term structure of interest rates. What will be your hedging strategy?

(e) Show that with the hedging position in the futures, the value of your total position (the asset plus the futures position) is insensitive to the change in interest rate.

21. The term structure of spot interest rates is given in the table below:

Maturity (years)	1	2	3	4	5	6
Interest rate (%)	3.5	3.0	4.0	4.0	4.0	4.0

You have just signed a lease on an office building with a rental payment of $1 million per year forever. The first payment is due one year from now.

(a) What is the present value of the lease?

(b) New inflation figures imply that expected inflation will be 0.5% percent higher. As a result, interest rates for all maturities now increase by 0.5%. What is the PV of the lease under the new market conditions?

22. The following is a list of prices for zero-coupon bonds of various maturities. Calculate the yields to maturity of each bond and the implied sequence of forward rates.

Maturity (Years)	Price of Bond ($)
1	943.40
2	898.47
3	847.62
4	792.16

23. You have accounts receivable of $10 million due in one year. You plan to invest this amount in the Treasury market for one year after receiving it. You would like to lock into an interest rate today for this future investment. Current yields on Treasury STRIPS are as follows:

Maturity	Yield (%)
1	5.25
2	5.50
3	5.75

(a) Your bank quotes you a forward rate of 5.50%. Is this in line with the forward rate implied by market interest rates?

(b) Suppose that you can buy or sell short the STRIPS at the above yields without additional costs and the STRIPS have face value of $1,000. How can you use the STRIPS to structure the forward investment you wanted?

24. Refer to Table 1 (p12), use the quoted yields to calculate the present value for the cash payments on the

(a) August 2011 strip.

(b) August 2011 note.

Assume that the first note coupon comes exactly six months after August 20, 2002, and that principal is repaid after exactly 9 years. (This timing assumption is not exactly right. Also, the quoted yields are rounded. Your PV will not match the Asked Price exactly.)

25. Spot Rates and Forward Transactions.

Suppose you have the following bonds, which pay coupons at the end of each year:

Maturity (yrs)	YTM (%)	Coupon (%)
1	4.0%	4%
2	4.2%	5%
3	4.8%	5%

(a) Determine the price of each bond per $100 face value.

(b) What are the spot rates for years 1, 2 and 3?

26. Which of the following statements are correct?
With today's Yield Curve, you can compute exactly:

(a) The price at which a 5-year T-Strip with $1, 000 face value trades today.

(b) The spot rates that will prevail in two years.

(c) The price at which a 5-year T-bond with 7% coupon and $1,000 face value will trade in one year.

(d) The forward rates that prevail today.

(e) The forward rates that will prevail in two years.

27. Suppose you are given the following prices for two U.S. Treasury strips.

Maturity date	Price	Yield to maturity
April 2033	91.21	0.71%
April 2034	89.94	0.76%

Assume for simplicity that the maturity dates are exactly 13 and 14 years from now (April). Calculate the forward rate of interest between April 2033 and April 2034.

28. Here are closing quotes for 4 Treasury securities on October 11, 2002.

	Coupon	Maturity	Asked Price	Asked Yield
(Note)	6.5	Feb. 2010	119:08	3.50%
(Note)	5.0	Feb. 2011	109:20	3.65%
(Strip)	0	Feb. 2010	76:22	3.65%
(Strip)	0	Feb. 2011	73:08	3.77%

(a) Suppose you buy the Feb. 2011 note and hold it to maturity. How much would you have to pay (approximately)? What cash flows would you receive, on what dates?

(b) What are the spot interest rates for February 2010 and February 2011?

(c) What is the forward rate of interest between February 2010 and February 2011?

(d) Which of these securities has the shortest duration? Explain.

29. Yankee Inc. has sold the Super Coupon Absolute Marvel (SCAM) security to raise new funds. Unlike ordinary bonds, it pays no par value/face value at the end of its life. It only pays coupons every year as follows: $\$100(1 + 0.05)$ at the end of year one, $\$100(1 + 0.05)^2$ at the end of year two, and so on. This security lasts for 4 years (i.e., makes 4 payments). The current interest rate is 5% for all maturities.

(a) What is the price today of SCAM?

(b) What is the duration today of SCAM?

(c) Yankee Inc. sold $10 million worth of SCAM. It plans to invest the proceeds in two assets, A1 and A2, for the short run. A1 is a 12-month T-Bill, whereas A2 is a 4-year STRIPS. How much should Yankee Inc. invest in A1 and A2 to avoid interest rate risks?

30. You manage a pension fund, and your liabilities consist of two payments as follows:

Time	Payment
10 years	$20 million
30 years	$30 million

Your assets are $18 million. The term structure is currently flat at 5%.

(a) Compute the present value of your liabilities.

(b) Compute modified duration of your liabilities.

(c) Compute an approximate change in the present value of your liabilities, using duration, when interest rates fall by 0.25%.

(d) Suppose that you invest the $18 million in 1-year Treasury bills (i.e., 1-year zero-coupon bond) and in a Treasury bond with modified duration of 20. How would you allocate your assets to avoid interest rate risk of your portfolio, which includes both assets and liabilities?

31. As a mid-size company, you have a pension plan which pays out $10 million a year forever. The first payment is exactly one year from now. The term structure is currently flat at 5%.

(a) Compute the present value of your pension liabilities.

(b) Suppose that the interest rate goes down by 0.1%. How does the value of your liability change?

(c) Given your answer to (b), what is the modified duration of your pension liability?

(d) Suppose that the pension plan is fully funded (i.e., the value of your assets equal the value of pension liabilities). You want to invest all your assets in bonds to avoid any interest rate risk. What should the duration of your bond portfolio be?

(e) Suppose that this portfolio is a single zero-coupon bond. What should its maturity and total par value be?

32. On a job interview, you were handed the following quotes on U.S. Treasuries:

Bond	Maturity (years)	Coupon Rate	Yield to Maturity
1	1	5%	4.5%
2	2	5%	5.0%
3	3	0%	5.5%

Assume that the par value is $100 and coupons are paid annually, with the first coupon payment coming in exactly one year from now. The yield to maturity is also quoted as an annual rate. You are then asked the following questions:

(a) What should be the price of a bond with a maturity of 3 years and coupon rate of 5%, given the above information?

(b) What should be the 1-year forward rate between years 2 and 3?

(c) What is the modified duration of a bond portfolio with 30% invested in bond 1 and 70% invested in bond 3?

(d) How much would the value of the portfolio in (c) change if the yields of all bonds increased by 0.15%?

33. You have the following data on Treasury bonds. Assume that there are no taxes, only annual coupon payments are made and the first coupon payment occurs a year from now.

Bond	Year of Maturity	Coupon	Face Value at T	Price Today
A	1	25	100	100.00
B	2	50	500	422.61
C	3	20	300	232.28
D	10	0	1000	192.31

(a) Calculate the following four annualized forward rates: f from 0 to 1, f from 1 to 2, f from 2 to 3, and f from 3 to 10.

(b) Is it a good investment if it costs $21 million now and yields the following risk-free cash inflows?

Year	1	2	3
Cash Flow (in million dollars)	9	10	11

34. Which of the following investments is most affected by changes in the level of interest rates? Suppose interest rates go up or down by 50 basis points (± 0.5%). Rank the investments from most affected (largest change in value) to least affected (smallest change in value).

(a) $1 million invested in short-term Treasury bills.

(b) $1 million invested in Treasury strips (zero coupons) maturing in December 2016.

(c) $1 million invested in a Treasury note maturing in December 2016. The note pays a 5.5% coupon.

(d) $1 million invested in a Treasury bond maturing in January 2017. The bond pays a 9.25% coupon.

Explain your ranking briefly.

35. Valerie Smith is attempting to construct a bond portfolio with a duration of 9 years. She has $500,000 to invest and is considering allocating it between two zero coupon bonds. The first zero coupon bond matures in exactly 6 years, and the second zero coupon bond matures in exactly 16 years. Both of these bonds are currently selling for a market price of $100. Suppose that the yield curve is flat at 7.5%. Is it possible for Valerie to construct a bond portfolio having a duration of 9 years using these two types of zero coupon bonds? If so, how? (Describe the actual portfolio.) If not, why not?

36. Given the bond prices in the question above, you plan to borrow $15 million one year from now (end of year 1). It will be a two-year loan (from year 1 to year 3) with interest paid at the ends of year 2 and 3. The cash flow is as follows:

Year 1	Year 2	Year 3
Borrow $15M	Pay interest	Pay interest plus principal of $15M$

Explain how you could arrange this loan today and "lock in" the interest rate on the loan. What transactions today would be required? What would the interest rate be? You can buy or sell any of the bonds listed above (in the previous question).

37. You purchased a 3 year coupon bond one year ago. Its par value is $1,000 and coupon rate is 6%, paid annually. At the time you purchased the bond, its yield to maturity was 6.5%. Suppose you sell the bond after receiving the first interest payment.

(a) What is the total rate of return from holding the bond for the year if the yield to maturity remains at 6.5% when you sell it?

(b) What if the yield to maturity becomes 6.0% when you sell it?

38. You manage a pension fund, which provides retired workers with lifetime annuities. The fund must pay out $1 million per year to cover these annuities. Assume for simplicity that these payments continue for 20 years and then cease. The interest rate is 4% (flat term structure). You plan to cover this obligation by investing in 5- and 20-year maturity Treasury strips.

(a) What is the duration of the funds 20-year payout obligation?

(b) You decide to minimize the funds exposure to changes in interest rates. How much should you invest in the 5- and 20- year strips? What will be the par value of your holdings of each strip?

(c) After three months, you reexamine the pension funds investment strategy. Interest rates have increased. You still want to minimize exposure to interest rate risk. Will you invest more in 20-year strips and less in 5-year strips? Explain briefly.

39. Duration and Convexity.

Consider a 10 year bond with a face value of $100 that pays an annual coupon of 8%. Assume spot rates are flat at 5%.

(a) Find the bond's price and duration.

(b) Suppose that 10yr yields increase by 10bps. Calculate the change in the bond's price using your bond pricing formula and then using the duration approximation. How big is the difference?

(c) Suppose now that 10yr yields increase by 200bps. Repeat your calculations for part (b).

(d) Given that the bond has a convexity of 33.8, use the convexity adjustment and repeat parts (b) and (c). Has anything changed?

40. The yield to maturity of a 10-year zero-coupon bond is 4%.

(a) Suppose that you buy the bond today and hold it for 10 years. What is your return? (Express this return as an annual rate.)

(b) Given only the information provided, can you compute the return on the bond if you hold the bond only for 5 years? If you answered yes, compute the return. If you answered no, explain why.

41. Refer to Table 2.

(a) What was the quoted ask price (in dollars) for the 8.75s of 2020? Assume par value = $10,000. You can ignore accrued interest.

(b) What cash flows would you receive if you bought this bond on August 13, 2006 and held it to maturity? Specify amounts and timing (by month).

(c) Suppose you buy $10 million (par value) of the 4.125s of 2008 and sell short $10 million (par value) of the 3.25s of 2007. You hold each trade until the bond matures. What cash flows would you pay or receive? Specify amounts and timing. You can ignore any fees or margin requirements for the short sale.

Table 2: Treasury Prices and Yields, August 3, 2006

Coupon Rate	Maturity	Bid	Asked	Asked yield
Bonds and Notes:				
3.25	Aug. 07	98:04	98:05	5.09%
4.125	Aug. 08	98:16	98:17	4.89
6.0	Aug. 09	103:00	103:00	4.92
5.75	Aug. 10	103:06	103:07	4.86
4.375	Aug. 12	97:11	97:12	4.88
12.5	Aug. 14	121:08	121:09	4.86
8.75	Aug. 20	136:02	136:03	5.11
6.125	Aug. 29	113:18	113:19	5.11
Strips:				
	Aug. 07	94:30	94:31	5.08%
	Aug. 08	90:18	90:19	4.93
	Aug. 09	86:10	86:10	4.91
	Aug. 10	82:18	82:19	4.80
	Aug. 12	74:25	74:26	4.87
	Aug. 14	67:21	67:22	4.92
	Aug. 16	60:20	60:21	5.05

42. Refer again to Table 2.

    (a) What were the 1, 2, 3, 4, 6 and 10-year spot interest rates?

    (b) What was the forward interest rate from August 2007 to August 2008? From August 2009 to August 2010?

    (c) The 8.75s of August 2020 will pay a coupon in August 2010. What was the PV of this payment in August 2006?

    (d) What did the slope of the term structure imply about future interest rates? Explain briefly.

    Express your answers to (a) and (b) as effective annual interest rates.

43. Refer again to Table 2. Use the quoted yield on the August 2012 note to calculate the present value for the cash payments on the August 2012 note.

    Assume that the first note coupon comes exactly six months after August 13, 2006. Note: The quoted yields are rounded. Your PV may not match the Asked Price exactly.

44. In August 2006 you learn that you will receive a $10 million inheritance in August 2007. You have committed to invest it in Treasury securities at that time, but worry that interest rates may fall over the next year. Assume that you can buy or sell short any of the Treasuries in Table 2 at the prices listed in the table.

    (a) How would you lock in a one-year interest rate from August 2007 to August 2008? What transactions would you make in August 2006? Show how the transactions that lock in the rate.

    (b) Suppose you wanted to lock in a 5-year interest rate from August 2007 to August 2012. How does your answer to part a change?

45. Assume the yield curve is flat at 4%. There are a 3-year zero coupon bond and a 3-year coupon bond that pays a 5% coupon annually.

    (a) What are the YTMs of these two bonds?

    (b) Suppose the yield curve does not change in the future. You invest $100 in each of the two bonds. You re-invest all coupons in zero coupon bonds that mature in year 3. How much would you have at the end of year 3?

46. The attached chart shows the fixed obligations of the Edison Mills pension plan, which is also managed by the Renssalear Advisors.

Year	Benefits ($MM)
2000	$10.60
2001	$11.24
2002	$11.91
2003	$12.62
Total	$46.37

Using an assumed interest rate of 6%, the present value of this stream of fixed cash outflows is $40 million. You are given $40 million to invest in U.S. Treasury bonds for the pension plan. Your boss insists that you only invest in 1 year and 10 year STRIPS. Your task is to minimize the exposure of the Edison Mills pension fund to unexpected changes in the level of interest rates. Your performance will be evaluated after one year.

Answer the following questions. Use the backs of this page and the next page if needed to complete your answer.

(a) What is duration of your obligation?

(b) Describe – step by step – how you would choose and manage the portfolio of 1-year and 10-year STRIPS to minimize the exposure to interest rate risk.

47. Bond underwriting.

Bond underwriters agree to purchase a corporate client's new bonds at a specific price, usually near 100% of face value, and then attempt to resell the bonds to the public. The act of reselling takes some time. Underwriting fees increase with the maturity of the bonds. Provide an explanation for this pattern of fees.

48. You have just been given the following bond portfolio:

Bond	Maturity (yrs)	Coupon rate (%)	Holdings ($ million)
A	2	7.00	10
B	5	7.25	20
C	10	7.50	20
D	20	8.00	10

Coupons are paid semi-annually. The current yield curve is flat at 6%.

(a) What is duration for each of the bonds in your portfolio?

(b) What is the duration of your total portfolio?

(c) What is the percentage change in the value of your portfolio if the yield moves up by 20 basis points?

49. A U.S. Treasury bond makes semi-annual payments of $300 for 10 years. (The investor receives 20 $300 payments at 6-month intervals.) At the end of 10 years, the bonds principal amount of $10,000 is paid to the investor.

(a) What is the present value of the bond if the annual interest rate is 5%?

(b) Suppose the bond is observed trading at $11,240. What discount rate are investors using to value the bonds cash flows? (This discount rate is called the bonds "yield to maturity.")

50. A savings bank has the following balance sheet ($ millions, market values).

Assets	Liabilities
Treasuries: $200	Deposits: $900
Floating rate mortgage loans: $300	Equity: $100
Fixed rate mortgage loans: $500	
Total: $1,000	Total: $1,000

Durations are as follows:

Treasuries	6 months
Floating rate mortgage loans	1 year
Fixed rate mortgage loans	5 years
Deposits	1 year

(a) What is the duration of the banks equity? Briefly explain what this duration means for the banks stockholders.

(b) Suppose interest rates move from 3% to 4% (flat term structure). Use duration to calculate the change in the value of the banks equity. Will the actual change be more or less than your calculated value? Explain briefly.

51. Fixed Income Management:

A pension fund has the following liability:

A 20-yr annuity, that will pay coupons of 7% at the end of each year (t=1...t=20). The pension fund's liability has a face value of 100. The yield curve is flat at 5%.

(a) Calculate the PV and duration of this liability.

(b) The same pension fund has the following assets: a 1-yr discount bond with face value 100, and a 20-yr discount bond which also has a face value of 100.

Calculate the PV and duration of the portfolio of assets.

(c) How would you change the portfolio composition of assets (keeping the PV of assets the same), so that the NPV of the firm, defined as $PV_A - PV_L$, that is Present Value of assets minus the Present Value of liabilities, is unaffected by interest rate changes?

(d) After making the change above in (c), what is the change in the NPV of the firm if interest rates increase by 10 basis points?

52. Three bonds trade in London and pay annual coupons

Bond	Coupon	Maturity	Price
A	5%	1	100.96%
B	6.5%	3	106.29%
C	2%	3	93.84%

Prices are in decimals, not 32nds.

(a) What is each bond's yield to maturity?

(b) What are the 1, 2 and 3-year spot rates? What are the forward rates?

53. Assume the spot rates for year 1, year 2 and year 3 are 3.5%, 4% and 4.5%, respectively. There are a 3-year zero coupon bond and a 3-year coupon bond that pays a 5% coupon annually.

(a) What are the YTMs of the bonds?

(b) Calculate all 1-year forward rates.

(c) Calculate the realized returns of the two bonds over the next year if the yield curve does not change. (In year 1 the 1-year spot rate is 3.5%, the 2-year spot rate is 4% and the 3-year spot rate is 4.5%.)

54. A pension plan is obligated to make disbursements of $1 million, $2 million and $1 million at the end of each of the next three years, respectively. Find the durations of the plan's obligations if the interest rate is 10% annually.

55. A local bank has the following balance sheet:

Asset	Liability
Loans $100 million	Deposits $90 million
	Equity $10 million

The duration of the loans is 4 years and the duration of the deposits is 2 years.

(a) What is the duration of the bank's equity? How would you interpret the duration of the equity?

(b) Suppose that the yield curve moves from 6% to 6.5%. What is the change in the bank's equity value?

56. The term structure is flat at 6%. A bond has 10 years to maturity, face value $100, and annual coupon rate 5%. Interest rates are expressed as EARs.

(a) Compute the bond price.

(b) Compute the bond's duration and modified duration.

(c) Suppose the term structure moves up to 7% (still staying flat). What is the bond's new price?

(d) Compute the approximate price change using duration, and compare it to the actual price change.

57. Suppose Microsoft, which has billions invested in short-term debt securities, undertakes the following two-step transaction on Dec. 30, 2009. (1) Sell $1 billion market value of 6-month U.S. Treasury bills yielding 4% (6 month spot rate). (2) Buy $1 billion of 10-year Treasury notes. The notes have a 5.5% coupon and are trading at par. Microsoft does not need the $1 billion for its operations and will hold the notes to maturity.

(a) What is the impact of this two-step transaction on Microsoft's earnings for the first 6 months of 2010?

(b) What is the transaction's NPV?

Briefly explain your answers.

58. The Treasury bond maturing on August 15, 2017 traded at a closing ask price of 133:16 (i.e., $133 16/32) on August 31, 2007. The coupon rate is 8.875%, paid semi-annually. The yield to maturity was 4.63% (with semi-annual compounding).

   (a) Explain in detail how this yield to maturity was calculated.

   (b) Discount the bond's cash flows, using the yield to maturity. Can you replicate the ask price? (The replication should be close but won't be exact.) Show your calculations.

59. The following questions appeared in past CFA Examinations. Give a brief explanation for each of your answers.

   (a) Which set of conditions will result in a bond with the greatest volatility?

      i. A high coupon and a short maturity.
      ii. A high coupon and a long maturity.
      iii. A low coupon and a short maturity.
      iv. A low coupon and a long maturity.

   (b) An investor who expects declining interest rates would be likely to purchase a bond that has a .... coupon and a ...... term to maturity.

      i. Low, long.
      ii. High, short.
      iii. High, long.
      iv. Zero, long.

   (c) With a zero-coupon bond:

      i. Duration equals the weighted average term to maturity.
      ii. Term to maturity equals duration.
      iii. Weighted average term to maturity equals the term to maturity.
      iv. All of the above.

60. Please circle your answer to the following questions and provide a one-line explanation.

   (a) Holding the one-year real interest rate constant, if the nominal one-year interest rate where to increase by 1%, it would imply that the inflation rate over the same period

      i. Increased.
      ii. Declined.
      iii. Stayed the same.
      iv. It can go either way, impossible to tell from the provided data.

(b) Consider two treasury bonds, A and B. Both have 5 years to maturity, A pays a 5% coupon rate, B pays a 7% coupon rate. Which of bonds A and B has higher modified duration,

i. A.

ii. B.

iii. The same for A and B.

iv. It can go either way, impossible to tell from the provided data.

(c) A ten-year bond with a coupon rate of 6% and a face value of $100 is priced at $98. Let the yield to maturity be denoted by y. Which of the following statements is true:

i. $y > 6\%$.

ii. $y < 6\%$.

iii. $y = 6\%$.

iv. It can go either way, impossible to tell from the provided data.

(d) (This may require a calculation.) Suppose the one-year spot rate $r_1 = 5\%$ and the two-year rate $r_2 = 6\%$. At time 0 you enter into a forward contract to buy, in exactly one year from now, a one-year zero-coupon bond. Suppose that in one year from now the term structure of interest rates changes, so that a one year rate becomes 6%. Will you experience a profit or a loss on your forward contract?

i. Profit.

ii. Loss.

iii. No effect.

iv. It can go either way, impossible to tell from the provided data.

61. Consider two bonds (i) 3 year bond with zero coupons, and (ii) 3 year bond with 5% annual coupon. Assume the yield curve is flat at 5.5%.

(a) Calculate the price and modified duration of each bond.

(b) Suppose the yield curve shift up by 0.1%. What are the new prices of each bond? Check that the % change is close to MD times the yield change.

(c) Suppose the yield curve shifts up by 2%. Show that the approximation is not very close.

(d) You have $10 million and invested 40% of them in the zero coupon bond and 60% in the 5-year coupon bond. What is the modified duration of your portfolio?

62. You manage a pension fund that will provide retired workers with lifetime annuities. You determine that the payouts of the fund are (approximately) level perpetuities of $1 million per year. The interest rate is 10%. You plan to fully fund the obligation using 5-year maturity and 20-year maturity zero-coupon bonds.

(a) How much *market value* of each of the zeros will be necessary to fund the plan if you desire an immunized position?

(b) What must be the *face value* of the two zeros to fund the plan?

63. Do agree with the following statements? Explain your reason.

(a) Higher YTM means higher bond return.

(b) If the forward rates are lower than the current short term spot rate, then we should not enter into the forward rate agreement to lend money because the rates we get are too low.

(c) The market expects a rate cut in next month's Fed meeting, therefore I should load up on bonds to take advantage of the opportunity.

64. You will be paying $10,000 a year in tuition expenses at the end of the next two years. Bonds currently yield 8%.

(a) What is the present value and duration of your obligation?

(b) What maturity zero-coupon bond would immunize your obligation?

(c) Suppose you buy a zero-coupon with value and duration equal to your obligation. Now suppose that rates immediately increase to 9%. What happens to your net position, that is, to the difference between the value of the bond and that of your tuition obligation? What if rates fall to 7%?

65. You bought a 5-year treasury with 5% annual coupon. You decide to hold it until maturity. The yield curve is flat at 5%. The current projected inflation is 2% per year.

(a) Suppose inflation is 2% per year as forecasted. What will be your real return?

(b) Suppose inflation jumps to 3% right after you buy the bond and stay at 3% for the next 5 years. What would your real return be?

(c) TIPS are government risk-free bond that provides protection for inflation. Let's look at a 5-year TIPS with 2% real coupon (and $100 principal). The way TIPS work is that the $100 principal is expressed in real terms. Therefore in nominal (dollar), the principal increases each year with inflation. Therefore if inflation is 2% each year, then the principal becomes $102 (100*(1+2%))in year 1, and $102*1.02 = $104.04 in year 2, etc. The coupon payment each year is based on the new principal adjusted for inflation. For this 5-year TIPS, calculate the cash flow from the bond each year if inflation is (i)2%, and (ii) 3%, respectively.

2. Fixed Income Securities Solutions

1. (a) TRUE

Coupon payments accelerate cash flows received from the investment reducing the duration.

(b) FALSE

Higher yield to maturity (YTM) does not mean that the bond is a good investment; NPV=0 in any case.

Two good answers to this question were:

1. Bonds with longer duration have usually higher yield to maturity (YTM) as a result of increasing yield curve.

2. Bonds with higher probability of default have higher YTM to compensate for credit risk.

Full answer must mention at least one of these two points. Many students concentrated on reinvestment risk only.

(c) FALSE

"On the run" refers to a newly issued bond. These bonds are usually more liquid and trade at a premium.

2. (a) False: yield of longer term bond also incorporates the expectation of future short term interest rates and depending on the this factor, the term structure may be up- or down-ward sloping.

(b) False: If interest rates rise, future coupons of such bonds fall, and so their price (which is the present value of future coupons) falls more than the price of its straight bond counterpart. If interest rates fall, future coupons of such bonds rise, and so their price (which is the present value of future coupons) rises more than the price of its straight bond counterpart. So these bonds are more interest rate sensitive.

3. (a) False. The term structure of interest rates depends on expected future interest rates, if short term interest rates are expected to decrease substantially, the term structure can be inverted, flat, or downward sloping even when there is a liquidity premium for long term interest rates.

(b) False. Bonds with higher duration are more sensitive to changes in interest rates. If we examine coupon bonds, all things equal a bond with a higher coupon rate will have a lower duration and thus be less sensitive to changes in interest rates.

4. False. The term structure depends on the expected path of interest rates (among other factors): if interest rates are expected to fall, the term structure will slope downward.

5. False. If there is liquidity premium, then a flat term structure means that investors expect interest rates to fall.

6. No. To minimize interest rate risks, we want MD(A)×V(A) − MD(L)×V(L) = 0. If V(A)>V(L), we want MD(A) < MD(L). That means we should invest in assets with shorter duration.

7. (a) Effective annual rate on 3-month T-bill:

$$\left(\frac{100,000}{97,645}\right)^4 - 1 = (1.02412)^4 - 1 = .10 \text{ or } 10\%.$$

   (b) Effective annual interest rate on coupon bond paying 5% semiannually:

$$(1.05)^2 - 1 = .1025 \text{ or } 10.25\%.$$

   Therefore, the coupon bond has the higher effective annual rate.

8. The qoutes yield is calculated as

$$y = \frac{1}{t}\frac{F-P}{P}$$

   where t is in fraction of year, i.e. 1/6 in this example. Using $F = \$100,000$ the current price is $\$99,009$. So the effective annual rate is $\left(\frac{\$100,000}{\$99,009}\right)^6 - 1 = 6.15\%$

9. (a) $1 + EAR = (100/98.058)^2$ so EAR=4%

   (b) Bond at par, so yield=coupon. So $1 + EAR = (1 + 0.042/2)^2$. So EAR=4.24%
   The coupon bond has the higher EAR.

10. (a) $r_1 = \frac{\$100}{\$96.2} - 1 = 3.95\%$, $r_2 = \left(\frac{\$100}{\$91.6}\right)^{1/2} - 1 = 4.45\%$ $r_3 = \left(\frac{\$100}{\$86.6}\right)^{1/3} - 1 = 5.11\%$

   (b) $PV = \$300M \times \frac{96.2}{100} + \$210M \times \frac{91.6}{100} + \$400M \times \frac{86.1}{100} - \$600M = \$225.36$

11. (a) Price $= \frac{100}{1.04^3} = 88.90$; YTM $= 0.040$.

   (b) Price $= \frac{5}{1.03^1} + \frac{105}{1.035^2} = \$102.87$; YTM solves $\frac{5}{(1+y)^1} + \frac{105}{(1+y)^2} = \$102.87$: $y = 0.03488$.

   (c) Price $= \frac{6}{1.03^1} + \frac{6}{1.035^2} + \frac{6}{1.04^3} + \frac{106}{1.045^4} = 105.65$;
   YTM solves $\frac{6}{(1+y)^1} + \frac{6}{(1+y)^2} + \frac{6}{(1+y)^3} + \frac{106}{(1+y)^4} = 105.65$: $y = 0.04428$.

12. The effective annual yield on the semi-annual coupon bonds is 8.16%. If the annual coupon bonds are to sell at par, they must offer the same yield, which will require an annual coupon of 8.16%.

13. (a) $P_{1-year} = \frac{\$100}{1+5.25\%} = \$95.01$
   $P_{2-year} = \frac{\$5}{(1+5.50\%)} + \frac{\$100}{(1+5.50\%)^2} = \$99.08$
   $P_{3-year} = \frac{\$6}{(1+6\%)} + \frac{\$6}{(1+6\%)^2} + \frac{\$106}{(1+6\%)^2} = \$100$

   (b) The first year, is easy. It is simply the yield of the 1-year note so, $r_1 = 5.25\%$. Then solve for r_2 and r_3 recursively. $r_2 = 5.505\%$ and $r_3 = 5.7\%$.

(c) Use the fact that $\left(\frac{1}{1+r_3}\right)^3 = \left(\frac{1}{1+r_2}\right)^2 \times (1 + f_{2,3})$ so $f_{2,3} = 6.1\%$

14. (a) Spot rate at 2 years is 2.0%. At 3 years is 2.58%. At 8 years is 4.24%.

(b) Use this relation: $(1 + r_{t-1})^{t-1}(1 + f_{t-1,t}) = (1 + r_t)^t$
The one year forward rate from Aug 2004 to Aug 2005 is 3.74%.
Similarly the one year forward rate from Aug 20010 to Aug 20011 is 6.33%.

(c) Under the Expectation Hypothesis, the upward slope implies that interest rates are expected to go up. In practice, the long-term average term structure is upward sloping. However, the August 20, 2002 slope is very steep. So interest rates are expected to rise.

15. (a) $(1 + r_2)^2 = (1 + r_1)^1(1 + f_{1,2}) \rightarrow f_{1,2} = 3.5\%$

(b) $(1 + r_3)^3 = (1 + r_1)^1(1 + f_{1,3})^2 \rightarrow f_{1,3} = 3.95\%$

(c) Let's calculate the forward rate first. $(1+r_4)^4 = (1+r_3)^3(1+f_{3,4}) \rightarrow f_{1,3} = 6.02\%$
So the TA is offering too high of a rate. What you need to do is as follows:

i. Enter into a contract to lend him money at 6.3% between years 3 and 4.
ii. Short a 4 year strip
iii. Use the proceeds form the shorting to buy 3 year strip. The amount should be based on how much you enter into a contract with the TA for.

If you follow these steps you will have more than enough to cover the short position in year 4 (since the rate you lend at is higher than the fair forward rate). Hence, this is an arbitrage. The risks you are exposed are contract and counter-party risk, for example the TA may go bankrupt before year 3 if he continues to enter into contracts of this form and may not be able to honor his/her end. In that case, you will end up with hanging position and expose to various interest rate risks.

16. (a) A: $\frac{100}{1.05} = 95.24$.
B: $\frac{5}{1.055} + \frac{105}{1.055^2} = 99.08$.
C: 100 (coupon rate = YTM).

(b) $r_{0,1} = 0.05$.

$$r_{0,2} = \left[\frac{105}{99.08 - 5/1.05}\right]^{0.5} - 1 = 0.05513.$$

$$r_{0,3} = \left[\frac{106}{100 - 6/1.05 - 6/1.05513^2}\right]^{1/3} - 1 = 0.06041.$$

(c) Long $100\frac{100}{106} = 0.9434$ C
Short $0.9434\frac{6}{105} = 0.05391$ B
Short $\frac{1}{100}[6 \times 0.05513 - 5 \times 0.06041] = 0.05391$ A

(d) No arbitrage means that the bond in (c) should cost $0.9434 \times 100 - 0.05391 \times 99.08 - 0.05391 * 95.24 = 83.86$.
Note that $\left[\frac{100}{83.86}\right]^{1/3} - 1 = 0.06041$.

17. (a)

$$r_1 = \frac{110}{106.8} - 1 = 3.00\%$$

$$101.93 = \frac{5}{1+r_1} + \frac{105}{(1+r_2)^2} \Rightarrow r_2 = 4.00\%$$

$$111.31 = \frac{10}{1+r_1} + \frac{10}{(1+r_2)^2} + \frac{110}{(1+r_3)^3} \Rightarrow r_3 = 6.00\%$$

(b)

$$f_2 = \frac{(1+r_2)^2}{1+r_1} - 1 = 5.0\%$$

18. (a)

$$\begin{aligned} 95.92 &= \frac{100}{1+r_1} \\ 92.01 &= \frac{100}{(1+r_2)^2} \\ 87.00 &= \frac{100}{(1+r_3)^3} \end{aligned}$$

$$\begin{aligned} r_1 &= \frac{100}{95.92} - 1 = 4.25\% \\ r_2 &= \left(\frac{100}{92.01}\right)^{1/2} - 1 = 4.25\% \\ r_3 &= \left(\frac{100}{87.00}\right)^{1/3} - 1 = 4.75\% \end{aligned}$$

(b)

$$\begin{aligned} f_{2,3} &= \frac{(1+r_3)^3}{(1+r_2)^2} - 1 \\ &= 5.76\% \end{aligned}$$

(c) The yield to maturity is simply 4.25% since the one year and two year spot rates are roughly equivalent, more specifically when we calculate the present value of a coupon bond with a coupon rate of 4.25% this bond has a current price at par.

$$\begin{aligned} PV &= \frac{4.25}{1+r_1} + \frac{104.25}{(1+r_2)^2} \\ &= \frac{4.25}{1.0425} + \frac{104.25}{(1.0425)^2} = 100 \end{aligned}$$

19. (a) $PV = 10 \times (0.9756 + 0.9518 + 0.9286) = \$28.56MM$

(b) Duration of liability
$D = 10 \times (0.9756 + 2 \times 0.9518 + 3 \times 0.9286)/28.56 = 1.98$ years

(c) To hedge interest rate risk $\Delta P/P = -D\Delta y/(1+y)$, so if we want $\Delta P(liability) = \Delta P(asset)$, i.e. perfect hedge, then $D(asset)P(asset) = D(liability)P(liability)$ Leading to $20 \times D(asset) = 1.98 \times 28.56 \rightarrow D(asset) = 2.83 year$ To achieve such a duration, invest $x\%$ in 2 year strips and (1-x)% in 3 year strips such as $2x + 3(1-x) = 2.83 \rightarrow x = 17\%$

(d) Impact on net liability
Since by investing in portfolio calculated in c, you are perfectly hedge, the change in interest rate has no impact on your net liability. There will be some effect due to convexity but there is not first order effect.

20. (a) P = 20(0.961538+0.924556) = \$37.72.

(b) Duration is given by:

$$\frac{20 \times 0.961538 + 20 \times 0.924556 \times 2}{37.72188} = 1.490.$$

The yield is 4% (we have a flat term structure) so the modified duration is:

$$\frac{1.490196141}{1.04} = 1.433.$$

(c) Asset value falls by 0.1433%, i.e. by \$54,050.96.

(d) Let F be the dollar value of the futures contracts that we wish to short.

$$5F = 37.72188 \times 1.433.$$

Hence $F =$ \$10.810 million so you will short 108 futures contracts. (You can only long or short whole numbers of futures contracts.)

(e) Let the yield change by y percentage points. Portfolio value will change by

$$-5 \times 10.8 \times \Delta y + 37.72188 \times 1.433 \times \Delta y \approx 0.$$

21. (a) $PV_1 = \frac{1}{1+3.5\%} = 0.9662, PV_2 = \frac{1}{(1+3\%)^2} = 0.9426$
From PV_3 on, sum PV $= 1/(1+4\%)^3 + 1/(1+4\%)^4 + \ldots$
$= \frac{1}{(1+4\%)^2}(\frac{1}{1.04} + \frac{1}{1.042} + \frac{1}{1.043} \cdots)$
$= \frac{1}{(1+4\%)^2} \times \frac{1}{0.04} = 23.1139$
Therefore the total = 25.02268

(b) If you increase all rates by 0.5%, then the new values become 0.9615, 0.9335 and 20.3500. The total is 22.2446.

22.

Maturity	Price	YTM	Forward Rates
1	\$943.40	6.00%	
2	\$898.47	5.50%	$(1.055^2/1.06 - 1) = 5\%$
3	\$847.62	5.67%	$(1.0567^3/1.055^2 - 1) = 6\%$
4	\$792.16	6.00%	$(1.06^4/1.0567^3 - 1) = 7\%$

23. (a) No, the rate of 5.50% is too low. To see why, use the identity that

$$(1+r_t)^t \times f_{t,t+1} = (1+r_{t+1})^{t+1}$$

So the forward rate for year 1 to 2 should be:

$$f_{1,2} = \frac{1.055^2}{1.0525} - 1 = 5.75\%$$

(b) Borrow PV of \$10 million due in one year now for one year. Invest the amount for two years. The amount you have to borrow is $\$10M/(1+5.25\%) = \$9,501,187$. In one year, you willl have enough to pay back your loan. To borrow for one year, you can short sell the 1-year strip and to invest for 2-years, you can buy the 2-year strip.

24. (a) $P = \frac{100}{(1+y/2)^{18}}$

So P=\$67.17

(b) $P = \frac{100}{(1+y/2)^{18}} + \frac{c/2}{y/2}(1 - \frac{1}{(1+y/2)^{18}})$

So P=\$106.79

25. Not available but identical (different numbers) to question 000202

26. (a), (d)

27. $f_{n-33,n-34} = \left(\frac{91.21}{89.94} - 1\right) = 1.41\%$

28. (a) Assuming a par value of \$100. The price is the quoted price plus accrued interests: $\$109 + 20/32 + 5 * 41/365 = \$110.2$ (Any reasonable approximation to the accrued interest was considered correct).

You will receive \$2.5 in Feb and August every year from 2003 to 2010. In Feb 2011, you will receive \$102.5 Common mistake: Most people didnt take into account the difference between clean and dirty price

(b) You were expected to give EAR. Using the quoted yield (semi-annually compounded rate): $r_{2010} = (1+0.0365/2)^2 - 1 = 3.68\%$

$r_{2011} = (1+0.0377/2)^2 - 1 = 3.81\%$

Common mistake: give the APR (-2/4), or compute it using prices without taking correct time to maturity (-2/4).

(c) $f_{2010,2011} = \frac{1.0381^{8.4}}{1.0368^{7.4}} - 1 = \frac{B_{2010}}{B_{2011}} = \frac{76.6875}{73.25} = 4.6\%$

(d) The Feb 2010 Note with 6.5 coupon rate has the shortest duration, because its coupons (given out early) are bigger than the Feb 2011 bond and the principal is also given earlier.

29. (a) Price = PV of cash flows = $100(1+r)/(1+r) + 100(1+r)^2/(1+r)^2 + \cdots = 400.00$

(b) $Duration = (1+2+3+4)/4 = 2.5$ years

(c) Put 50% in each. Then the duration of your liability = 2.5 = duration of your assets = 0.5(1)+0.5(4)

30. (a)

$$PV = \frac{20}{(1+5\%)^10} + \frac{20}{(1+5\%)^30} = 12.28 + 6.94 = 19.22\text{million}$$

(b)

$$D = \frac{12.28 * 10 + 6.94 * 30}{19.22} = 17.22$$

$$MD = \frac{D}{1+y} = 16.40$$

y = 5% because the yield curve is flat

(c) When rates drop by 0.25%, the PV of liabilities will go up by $0.25\% * MD * PV =$ 0.7881 million

(d) First, you calculate the desired MD of your assets. We want:
$MD_{asset} * PV_{asset} = MD_{liabilites} * PV_{liabilities}$
Therefore, $MD_{asset} = \frac{16.4*19.22}{18} = 17.51$
Now we can determine the allocation of our portfolio. Suppose we invest a fraction of x of our portfolio into 1-year bond and the rest into treasury bond, then the MD of our portfolio will be:
$MD_{portfolio} = x * MD_{1yrbond} + (1-x) * MD_{tbond} = x * \frac{1}{1+5\%} + (1-x) * 20$
Equating $MD_{portfolio} = 17.51$, we get x = 13.05%

31. (a) Using the perpetuity formula

$$PV_{Liability} = \frac{10M}{r} = \frac{10M}{0.05} = 200M$$

(b)

$$PV_{Liability} = \frac{10M}{r} = \frac{10M}{0.049} = 204.0816M$$

The value of the liabilities would increase by 4.0816M.

(c)

$$P_{new} = P_{old} - P_{old} \times MD \times \Delta y$$

$$\rightarrow MD = \frac{P_{old} - P_{new}}{P_{old} \times \Delta y} = \frac{200 - 204.0816}{200 \times -0.001} = 20.4082$$

(d) You should match the modified duration to neutralize first order interest rate risk. MD=20.4082

(e) We must match both the modified duration and the present value of the zero coupon bond. Let PAR be the par value of the zero coupon bond and t be the maturity. Since the duration of a zero coupon bond is its maturity $t = 21.4286$. We can then solve for the PAR value

$$
\begin{aligned}
200M &= \frac{PAR}{(1.05)^t} \\
\rightarrow PAR &= 200(1.05)^{20.4082} = 541.331M
\end{aligned}
$$

32. (a) $r_1 = 4.5\%, r_3 = 5.5\%$
$B_2 = 100$ (coupon = YTM), so $100 = 5/(1+r_1) + 105/(1+r_2)^2$, so $r_2 = 5.0126\%$
Therefore the price of the 5% 3-year bond is
$5/(1+4.5\%) + 5/(1+5.0126\%)2 + 105/(1+5.5\%)3 = 98.7382$

(b) $f_3 = (1+r_3)^3/(1+r_2)^2 - 1 = 6.4817\%$

(c) MD(bond 1) = 1/(1+4.5%) = 0.9569, MD(bond 3) = 3/(1+5.5%) = 2.8436, so MD(portfolio) = 30%*0.9569+70%*2.8436 = 2.2776

(d) That means the portfolio will decrease in value by 0.15%*2.2776 = 0.3416%

33. (a) We have the following system of equations:
$100 = (100+25)/(1+f_{0,1})$
$422.61 = 50/(1+f_{0,1}) + (500+50)/((1+f_{0,1})(1+f_{1,2}))$
$232.28 = 20/(1+f_{0,1}) + 20/((1+f_{0,1})(1+f_{1,2})) + (300+20)/((1+f_{0,1})(1+f_{1,2})(1+f_{2,3}))$
$192.31 = 1000/((1+f_{0,1})(1+f_{1,2})(1+f_{2,3})(1+f_{3,10})^7$
The solutions are
$f_{0,1} = 0.25$,
$f_{1,2} = 0.15$,
$f_{2,3} = 0.10$,
$f_{3,10} = 0.18.54$.

(b) $NPV = -21 + 9/1.25 + 10/((1.25)(1.15)) + 11/((1.25)(1.15)(1.10)) = 12$

34. (b) the entire value of the treasury strip is in the principal repayment in the distant future; it has the highest duration and is most sensitive to a change in the interest rate.
(c) despite having the same maturity as (b), (c) has 5.5% coupon payments thatdampen its sensitivity to interest rate changes.
(d) higher coupon payment $\rightarrow$ lower duration
(a) T-bills are only for 1 to 6 months; they have the smallest duration.

35. Since we want portfolio $D = 9$, we can invest x in the 6-year bond ($D = 6$) and $(1-x)$ in 16-year bond ($D = 16$).
Then $D_p = x * 6 + (1-x) * 16, x = 70\%$ (*)
Therefore invest 70% (350,000 or 3500 shares) in 6-year bond and 30% (150,000 or

1500 shares) in 16-year bond.
*: This equation holds for D here because the bonds have the same YTM.

36. Let the annual interest payment for the two-year forward loan be C. You can arrange the loan and lock in the forward interest rate loan as follows

- Buy \$15M in par value of 1-year zero, costing (15M)(943.40/1000) = \$14,151,000.
- Sell $\$C$ in par value of 2-year zero, receiving $\$C \times (898.47/1000)$
- Sell $\$C + 15\text{M}$ in par value of 3-year zero, receiving $\$(C + 15\text{M})(847.62/1000)$
- Choose C such that proceeds from the selling of 2-year and 3-year zeros just cover the cost of buying the 1-year zero.

Thus,

$$(15\text{M})(943.40/1000) = C(898.47/1000) + (C + 15\text{M})(847.62/1000)$$

which gives $C = \$822,810$. The CF's of the transactions are:

Transaction	CF today	CF in year 1	CF in year 2	CF in year 3
Buy 1-year zero	-14,151,000	15,000,000	0	0
Sell 2-year zero	739,270		-822,810	0
Sell 3-year zero	13,411,730	0	0	-15,822,810
Net	0	15,000,000	-822,810	-15,822,820

The forward interest rate for the 2-year forward loan is

$$f = 822,810/15,000,000 = 5.4854\%$$

which is locked in now.

37. (a) The current price of the bond is

$$P_0 = \frac{\$60}{1.065} + \frac{\$60}{1.065^2} + \frac{\$1060}{1.065^3} = \$986.76$$

You can sell it in one year for

$$P_1 = \frac{\$60}{1.065} + \frac{\$1060}{1.065^2} = \$990.90$$

But there is also the \$60 of coupon. So the total return is

$$RoR = (\$990.90 + \$60)/\$986.76 - 1 = 6.5\%$$

(b) The bond now has the same coupon rate as the yield to maturity so it is trading at par. So the new price $P_1 = \$1,000$ and

$$RoR = (\$1,000 + \$60)/\$986.76 - 1 = 7.4\%$$

38. (a) $D = \frac{\sum i=1^{20} \frac{i}{1.04^i}}{\sum i=1^{20} \frac{1}{1.04^i}} = 9.21$ years

(b) Need to match the duration and also the value of investment today should be equal to the total liabilities. So have the following two equations:
$V_5 * 5 + V_{20} * 20 = D * (V_5 + V_{20})$
$V_5 + V_{20} = \$13.59M$ Annuity formula
Solving gives $V_5 = \$9.78M$ and $V_{20} = \$3.81M$
$P_5 = V_5 * (1.04)^5 = \$11.9M$
$P_{20} = V_{20} * (1.04)^{20} = \$8.36M$

(c) No. Interest rates increase, so bigger fraction of present value given early. So duration decreases. So more in 5 year strips and less in 20 year strips.

39. (a) $P = \frac{\$8}{1.05} + \frac{\$8}{1.05^2} + \cdots + \frac{\$108}{1.05^{10}} = \$123.16$
$D = \frac{\$8}{1.05}\frac{1}{\$123.16} + \frac{\$8}{1.05^2}\frac{2}{\$123.16} + \cdots + \frac{\$108}{1.05^{10}}\frac{10}{\$123.16} = 7.54$

(b) Using pricing equation, the price will change to $P_{new} = \$122.28$.
Recall the approximation that

$$\Delta P/P \approx -MDy + CX \times (\Delta y)^2$$

$MD = D/(1+y) = 7.54/1.05 = 7.18$
Usign the first two terms only we have

$$\Delta P = -P \times MD \times \Delta y = -\$123.16 \times 7.18 \times 10 \times 10^{-5} = 0.88 \rightarrow P_{new} = \$122.28$$

(c) Using pricing equation: $P_{new} = \$107.02$.
Using duration approximation: $P_{new} = \$105.47$.

(d) Addign the convexity term the new prices are
$P_{new} = \$122.28$, and $P_{new} = \$107.13$
So not much has changed in the first case (part b) but the approximation is not much better for part c. In short, convexity is more important issue for large interest rate movements and hence one has to be hedged in terms of convexity in order to be immune to large interest rate movements.

40. (a) 4% per year

(b) No. The spot rates at $t = 5$ are not known at $t = 0$.

41. (a) $10000 \times (136\frac{3}{32}\%) = 13609.38$

(b) \$437.50 in Feb. 2007, Aug. 2007, Feb. 2008, ..., Aug. 2020
\$10,437.50 in Aug. 2020.

(c) \$43,750 in Feb. 2007
\$-9,956,250 in Aug. 2007
\$206,250 in Feb. 2008
\$10,206,250 in Aug. 2008.

42. (a) The 1, 2, 3, 4, 6 and 10-year spot interest rates are 5.08%, 4.93%, 4.91%, 4.80%, 4.87% and 5.05%, respectively.

(b) $f_{1,2} = 1.0493^2/1.0508 - 1 = 0.0478.$
$f_{3,4} = 1.0480^4/1.0491^3 - 1 = 0.0447.$

(c) $PV = 437.50/(1.0480^4) = 362.69.$

43. $P = Annuity(218.75, (1.0488)^0.5, 12) + 10000/(1.0488)^6 = \$9,769.56.$

44. (a) Short $\frac{10}{1.0508} = \$9.5166$ million worth of one-year strip.
Long \$9.5166 million worth of two-year strip.
Cash flow at $t = 1$: 0.
Cash flow at $t = 2$: $9.5166 \times (1.0493)^2 = \$10.478$ million.

(b) Short $\frac{10}{1.0508} = \$9.5166$ million worth of one-year strip.
Long \$9.5166 million worth of six-year strip.
Cash flow at $t = 1$: 0.
Cash flow at $t = 5$: $9.5166 \times (1.0487)^6 = \$12.659$ million.

45. (a) 4%

(b) $200 \times (1.04)^3 = 224.97.$

46. Not available but we have many other pension questions

47. Bond underwriting

If the underwriter purchases the bonds from the corporate client, then it assumes the full risk of being unable to resell the bonds at the stipulated offering price. In other words, the underwriter bears the risk of interest rate movement between the time of purchase and the time of resale. For long maturity bonds, it is generally true that its duration is also long. Thus, bonds with long maturities are more exposed to interest rate movement risk. Therefore, the underwriter demands a larger spread (higher underwriting fees) between the purchase price and stipulated offering price.

48. (a) Use the formula that

$$D = \sum_{t=1}^{T} t \times w_t \quad \text{where } w_t = \frac{CF_t/(1+y)^t}{BondPrice}$$

Durations are 1.97, 4.61, 8.35, 14.13 for A through D, respectively.

(b) You can either do this question by aggregating the cashflows as each time and using an approach identical to what we did in part a. Here is an alternative.

To calculate the duration of a portfolio, it is easier to calculate its modified duration first. Recall that modified duration if $MD = D/(1+y)$ and that $\Delta P/P = -MD \times \Delta y$

Is is not have to show that the modified duration of a portfolio is simply the weighted sum of the modified durations of the individual bonds where the weights are proportional to the value of the bond to the total value.

In this case, the modified durations A through D are 1.85, 4.35, 7.88, and 13.33, respectively. For the portfolio we have:

$$MD_{portfolio} = \frac{10}{60} \times 1.85 + \frac{20}{60} \times 4.35 + \frac{20}{60} \times 7.88 + \frac{10}{60} \times 13.33 = 6.6$$

Hence the duration is

$$D_{portfolio} = 6.6 \times 1.06 = 7$$

(c)

$$\Delta P/P = -MD \times \Delta y = 7 \times 20bps = 140bps = 0.14\%$$

49. (a) PV bond = PV semiannual payments + PV principal payment
$= 300 \times \frac{1}{0.05/2} \times (1 - \frac{1}{(1+0.05/2)^{20}}) + \frac{10{,}000}{1.025^{20}} = \$10,779.45$
Note: if 5% is an effective annual rate, the semiannual rate is $1.05 = (1 + r/2)^2$ so $r/2 = 2.47\%$
Then, PV bond $= 300 \times \frac{1}{0.0247} \times (1 - \frac{1}{(1+0.0247)^{20}}) + \frac{10{,}000}{1.0247^{20}} = \$10,828.57$

(b) To calculate the IRR sovle this following equation
$0 = -11240 + \frac{300}{(1+IRR/2)^1} + \frac{300}{(1+IRR/2)^2} + \cdots + \frac{300}{(1+IRR/2)^{20}}$
So: $IRR/2 = 2.23\%$

50. (a) $D = \frac{200*0.5+300*1+500*5-900*1}{100} = 20$
It means that the equity has the same sensibility to interest rates as a 20 year strip

(b) $dP/P = -D/(1 + y) * dy = -20/1.03 * 0.01 = -0.1941$. Given the \$100 value, $dP = -\$19.41M$
So the new price is \$80.58M. Because of convexity, the change will be smaller.

51. Not available but there are several other pension related question dealing with duration matching.

52. (a) A: $y = \frac{105}{100.96} - 1 = 0.040016$.
B: y solves $\frac{6.5}{(1+y)^1} + \frac{6.5}{(1+y)^2} + \frac{106.5}{(1+y)^3} = 106.29$; $y = 0.042238$.
C: y solves $\frac{2}{(1+y)^1} + \frac{2}{(1+y)^2} + \frac{102}{(1+y)^3} = 93.84$; $y = 0.042294$.

(b) $r_{0,1} = \frac{105}{100.96} = 1.0400$.
$r_{0,2}$: consider the portfolio of +1 B, $-\frac{106.5}{102} = -1.0441$ C and $\frac{2\times1.0441-6.5\times1}{105} = -0.042017$ A. The portfolio pays 0 at time $t = 1, 3$ and $6.5\times1-2\times1.0441 = 4.4118$ at time $t = 2$. The portfolio costs $100.96 \times (-0.042017) + 106.29 \times 1 - 93.84 \times 1.0441 = 4.0680$. Then, $r_{0,2} = \left[\frac{4.4118}{4.0680}\right]^{1/2} = 1.041398$.
$r_{0,3} = \left[\frac{106.5}{93.84-6.5/(1+r_{0,1})-6.5/(1+r_{0,2})^2}\right]^{1/3} - 1 = 0.042323$.
$f_{0,1} = 0.040016$.
$f_{1,2} = 1.041398^2/1.040016 - 1 = 0.042781$.
$f_{2,3} = 1.042323^2/1.041398^2 - 1 = 0.044175$.

53. (a) Zero-coupon bond: $y = 4.5\%$.
Coupon bond: $P = \frac{5}{1.035} + \frac{5}{1.04^2} + \frac{105}{1.045^3} = 101.46$; y solves $\frac{5}{1+y} + \frac{5}{(1+y)^2} + \frac{105}{(1+y)^3} = 101.46$; $y = 0.044675$.

(b) Zero-coupon bond: $P_0 = \frac{100}{1.045^3} = 87.63$; $P_1 = \frac{100}{1.04^2} = 92.46$; realized return $= \frac{92.46}{87.63} - 1 = 0.055072$.
Coupon bond: $P_1 = \frac{5}{1.035} + \frac{105}{1.04^2} = 101.91$; realized return $= \frac{101.91+5}{101.46} - 1 = 0.053659$.

54.

(1)	(2)	(3)	(4)	(5)
Yrs	Pmt	PV of Pmt	Wt of Pmt	(1) × (4)
1	\$1 M	\$0.9091 M	0.2744	0.2744
2	\$2 M	\$1.6529 M	0.4989	0.9978
3	\$1 M	\$0.7513 M	0.2267	0.6801
Total		\$3.3133 M	1.0000	1.9523

Duration is 1.9523 years.

55. (a) 22 years; (b) \$-1.1 million

56. (Assuming semiannual coupon payment: YTM per six months is $(1.06)^0.5 = 1.02956$.)

(a) $P = Annuity(2.5, 0.02956, 20) + 100/1.06^10 = \$93.18$.

(b) $D = \frac{1\frac{2.5}{1.02956} + 2\frac{2.5}{1.02956^2} + \ldots + 20\frac{2.5}{1.02956}}{\frac{2.5}{1.02956} + \frac{2.5}{1.02956^2} + \ldots + \frac{2.5}{1.02956}} = 15.8067$ periods (of 6 months).
$MD = \frac{15.8067}{1.02956} = 15.3529$.

(c) $\Delta P = 93.18(1 - 15.3529 \times (1.07^{0.5} - 1.06^{0.5})) = \$86.25$.

(d) $P = Annuity(2.5, (1.07)^{0.5}, 20) + 100/1.07^{10} = \$86.56$.

57. (a) The net cash flow in May-June, 2010 will be $-(1.04)^{0.5} + 0.055/2 = -\$0.99230$ billion.

(b) The transaction's NPV is 1-1=0.

58. (a) There is a one-to-one mapping between a bond's price and YTM (y). y is the solution to $P = Annuity(4.4375, y, 20) + 100/(1 + y)^{20}$.

(b) $Annuity(4.4375, 0.02315, 20) + 100/(1.02315)^20 = 133.67$.

59. **(a)** iv; **(b)** iv; **(c)** iv.

60. (a) (i) Increased. $r \approx R - i$.

(b) (i) A. Bond A has a smaller coupon payment, so its value depends more heavily on the principal repayment, which occurs far into the future. Therefore, it is more sensitive to change in the yield.

(c) (i) $y > 6\%$. Since the bond is sold at a discount, $y > c$.

(d) (i) Profit. $f_{1,2} = 1.06^2/1.05 - 1 = 0.07010 > 0.06$. The price has gone up.

61. (a) Zero-coupon bond: $P = \frac{100}{1.055^3} = 94.79$; $D = 3$; $MD = \frac{3}{1.055} = 2.844$.

Coupon bond: $P = \frac{5}{1.055} + \frac{5}{1.055^2} + \frac{105}{1.055^3} = 98.65$; $MD = \frac{1}{1.055}\left[\frac{1\frac{5}{1.055}+2\frac{5}{1.055^2}+3\frac{105}{1.055^3}}{94.79}\right] =$ 2.7094.

(b) Zero-coupon bond: $\Delta P = -2.844 \times 0.001 = -0.0028436$. Actual % change $= \frac{100/1.056^3 - 94.79}{94.79} = -0.0028382$.

Coupon bond: $\Delta P = -2.7094 \times 0.001 = -0.0027094$. New price = \$98.38; actual % change $= \frac{98.38-98.65}{98.65} = -0.0027043$.

(c) Zero-coupon bond: $\Delta P = -2.844 \times 0.02 = -0.056872$. Actual % change $= \frac{100/1.075^3 - 94.79}{94.79} = -0.054782$.

Coupon bond: $\Delta P = -2.7094 \times 0.02 = -0.054187$. New price = \$93.50; actual % change $= \frac{93.50-98.65}{98.65} = -0.052228$.

(d) Portfolio: $MD = 2.844 \times 0.4 + 2.7094 \times 0.6 = 2.7631$.

62. (a) The present value of the annuities is \$1 M /0.1 = \$10 M.

The duration is $1.10/0.10 = 11$ years.

Let x = weight of 5 year zeros, and $1 - x$ = weight of 20 year zeros. Then

$$11 = 5x + 20(1 - x)$$

and so $x = 0.60$ (in 5 year zeros), and $1 - x = 0.40$ (in 20 year zeros).

5 year zeros: \$10M × 0.60 = \$6M market value;

20 year zeros: \$10 M ×0.40 = \$4M market value.

(b) Face value of 5 year zeros: \$6M $\times (1.10)^5$ = \$9.66M; Face value of 20 year zeros: \$4M $\times (1.10)^{20}$ = \$26.91M.

63. (a) Depends. If the bond is held to maturity and all coupon payments are reinvested at the YTM, then higher YTM means higher return, ceteris paribus.

(b) No. Forward rates are expectations of future spot rate; they are unrelated to the current spot rate.

(c) No. Since the market expects a rate cut, it is already incorporated into today's prices.

64. (a) PV of obligation:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.08)^t} = \$17,832.65.$$

Duration of obligation:

(1)	(2)	(3)	(4)	(5)
Yr	Pmt	PV of Pmt	Wt of Pmt	(1) × (4)
1	\$10,000	\$9,259.26	0.51923	0.51923
2	\$10,000	\$8,573.39	0.48077	0.96154
Total		\$17,832.65	1.0000	1.48077

Duration of obligation is 1.4808 years.

(b) Zero coupon bond with a duration of 1.4808 years would immunize the obligation. The present value of this bond must be $\$17,832.65$, thus the face value (feature redemption value) must be:

$$\$17,832.65 \times (1.08)^{1.4808} = \$19,985.26.$$

(c) If interest rates increase to 9%, the value of the bond would be:

$$\frac{\$19,985.26}{(1.09)^{1.4808}} = \$17,590.92.$$

The tuition obligation would be:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.09)^2} = \$17,591.11.$$

The net position changes by only $0.19,
If interest rates decline to 7%:
The value of the zero coupon bond would increase to:

$$\frac{\$19,985.26}{(1.07)^{1.4808}} = \$18,079.99.$$

The tuition obligation would increase to

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.07)^2} = \$18,080.18.$$

The net position changes by $0.19.
As interest rates change, so does the duration of the stream of tuition payments, thus the slight net differences.

65. (a) Real return is $1.05/1.02 - 1 = 0.029412$.

(b) Real return is $1.05/1.03 - 1 = 0.019417$.

(c) (i) The cash flows at time $t = 1, 2, 3, 4$ and 5 are $5.10, 5.20, 5.31, 5.41$ and 115.93, respectively.
(ii) The cash flows at time $t = 1, 2, 3, 4$ and 5 are $5.15, 5.30, 5.46, 5.63$ and 121.72, respectively.

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