

1 Present Value

1. You can invest \$10,000 in a certificate of deposit (CD) offered by your bank. The CD is for 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years if the 4.5% is
 - (a) an EAR?
 - (b) a quarterly APR?
 - (c) a monthly APR?
2. (W) e-Money rates. An internet company, e-Money, is offering a money market account with an A.P.R. of 4.75%. What is the effective annual interest rate offered by e-Money if the compounding interval is
 - (a) annual
 - (b) monthly
 - (c) weekly
 - (d) continuously?
3. You can invest \$50,000 in a certificate of deposit (CD) offered by your bank. The CD is for 2 years and the bank quotes you a rate of 4%. How much will you have in 2 years if the 4% is
 - (a) an EAR?
 - (b) a quarterly APR?
 - (c) a monthly APR?
4. You can invest \$10,000 in a certificate of deposit (CD) offered by your bank. The CD is for 5 years and the bank quotes you a rate of 4.5%. How much will you have in 5 years if the 4.5% is
 - (a) an EAR?
 - (b) a quarterly APR?
 - (c) a monthly APR?
5. e-Money rates. An internet company, e-Money, is offering a money market account with an A.P.R. of 5.25%. What is the effective annual interest rate offered by e-Money if the compounding interval is
 - (a) annual
 - (b) monthly

- (c) daily
- (d) continuously?

6. True, false or “it depends” (give a brief explanation): PV of a bond is calculated by discounting its cash flow for several years, say from year 1 to N . Sometime investors do not expect to hold bonds to their maturity. Then one way calculate the value of the bond is discounting a forecasted terminal value at horizon date $T < N$. The choice of the horizon date can have a significant effect on PV of the bonds.
7. Suppose you invest \$10,000 per year for 10 years at an average return of 5.5%. The average future inflation rate is 2% per year.
- (a) The first investment is made immediately. What is your ending investment balance?
 - (b) What is its purchasing power in today's dollars?
8. Overhaul of a production line generates the following incremental cash inflows over the line's 5-year remaining life.

	C_1	C_2	C_3	C_4	C_5
Cash inflow (\$ million)	+1.5	+1.3	+1.05	+0.9	+0.75

- (a) What is the PV of the inflows? The cost of capital is 12%.
 - (b) Part (a) used a nominal discount rate and the cash inflows incorporated inflation. Redo Part (a) with real cash flows and a real discount rate. The forecasted inflation rate is 3% per year.
9. You have just inherited an office building. You expect the annual rental income (net of maintenance and other cost) for the building to be \$100,000 for the next year and to increase at 5% per year indefinitely. A expanding internet company offers to rent the building at a fixed annual rent for 5 years. After year 5, you could re-negotiate or rent the building to another tenant. What is the minimum acceptable fixed rental payments for this five-year agreement? Use a discount rate of 12%.
10. Two dealers compete to sell you a new Hummer with a list price of \$45,000. Dealer C offers to sell it for \$40,000 cash. Dealer F offers “0-percent financing:” 48 monthly payments of \$937.50. ($48 \times 937.50 = 45,000$)
- (a) You can finance purchase by withdrawals from a money market fund yielding 2% per year. Which deal is better?
 - (b) You always carry unpaid credit card balances charging interest at 15% per year. Which deal is better?
11. Your sales are \$10 million this and expected to grow at 5% in real terms for the next three years. The appropriate nominal discount rate is 10%. The inflation is expected

to be 2% per year during the same period. What is the present value of your sales revenue for the next three years?

12. Company ABC's after-tax cash flow is \$10 million (at the end of) this year and expected to grow at 5% per year forever. The appropriate discount rate is 9%. What is the value of company ABC?
13. You own three oil wells in Vidalia, Texas. They are expected to produce 7,000 barrels next year in total, but production is declining by 6 percent every year after that. Fortunately, you have a contract fixing the selling price at \$15 per barrel for the next 12 years. What is the present value of the revenues from the well during the remaining life of the contract? Assume a discount rate of 8 percent.
14. A geothermal power station produces cash flow at a current rate of \$14 million per year, after maintenance, all operating expenses and taxes. All the cash flow is paid out to the power stations owners. The cash flow is expected to grow at the inflation rate, which is forecasted at 2% per year. The opportunity cost of capital is 8%, about 3 percentage points above the long-term Treasury rate. (Assume this is an annually compounded rate.)

The power station will operate for a very long time. Assume for simplicity that it will last forever.

 - (a) What is the present value of the power station? Assume the first cash flow is received one year hence.
 - (b) Now assume that the power station's cash flow is generated in a continuous stream, starting immediately. What is the present value?
15. A foundation announces that it will be offering one MIT scholarship every year for an indefinite number of years. The first scholarship is to be offered exactly one year from now. When the scholarship is offered, the student will receive \$20,000 annually for a period of four years, beginning from the date the scholarship is offered. This student is then expected to repay the principal amount received (\$80,000) in 10 equal annual installments, interest-free, starting one year after the expiration of her scholarship. This implies that the foundation is really giving an interest-free loan under the guise of a scholarship. The current interest is 6% for all maturities and is expected to remain unchanged.
 - (a) What is the PV of the first scholarship?
 - (b) The foundation invests a lump sum to fund all future scholarships. Determine the size of the investment today.
16. You signed a rental lease for an office space in the Back Bay for five years with an annual rent of \$1 million, paid at the beginning of each year of the lease. Just before you pay your first rent, the property owner wants to use the space for another purpose and proposes to buy back the lease from you. The rent for similar space is now \$1.25 million per year. What would be the minimum compensation that you would ask from

the property owner? Assume the interest rate to be 6%.

17. The annual membership fee at your health club is \$750 a year and is expected to increase at 5% per year. A life membership is \$7,500 and the discount rate is 12%. In order to justify taking out the life membership, what would be your minimum life expectancy?
18. You are considering buying a car worth \$30,000. The dealer, who is anxious to sell the car, offers you an attractive financing package. You have to make a down-payment of \$3,500, and pay the rest over 5 years with annual payments. The dealer will charge you interest at a constant annual interest rate of 2%, which may be different from the market interest rate.
- (a) What is the annual payment to the dealer?
 - (b) The dealer offers you a second option: you pay cash, but get a \$2,500 rebate. Should you go for the loan or should you pay cash? Assume that the market annual interest rate is constant at 5%.

Note: the tradeoff between the two options is that in the first case, you can finance your purchase at a relatively low rate of interest. In the second case, you receive a lump-sum cash rebate.

19. Your brother-in-law asks you to lend him \$100,000 as a second mortgage on his vacation home. He promises to make level monthly payments for 10 years, 120 payments in all. You decide that a fair interest rate is 8% compounded annually. What should the monthly payment be on the \$100,000 loan?
20. Your cousin is entering medical school next fall and asks you for financial help. He needs \$65,000 each year for the first two years. After that, he is in residency for two years and will be able to pay you back \$10,000 each year. Then he graduates and becomes a fully qualified doctor, and will be able to pay you \$40,000 each year. He promises to pay you \$40,000 for 5 years after he graduates. Are you taking a financial loss or gain by helping him out? Assume that the interest rate is 5% and that there is no risk.
21. You are awarded \$500,000 in a lawsuit, payable immediately. The defendant makes a counteroffer of \$50,000 per year for the first three years, starting at the end of the first year, followed by \$60,000 per year for the next 10 years. Should you accept the offer if the discount rate is 12%? How about if the discount rate is 8%?
22. You are considering buying a Back Bay two-bedroom apartment for \$800,000. You plan to make a \$200,000 down payment and take a \$600,000 30-year mortgage for the rest. The interest rate on the mortgage is 6% monthly APR. Payments are due at the end of every month.
- (a) What is the effective annual rate?
 - (b) What is the monthly payment?
 - (c) Suppose that exactly five years have passed, interest rates are now 5% and you decided to re-finance your mortgage. You have to pay the remaining portion of

the principal on the mortgage to the bank. Exactly how much do you owe to the bank at that point?

Hint: There is a very quick and a very slow way to answer part (c).

23. True, false or “it depends” (give a brief explanation): U.S. Treasury securities have no risk because they give sure payoffs at fixed future dates.

24. A 10-year German government bond (bund) has face value of 10,000 and an annual coupon rate of 5%. Assume that interest rate (in euros) is equal to 6% per year.

(a) What is the bond’s PV?

(b) Suppose instead that the bund paid interest semiannually like a U.S. bond. (The bond would pay $.025 \times 10,000 = 250$ every 6 months.) What is the PV in this case?

25. You are considering buying a two bedroom apartment in Back Bay for \$600,000. You plan to make a \$100,000 down payment and take out a \$500,000 30-year mortgage for the rest. The interest rate on the mortgage is 8.5% monthly APR.

(a) What is the effective annual rate (EAR)?

(b) What is the monthly payment?

(c) How much do you owe the bank immediately after the 60th monthly payment?

26. John is 30 years old at the beginning of the new millennium and is thinking about getting an MBA. John is currently making \$40,000 per year and expects the same for the remainder of his working years (until age 65). If he goes to a business school, he gives up his income for two years and, in addition, pays \$20,000 per year for tuition. In return, John expects an increase in his salary after his MBA is completed. Suppose that the post-graduation salary increases at a 5% per year and that the discount rate is 8%. What is minimum expected starting salary after graduation that makes going to a business school a positive-NPV investment for John? For simplicity, assume that all cash flows occur at the end of each year.

27. After doing well in your finance classes, you landed a job at the IMF. Your salary is \$100,000, and your contract is for 5 years. Your salary will stay the same during the 5 years and, since you are at the IMF, you are not subject to taxes. If you do well (which we assume will happen with certainty), you will get a permanent contract. Under this contract, your salary will grow at the rate of 3% per year, until retirement. Retirement will occur in 30 years after your contract becomes permanent.

For simplicity, assume that your salary is paid at the end of each year. In other words,

(End of) Year	Salary
1	\$100,000
2	\$100,000
3	\$100,000
4	\$100,000
5	\$100,000
6	$\$100,000(1+3\%)$
7	$\$100,000(1+3\%)^2$
.	.
35	$\$100,000(1+3\%)^{30}$

We assume that the interest rate is 4% (and will stay at 4% forever).

- What is the value of your human capital? That is, what is the PV (as of today) of all your future earnings?
- Assume that you spend 70% of your salary, and deposit the remainder in a savings account, which pays the rate 4%. How much money will you have in the savings account just after you received your fifth salary (end of year 5)? (You deposit only 30% of that salary in the savings account.)

28. Retirement planning: Mr. Jones is contemplating retirement. He is 55 and his net worth now is \$2 million. He hopes that after retirement he can maintain a lifestyle that costs him \$100,000 per year in today's dollars (i.e., real dollars, inflation adjusted).

If he retires, he will invest all his net worth in government bonds that yield a safe annual return of 5%. Inflation is expected to be 2% per year. Ignore taxes.

- Is Mr. Jones rich enough to retire today if he lives until (i) 80 (ii) 100 (iii) 115?
- Mr. Jones thinks he will live until about 100. What advice will you give him about retiring?

29. Suppose you invest \$50,000 for ten years at a nominal rate of 7.5% per year. If the annual inflation rate is 3% for the next ten years, what is the real value of your investment at the end of ten years?

30. Fill in the blanks:

-% continuously compounded is equivalent to annual interest rate of 12%.
- 5% continuously compounded is equivalent to annual interest rate of %.
-% continuously compounded is equivalent to annual interest rate of 9%.

31. A 10-year U.S. Treasury bond with a face value of \$10,000 pays a coupon of 5.5% (2.75% of face value every 6 months). The semi-annually compounded interest rate is

- % (a 6-month discount rate of $5.2/2 = 2.6\%$).
- What is the present value of the bond?

- (c) Generate a graph or table showing how the bond's present value changes for semi-annually compounded interest rate between 1% and 15%.

32. The Reborn VW Beetle.

You are considering the purchase of a new car, the reborn VW Beetle, and you have been offered two different deals from two different dealers. Dealer A offers to sell you the car for \$20,000, but allows you to put down \$2,000 and pay back \$18,000 over 36 months (fixed payment each month) at a rate of 8% compounded monthly. Dealer B offers to sell you the car for \$19,500 but requires a down payment of \$4,000 with repayment of the remaining \$15,500 over 36 months at 10% compounded monthly. Which deal would you choose? (Hint: Find ranges of market interest rates that make one deal more attractive than the other.)

33. Dear Financial Adviser,

My spouse and I are each 62 and hope to retire in 3 years. After retirement we will receive \$5,000 per month after taxes from our employers pension plans and \$1000 per month after taxes from Social Security. Unfortunately our monthly living expenses are \$15,000. Our social obligations preclude further economies.

We have \$1,200,000 invested in a high-grade corporate-bond mutual fund. Unfortunately the after-tax return on the fund has dropped to 3.5% per year. We plan to make annual withdrawals from the fund to cover the difference between our pension and social security income and our living expenses. How long will the money last?

Sincerely,

Luxury Challenged

Marblehead, MA

34. The annually compounded discount rate is 5.5%. You are asked to calculate the present value of a 12-year annuity with payments of \$50,000 per year. Calculate PV for each of the following cases.

- (a) The annuity payments arrive at one-year intervals. The first payment arrives one year from now.
- (b) The first payment arrives in 6 months. Following payments arrive at one-year intervals, at 18 months, 30 months, etc.

35. IRA Accounts and Taxes.

An Individual Retirement Account (IRA) allows you to set aside a limited amount of money each year for retirement. These funds will have a special tax status that depends on several factors. (These factors include your marital status, whether you

have other sources of retirement savings, your income, etc.)

Suppose that you have \$2,000 in pretax income to contribute to the IRA at the end of each year (starting with the end of the current year, i.e., year 1). You will retire in 30 years, and your marginal tax rate will be 28% for all years. Suppose that the account returns a fixed 6% each year until you retire. For simplicity, assume that you withdraw all money at your retirement, and any tax-deferred income is taxed at that time.

- (a) How much money will you have in year 30 if neither the contribution nor the interest income is tax-deferred? (In this case, you can withdraw the money without paying any additional tax at year 30.)
- (b) How much money will you have in 30 years if the contribution is not tax-deferred but the interest income is? (In this case, only the cumulative interest is taxed at year 30.)
- (c) How much money will you have in 30 years if both the contribution and the interest income are tax-deferred?
- (d) Would you expect the benefit of tax deferral to increase or decrease as the tax rate increases? Why?

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1 Present Value Solutions

1. (a) $\$10,000 \times (1.045)^5 = \$12,461.82$.
 (b) $\$10,000 \times (1 + \frac{0.045}{4})^{(4 \times 5)} = \$12,507.51$.
 (c) $\$10,000 \times (1 + \frac{0.045}{12})^{(12 \times 5)} = \$12,517.96$.
2. Using the following formula $r_{EAR} = (1 + \frac{r_{APR}}{k})^k - 1$
 - (a) Annual: $\approx 4.75\%$
 - (b) Monthly: $\approx 4.85\%$
 - (c) Weekly: $\approx 4.86\%$
 - (d) Continuous using the following formula $r_{EAR} = e^{r_{APR}} - 1 \approx 4.86\%$
3. (a) $\$50,000 \times (1.04)^2 = \$54,080$
 (b) $\$50,000 \times (1.01)^8 = \$54,142.84$
 (c) $\$50,000 \times (1 + \frac{0.04}{12})^{(12 \times 2)} = \$54,157.15$
4. (a) $\$10,000 \times (1.045)^5 = \$12,461.82$.
 (b) $\$10,000 \times (1 + \frac{0.045}{4})^{(4 \times 5)} = \$12,507.51$.
 (c) $\$10,000 \times (1 + \frac{0.045}{12})^{(12 \times 5)} = \$12,517.96$.
5. **e-Money.** The effective annual interest rate for an A.P.R. of 5.25% is
 - (a) 5.2500% with annual compounding
 - (b) 5.3782% with monthly compounding
 - (c) 5.3899% with daily compounding
 - (d) 5.3903% with continuous compounding.

We used the following two formulas for this question: $r_{EAR} = (1 + \frac{r_{APR}}{k})^k - 1$ and $r_{EAR} = e^{r_{APR}} - 1$.

6. False. The choice of the horizon date should not affect the present value.
7. (a) $1.055 \times (\frac{\$10,000}{0.055})(1 - \frac{1}{1.055^{10}}) \times (1.055)^{10} = \$135,835$ (end of year 10)
 (b) $\frac{\$135,835}{(1+0.02)^{10}} = \$111,432$ (NPV=79,521)
8. (a) $PV = \frac{1.5}{1.12} + \frac{1.3}{1.12^2} + \frac{1.05}{1.12^3} + \frac{0.9}{1.12^4} + \frac{0.75}{1.12^5} = \4.12 million.
 (b) Inflation affects the cash flow and the discount rate equally, and the two effects cancel out in the PV calculation. Therefore, $PV = \$4.12$ million.

9. Assuming the rent payments are paid at the beginning of the year, the PV of the floating rent for the next 5 years is

$$\$100,000 + \frac{\$100,000 * 1.05^1}{1.12^1} + \dots + \frac{\$100,000 * 1.05^4}{1.12^4} = \$441,285.71$$

This must be equal to

$$\$441,285.71 = F + \frac{F}{1.12} + \frac{F}{1.12^2} + \frac{F}{1.12^3} + \frac{F}{1.12^4}$$

Solve for F , to get $F = \$109,300.67$

10. (a) Cash option PV = \$40,000

$$\text{Financing option PV } 2\% = 937.5 \times \frac{1}{1.02^{1/12}-1} \times \left(1 - \frac{1}{(1.02^{(1/12)})^{48}}\right) = \$43,228$$

Cash option costs less

- (b) Cash option PV = \$40,000

$$\text{Financing option } 15\% = 937.5 \times \frac{1}{1.15^{1/12}-1} \times \left(1 - \frac{1}{(1.15^{(1/12)})^{48}}\right) = \$34,271$$

Cash option costs more

11. Year 1: $10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^1 = 9.7364$

$$\text{Year 2: } 10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^2 = 9.4797$$

$$\text{Year 3: } 10 \times \left(\frac{1.05 \times 1.02}{1.10}\right)^3 = 9.2298$$

$$\text{PV: } 9.7364 + 9.4797 + 9.2298 = \$28.4458 \text{ million.}$$

12. The value of company ABC is the PV of the growing annuity of cash flow:

$$\text{PV} = \frac{10}{1.09-1.05} = \$250 \text{ million.}$$

13. Assume all the oil is sold at the end of each year; for example the PV of the production for the first year is $\frac{\$7000 \times \$15}{1.08} = \$97,222$. The PV of the second year production is $\frac{\$6580 \times \$15}{1.08^2} = \$84,619$. The PV of the total production in the first 12 years is \$608,254 and by then the annual production has declined to 3,544 barrels.

14. (a) Assuming first cash flow one year from now in real terms is \$14m:

$$r_{\text{real}} = (1.08/1.02) - 1 = 5.88\%$$

$$PV = 14/0.0588 = \$238m \text{ (no growth)}$$

$$\text{Using nominal values: } 14 * 1.02 / (0.08 - 0.02) = \$238m$$

Alternatively, if assuming first cash flow one year from now in real terms is

$$14/1.02 = \$13.73m, PV = (14/1.02)/0.0588 = \$233.33m$$

$$\text{Using nominal values: } 14/(0.08 - 0.02) = \$233.33m$$

Note: in order to get full credit, the assumptions made should have been clearly stated.

- (b) To calculate PV of cash flows generated in a continuous stream, starting immediately, we need to use the same perpetuity formula ($PV = C / r$), but substitute the continuously compounded rate

$$\text{New } r = \log(1.08/1.02) = 5.72\% \text{ and } PV = 14/0.0572 = \$244.93m$$

Common mistake: not using this compounding formula / forgetting that the inflation rate also must be adjusted. Using nominal values instead, $14/(\log 1.08 - \log 1.02) = \$244.93m$

Note: in order to get full credit, the assumptions made should have been clearly stated.

15. The PV of the first scholarship from the foundations point of view is

$$-(20,000/0.06)[1 - (1/(1.06)^4)] + (1/(1.06)^5)(8,000/0.06)[1 - (1/(1.06)^{10})] = -25,303$$

So it loses \$25,303 every year beginning from $t=0$. The PV of this perpetuity is $-25,303/0.06 = -421,717$

This implies that the investment needed to fund this is \$421,717

16. The market value of the lease is \$1.25 million for 5 years, with the first payment due right now. So the value is

$$1.25 + \frac{1.25}{1+6\%} + \frac{1.25}{(1+6\%)^2} + \frac{1.25}{(1+6\%)^3} + \frac{1.25}{(1+6\%)^4} = \$5.5814 \text{ million.}$$

The value of your current lease is \$1 million for 5 years, and the total PV is \$4.4651 million. Therefore the difference is \$1.1163 million, and this should be the compensation you asked from the owner.

17. Use growing annuity formula assuming that the payments are made at the beginning of each year and you pay in full for the year that you die (unfortunately).

The value if you have an expected life of T years is:

$$\begin{aligned} PV &= \$750 + \$750 * \left(\frac{1.05}{1.12} \right) + \dots + \$750 * \left(\frac{1.05}{1.12} \right)^T \\ &= \$750 + \$750 \left[\frac{1}{0.12 - 0.05} - \frac{1}{0.12 - 0.05} \times \left(\frac{1.05}{1.12} \right)^T \right] \end{aligned} \quad (1)$$

Solve for T . The breakeven point is $T \geq 16$

18. (a) Let the annual payment = C . The PV of all my payments, discounted at the dealer's rate, must equal to the price, i.e.,

$$3500 + \frac{C}{0.02} \left(1 - \frac{1}{1.02^5} \right) = 30000.$$

$$3500 + 4.71346C = 30000.$$

$$C = \$5,622.20.$$

- (b) Since I can save at a higher rate, the cost of the financing plan in (a) is only

$$3500 + \frac{C}{0.05} \left(1 - \frac{1}{1.05^5} \right) = \$27,841.17.$$

The cost of the second option is $30,000 - 2,500 = \$27,500$. I should pay cash.

19. The fair monthly rate is $(1 + 0.08)^{1/12} - 1 = 0.006434$.
Let C be the appropriate monthly payment. Then, $\frac{C}{0.006434} \left[1 - \frac{1}{1.006434^{120}} \right] = 100,000$.
 $C = \$1,198.58$.
20. The PV of all cash inflows and outflows is
 $PV = \frac{-65}{1.05} + \frac{-65}{1.05^2} + \frac{10}{1.05^3} + \frac{10}{1.05^4} + \frac{40}{1.05^5} + \dots + \frac{40}{1.05^9} = \$38,478.57$.
I am taking a financial gain by helping him out.
21. The PV of the offer at discount rate of 12% is \$359,921. With the discount rate of 8% the PV increases to \$446,821. In both cases accepting the award of \$500,000 is preferable. This calculation ignores other consideration such as difference tax treatment.
22. (a) $EAR = (1 + \frac{.06}{12})^{12} - 1 = 0.06168$.
(b) Let C be the monthly payment. The effective interest rate per month is $\frac{0.06}{12} = 0.005$. The PV of the mortgage annuity must equal the amount borrowed:
 $\frac{C}{0.005} \left[1 - \left(\frac{1}{1.005} \right)^{(12 \times 30)} \right] = 600,000$.
 $C = \$3,597.30$.
(c) $\frac{3597.30}{0.005} \left[1 - \left(\frac{1}{1.005} \right)^{(12 \times 25)} \right] = \$558,326.10$.
23. False. US treasury bond carries inflation risk.
24. (a) $PV = \frac{500}{0.06} \left[1 - \frac{1}{1.06^{10}} \right] + \frac{10000}{1.06^{10}} = \$9,263.99$.
(b) The effective interest rate per 6 months is $1.06^{1/2} - 1 = 0.02956$.
 $PV = \frac{250}{0.02956} \left[1 - \frac{1}{1.02956^{20}} \right] + \frac{10000}{1.06^{10}} = \$9,318.39$.
25. (a) $EAR = (1 + \frac{.085}{12})^{12} - 1 = 0.088391$.
(b) The effective interest rate per month is $\frac{.085}{12} = 0.0070833$.
Let C be the monthly payment. The PV of the mortgage annuity must equal the amount borrowed:
 $\frac{C}{0.0070833} \left[1 - \frac{1}{1.0070833}^{(12 \times 30)} \right] = 500,000$.
 $C = \$3,844.57$.
(c) $\frac{3844.57}{0.0070833} \left[1 - \frac{1}{1.0070833}^{(12 \times 30 - 60)} \right] = \$477,451.33$.
26. The PV of the current salary at the discount rate of 8% is \$468,687. It is assumed that salaries are paid at the end of each year (only an approximation to the more accurate monthly payment) and he works up to the end of his 65th year.

To calculate the break-even starting salary after MBA, one needs to include the negative NPV of the cost of MBA as well as the lost income for the 2-years spent in school and compare that against the positive NPV of the increased earning power. The negative NPV of the cost of MBA is \$35,665 and the loss in salary is \$71,330 for a total of \$-106,995.

Even if the starting salary after the MBA is as low as \$32,687 it still pays off to get the MBA. Of course, getting an MBA makes the skill set more flexible (such as the

general management skills or the network of people you know) which reduced the risk of the income as well. The aspect if not considered here but definitely would make the MBA even more valuable.

27. (a) PV at $t = 5$ of the growing annuity from $t = 6$ to 35:
 $100,000 \frac{1.03}{0.04-0.03} \left[1 - \left(\frac{1.03}{1.04} \right)^{30} \right] = \$2,591,785.58$.
 PV at $t = 0$ of the growing annuity:
 $2591785.58 \times (1.04)^{-5} = \$2,130,258.82$
 PV at $t = 0$ of the annuity from $t = 1$ to 5:
 $\frac{100,000}{0.04} \left[1 - \frac{1}{1.04^5} \right] = \$445,182.23$.
 Total human capital:
 $2,130,258.82 + 445,182.23 = \$2,575,441.06$.
- (b) FV at $t = 5$ of savings:
 $\frac{30,000}{0.04} [1.04^5 - 1] = \$162,489.68$.
28. (a) Assume that every year, Mr. Jones withdraws the year's living expense at the beginning of the year. If he retires today and lives for another T years, he would need the following amount:
 $S = 100,000 \frac{1.05}{1.05-1.02} \left[1 - \frac{1.02^T}{1.05} \right]$.
 (i) $T = 25 : S(25) = \$1,804,335.76$ (enough).
 (ii) $T = 45 : S(45) = \$2,550,363.68$ (not enough).
 (iii) $T = 60 : S(60) = \$2,885,219.04$ (not enough).
- (b) Mr. Jones can (i) work longer and retire later, (ii) downgrade his lifestyle and reduce living expense, or (iii) find better investment opportunities that give higher returns (though he must also consider the risk involved).
29. The nominal value is $\$50,000 \times (1.075^{10})$ Inflation reduced this by a factor of $(1.03)^{10}$. So the real value is only $\$76,680$.
30. (a) $r_{APR} = \ln(r_{EAR} + 1) = \ln(1.12) = 0.1133$
 (b) $e^{0.05} - 1 = 0.0513$
 (c) $\ln(1.09) = 0.08617$
31. (a) $PV = \frac{275}{0.026} \left[1 - \frac{1}{1.026^{20}} \right] + \frac{10000}{1.026^{20}} = \$10,231.64$.
 (b) (A convex, downward sloping curve)

32. The Reborn VW Beetle.

First use the annuity formula to determine the monthly payments C_a and C_b for dealerships A and B, respectively, ignoring the initial down payments:

- (a) Dealership A: $PV_a = \$18,000$, $r_a = 0.08/12$, and $t = 36$ months $\Rightarrow C_a = \$564.05$.
 (b) Dealership B: $PV_b = \$15,500$, $r_b = 0.10/12$, and $t = 36$ months $\Rightarrow C_b = \$500.14$.

If the monthly discount rate is currently r , then the net present values of the two packages are

$$\begin{aligned}\text{NPV}_a &= 2,000 + C_a \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right] \\ \text{NPV}_b &= 4,000 + C_b \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right].\end{aligned}$$

It is clearly more advantageous to accept dealership A's offer if and only if $\text{NPV}_a < \text{NPV}_b$. Substituting the expressions from above and simplifying, we have that $\text{NPV}_a < \text{NPV}_b$ if and only if

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right] < 31.29.$$

By trial and error, the cross-over point is at $r = 0.00778$. The conclusion is that if the current annual interest rate for a 36-month period (compounded monthly) is above 9.34%, you should choose dealership A. If the current annual interest rate for a 36-month period (compounded monthly) is below 9.34%, you should choose dealership B.

- 33.** Monthly amount to fund from the mutual fund = $15,000 - 6,000 = \$9,000$

PV of the fund in three years = $\$1,200,000 \times 1.035^3 = \$1,330,461$

Number of years that it will last:

$$\$1,330,461 = 1.035 \times 9000 \times 12 \times \frac{1}{0.035} \times \left(1 - \frac{1}{(1+0.035)^T} \right)$$

(assume you take the money out at the beginning of the year)

$$T = 16.4 \text{ years}$$

- 34.** (a) $\text{PV} = \frac{50000}{0.055} \left[1 - \frac{1}{1.055^{12}} \right] = \$430,925.89$.
 (b) $\text{PV}_{-0.5} = \frac{50000}{0.055} \left[1 - \frac{1}{1.055^{12}} \right] = \$430,925.89$.
 $\text{PV}_0 = 430,925.89 \times (1.055)^{0.5} = \$442,617.74$.

35. IRA Accounts and Taxes

- (a) Yearly contribution: $2000 \times (1 - 28\%) = \1440 :

$$\sum_{t=1}^{30} \{ 2000 \times (1 - 0.28) \times [1 + 0.06 \times (1 - 0.28)]^{30-t} \} = \$85,218.$$

- (b) To get the total amount at end of 30 years:

$$\sum_{t=1}^{30} \{ 2000 \times (1 - 0.28) \times [1 + 0.06 \times (1 - 0.00)]^{30-t} \} = \$113,843.$$

of which the principal is $30 \times 2000 \times (1 - 0.28) = \$43,200$, implying that the interest is $113,843 - 43,200 = \$70,643.79$. Tax on this is $70,643.79 \times 0.28$ implying that the take-home amount is $\$94,063$.

(c) The total amount is

$$\sum_{t=1}^{30} [2000 \times (1 + 0.06)^{30-t}] = \$158,116.37.$$

After taxes, this is only $158116.37 \times (1 - 0.28) = 113,843.789$. Note: The answers to (b) and (c) are identical. Given the after-tax interest rate, it doesn't matter whether your money is taxed on the way in or the way out.

(d) Increase, because more deferred tax can be used to accrue interest.