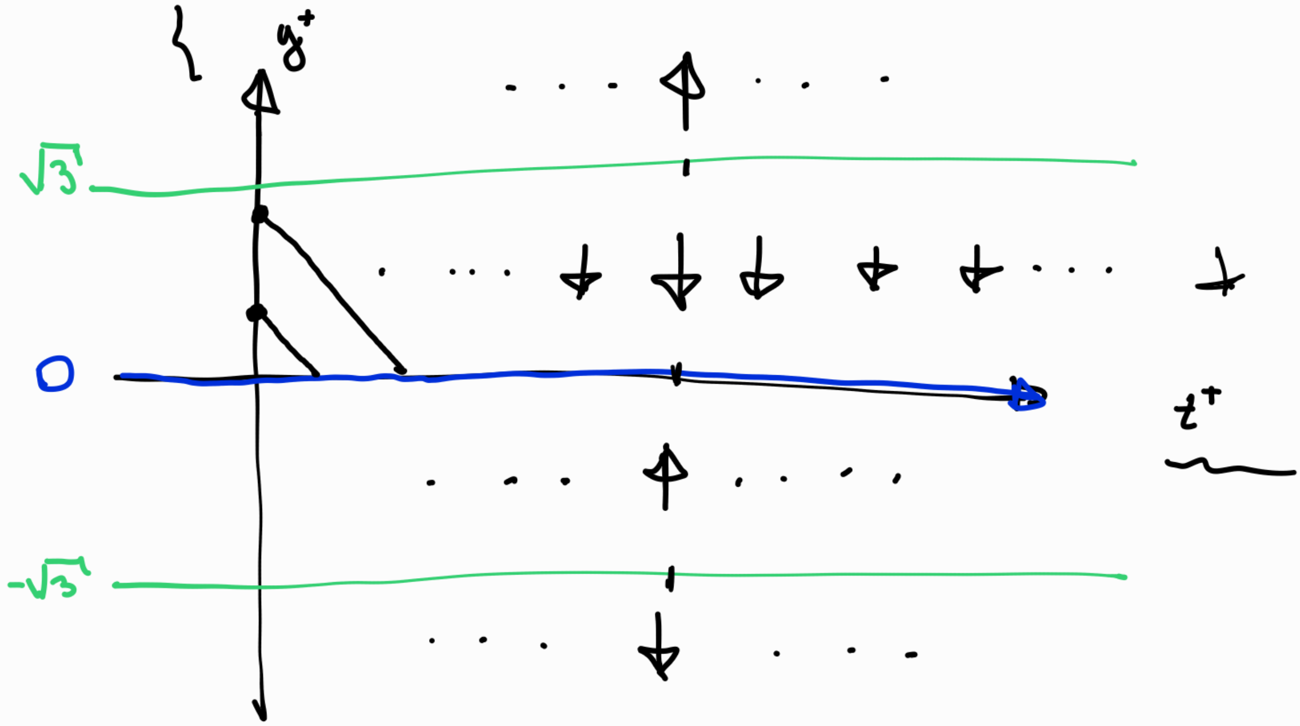
$y' = y^3 - 3y \longrightarrow \underbrace{F(y) = y^3 - 3y}_{\text{función pendiente}}$

$y' = F(t,y)$

Equilibrios

$y^3 - 3y = 0 \iff y(y^2-3) = 0 \iff y_0 \equiv 0,\ y_1 \equiv \sqrt{3},\ y_2 \equiv -\sqrt{3}$



$y \in (-\infty, -\sqrt{3})$

$\Rightarrow y^2 > 3 \Rightarrow y^2 - 3 > 0 \Rightarrow y(y^2-3) < 0.\quad F(y) < 0$

$y \in (-\sqrt{3}, 0)$

$y^2 < 3 \Rightarrow y^2 - 3 < 0 \Rightarrow (y^2-3)y > 0.\quad F(y) > 0$

$y \in (0, \sqrt{3})$

$y^2 < 3 \Rightarrow y^2 - 3 < 0 \Rightarrow y(y^2-3) < 0.\quad F(y) < 0$

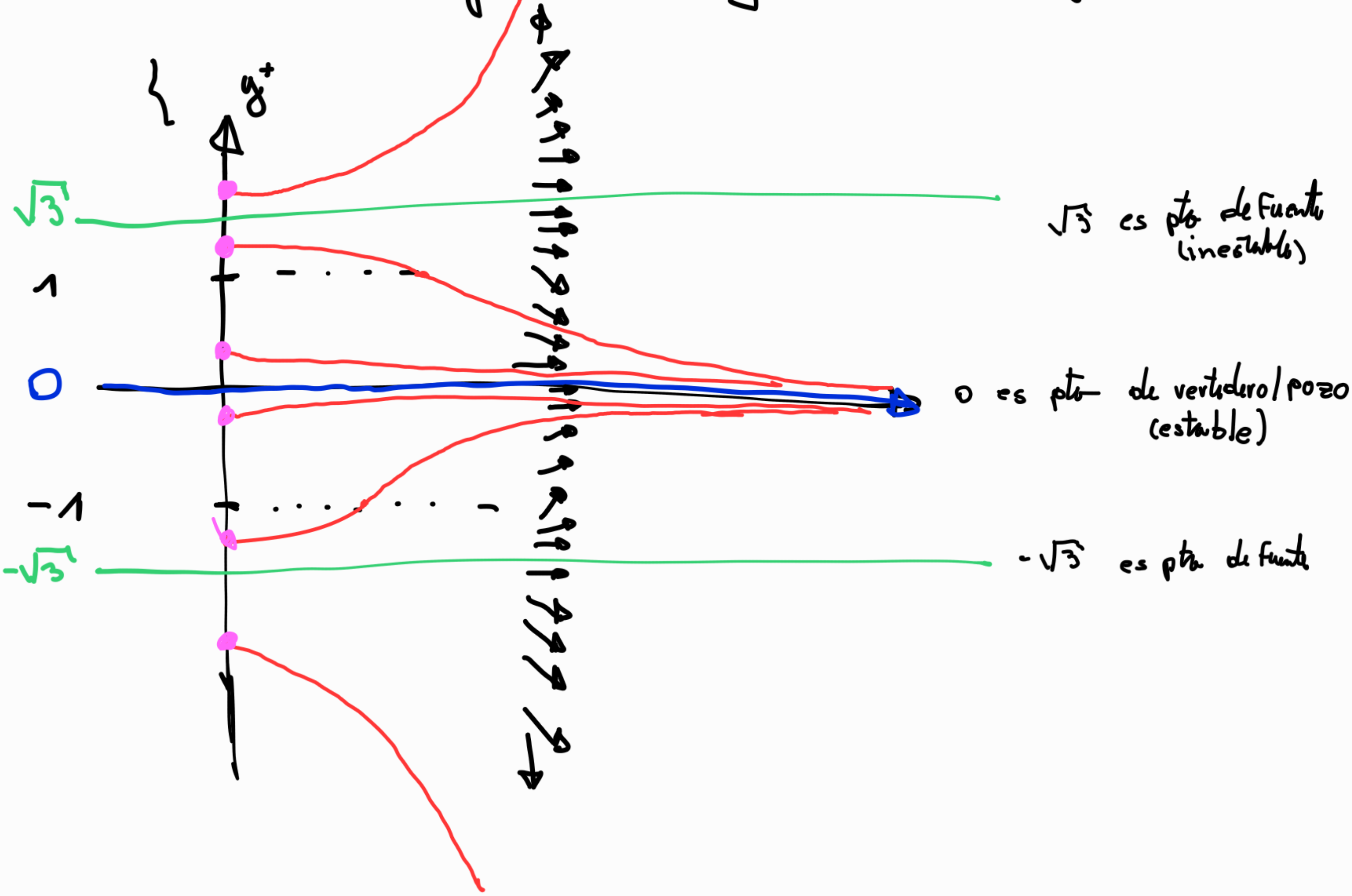
$y \in (\sqrt{3}, \infty)$

$y^2 > 3 \Rightarrow y^2 - 3 > 0 \Rightarrow y(y^2-3) > 0.\quad F(y) > 0$

$F(y) = y^3 - 3y \Rightarrow F'(y) = 3y^2 - 3 = 3(y^2-1)$

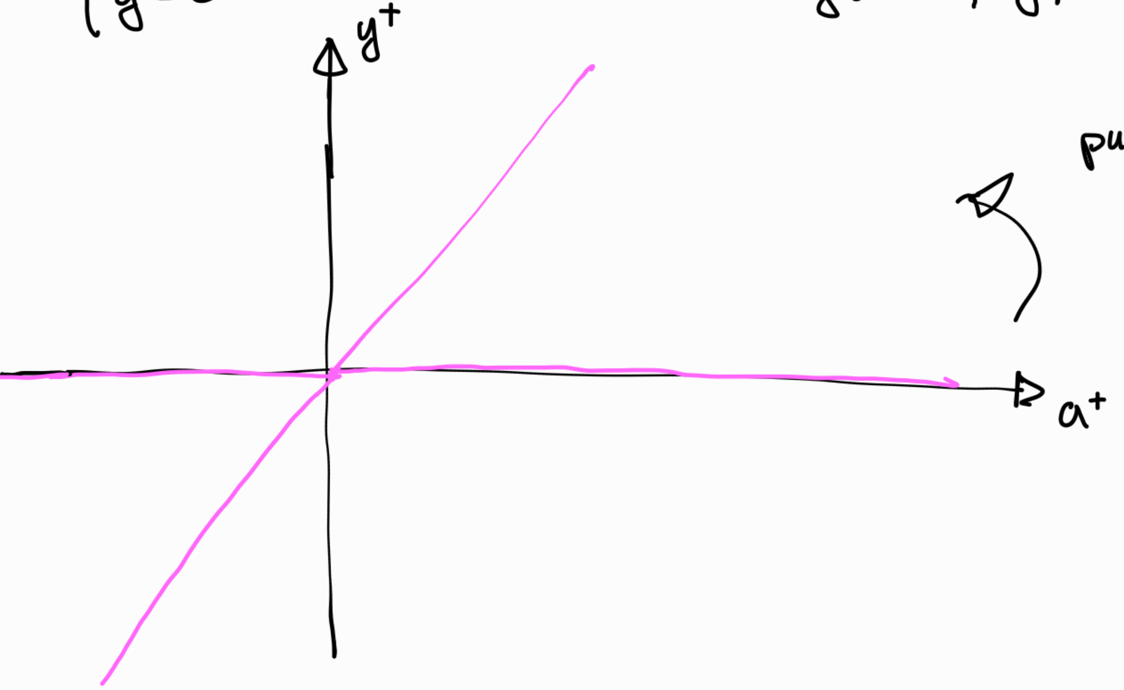
$y \in (-1,1) \Rightarrow F'(y) < 0$. F decrece

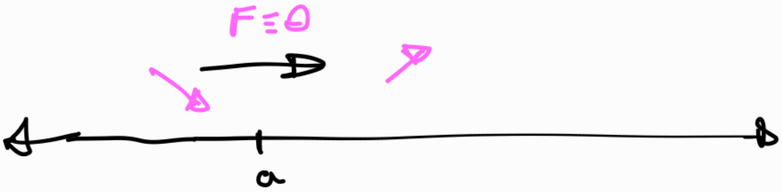
$|y| > 1 \Rightarrow F'(y) > 0$. F crece



P3) 1: $y' = y^2 - ay \Rightarrow y(y-a) = y'$

$$\begin{cases} y = a \\ y = 0 \end{cases} \qquad F(y) = 0 \Rightarrow y_0 \equiv 0,\ y_1 \equiv a$$



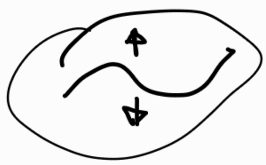


Estudiando la monotonía de F en a, conoceremos los signos de F al rededor del pto de equilibrio. La derivada de F en a nos ayudará.

$F(y) = y^2 - ay \Rightarrow F'(y) = 2y - a$. Ptos de eq $\begin{cases} F'(0) = -a \\ F'(a) = a \end{cases}$

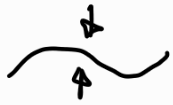
Si $\bar{x}$ es pto de eq, entonces

$F'(\bar{x}) > 0$ pendientes negativa $\to 0 \to$ pendientes pos



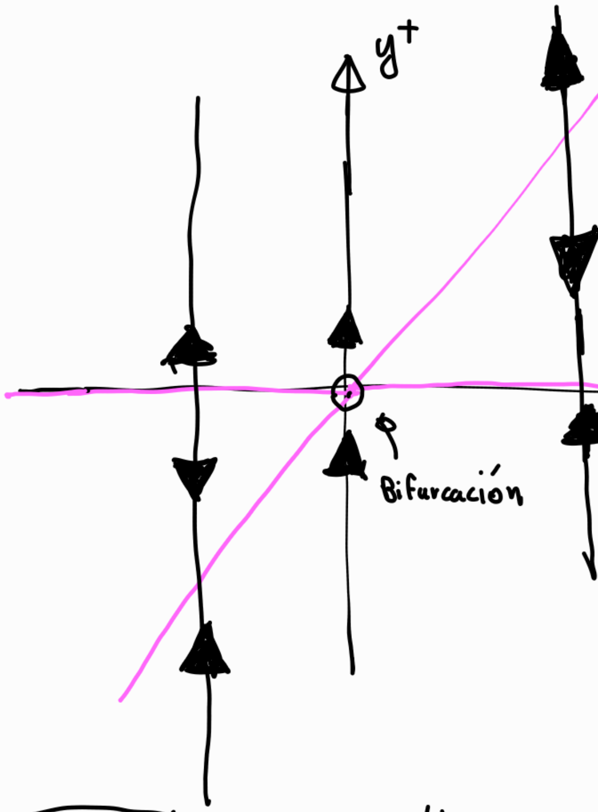
pto de eq inestable.

$F'(\bar{x}) < 0$ pendiente pos $\to 0 \to$ pendiente negativas



pto de equilib. estable

Ptos de eq $\begin{cases} F'(0) = -a & a<0 \Rightarrow \text{pto inestable},\ a>0 \Rightarrow \text{pto estable. } (0) \\ F'(a) = a & a<0 \Rightarrow \text{pto estable},\ a>0 \Rightarrow \text{pto inestable } (a) \end{cases}$



$\varepsilon > 0 \quad F'(y_0) > 0$

$y_0+\varepsilon \quad F(y_0+\varepsilon) > F(y_0) = 0$

$y_0 \quad F(y_0) = 0$

$y_0-\varepsilon \quad F(y_0-\varepsilon) < F(y_0) = 0$

$\boxed{\text{EDO 0-ésima}}$ (Caso borde)

$y' = \underbrace{y^2}_{0}, \quad F_0'(y) = 2y \qquad F_0'(0) = 0$ no concluyente

$y < 0 \Rightarrow F_0(y) > 0$

$y > 0 \Rightarrow F_0(y) > 0.$ (Conocido)