Resumen Auxiliar: Sumatorias

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Propiedades Básicas

Sean $\lambda \in \mathbb{R}$, $s \in \mathbb{N}$ y $(a_k)_{k\geq m}$, $(b_k)_{k\geq m}$ sucesiones.
Para todo $n \geq m$, se tiene:

- $\displaystyle\sum_{k=n}^{n} a_k = a_n$
- $\displaystyle\sum_{k=m}^{n} \lambda a_k = \lambda \sum_{k=m}^{n} a_k$
- $\displaystyle\sum_{k=m}^{n} (a_k \pm b_k) = \sum_{k=m}^{n} a_k \pm \sum_{k=m}^{n} b_k$
- $\displaystyle\sum_{k=m}^{n} a_k = \sum_{k=m}^{s} a_k + \sum_{k=s+1}^{n} a_k$
- Telescópica:
  $\displaystyle\sum_{k=m}^{n} (a_k - a_{k+1}) = a_m - a_{n+1}$
- Cambio de índices:
  $\displaystyle\sum_{k=m}^{n} a_k = \sum_{k=m+s}^{n+s} a_{k-s}$

Sumas Conocidas

- $\displaystyle\sum_{k=m}^{n} 1 = n - m + 1$
- $\displaystyle\sum_{k=0}^{n} k = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\displaystyle\sum_{k=0}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- Geométrica: $\displaystyle\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$, $r \neq 1$

Coeficientes Binomiales

Para $n, k \in \mathbb{N}$ con $0 \leq k \leq n$:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Propiedades:

- Si $k > n \geq 0$, entonces $\binom{n}{k} = 0$
- $\binom{n}{0} = \binom{n}{n} = 1$
- $\binom{n}{1} = \binom{n}{n-1} = n$
- Simetría: $\binom{n}{k} = \binom{n}{n-k}$
- Pascal: $\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$
- Triángulo de Pascal:
  $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$
- Binomio de Newton:
  $\forall x, y \in \mathbb{R}$ se tiene que

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$