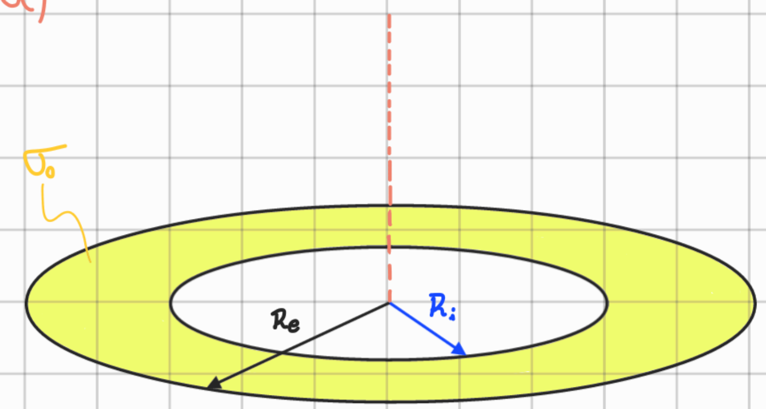
a)



$$\vec{r} = z\hat{z}$$

$$\vec{r}\,' = r'\hat{r}'; \quad r' \in [R_i, R_e]$$

$$dq = \sigma_0 r' dr' d\varphi'; \quad \varphi \in [0, 2\pi)$$

$\boxed{+1.5}$

$$|\vec{r} - \vec{r}\,'|^3 = \left[z^2 + r'^2\right]^{3/2}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi}\int_{R_i}^{R_e} \frac{z\hat{z} - r'\hat{r}}{\left[z^2 + r'^2\right]^{3/2}} \sigma_0 r' dr' d\varphi'}$$

$\boxed{+1.5}$

$$= \frac{\sigma_0}{4\pi\varepsilon_0}\left[\int_0^{2\pi}\int_{R_i}^{R_e} \frac{z\hat{z}}{\left[z^2 + r'^2\right]^{3/2}} r' dr' d\varphi' - \cancelto{0}{\int_0^{2\pi}\int_{R_i}^{R_e} \frac{r'\hat{r}}{\left[z^2 + r'^2\right]^{3/2}} r' dr' d\varphi'}\right]$$

Aquí basta con un argumento de simetría o decir que el seno y coseno del $\hat{r}$ serán 0 al integrar.

$\boxed{+0.5}$

$$= \frac{\sigma_0}{4\pi\varepsilon_0} \int_0^{2\pi}\int_{R_i}^{R_e} \frac{z\hat{z}}{\left[z^2 + r'^2\right]^{3/2}} r' dr' d\varphi'$$

$$= \frac{\sigma_0}{4\pi\varepsilon_0} \int_0^{2\pi}\int_{R_i}^{R_e} \frac{z\hat{z}}{\left[z^2 + r'^2\right]^{3/2}} r' dr' d\varphi'$$

$$= \frac{\sigma_0 z\hat{z}}{4\pi\varepsilon_0} \int_0^{2\pi}\int_{R_i}^{R_e} \frac{r' dr' d\varphi'}{\left[z^2 + r'^2\right]^{3/2}};$$

z y $\hat{z}$ son ctes. respecto a las variables de integración.

$$= \frac{\sigma_0 z \hat{z}}{4\pi\varepsilon_0} \underbrace{\int_0^{2\pi} d\varphi'}_{=2\pi} \times \int_{R_i}^{R_e} \frac{r'dr'}{\left[z^2 + r'^2\right]^{3/2}} = \frac{\sigma_0 z \hat{z}}{\cancel{4\pi}_{2} \varepsilon_0} \cancel{2\pi} \int_{R_i}^{R_e} \frac{r'dr'}{\left[z^2 + r'^2\right]^{3/2}}$$

$$= \frac{\sigma_0 z \hat{z}}{2\varepsilon_0} \int_{R_i}^{R_e} \frac{r'dr'}{\left[z^2 + r'^2\right]^{3/2}} \quad ;$$

C.V.

$u = z^2 + r'^2$

$du = 2r'dr'$

$$\Rightarrow \vec{E} = \frac{\sigma_0 z \hat{z}}{2\varepsilon_0} \times \frac{-1}{\sqrt{r'^2 + z^2}} \Bigg|_{R_i}^{R_e} = \frac{\sigma_0 z \hat{z}}{2\varepsilon_0} \times \frac{1}{\sqrt{r'^2 + z^2}} \Bigg|_{R_e}^{R_i}$$

$$\boxed{\vec{E} = \frac{\sigma_0}{2\varepsilon_0} \left(\frac{z}{\sqrt{R_i^2 + z^2}} - \frac{z}{\sqrt{R_e^2 + z^2}} \right) \hat{z}}$$

$\boxed{+1.0}$

b)

$R_e = R_i + \delta$

$$\Rightarrow \vec{E} = \frac{\sigma_0 z}{2\varepsilon_0} \left(\frac{1}{\sqrt{R_i^2 + z^2}} - \frac{1}{\sqrt{(R_i + \delta)^2 + z^2}} \right) \hat{z}$$

$$\delta \ll 1 \Rightarrow \vec{E} = \vec{E}(\delta = 0) + \delta \frac{d\vec{E}}{d\delta}\Bigg|_{\delta = 0} \quad (\star)$$

$\boxed{+1.0}$

$$\vec{E}(\delta = 0) = \frac{\sigma_0 z}{2\varepsilon_0} \left(\cancel{\frac{1}{\sqrt{R_i^2 + z^2}}} - \cancel{\frac{1}{\sqrt{R_i^2 + z^2}}} \right) \hat{z}$$

$$\vec{E}(\delta = 0) = 0$$

$$\frac{d\vec{E}}{d\delta} = \frac{\sigma_0 z}{2\varepsilon_0}\hat{z}\frac{d}{d\delta}\left(\frac{1}{\sqrt{R_i^2+z^2}} - \frac{1}{\sqrt{(R_i+\delta)^2+z^2}}\right) = -\frac{\sigma_0 z}{2\varepsilon_0}\hat{z}\frac{d}{d\delta}\left(\frac{1}{\sqrt{(R_i+\delta)^2+z^2}}\right)$$

$$= + \frac{\sigma_0 z}{2\varepsilon_0}\hat{z}\left[+\frac{1}{\cancel{2}}\frac{1}{\left[(R_i+\delta)^2+z^2\right]^{3/2}} \times \cancel{2}(R_i+\delta)\right]$$

$$\frac{d\vec{E}}{d\delta} = \frac{\sigma_0 z}{2\varepsilon_0}\hat{z}\left[\frac{R_i+\delta}{\left((R_i+\delta)^2+z^2\right)^{3/2}}\right] \Longrightarrow \left.\frac{d\vec{E}}{d\delta}\right|_{\delta=0} = \frac{\sigma_0 z}{2\varepsilon_0}\frac{R_i}{(R_i^2+z^2)^{3/2}}\hat{z}$$

$$(\star) \Longrightarrow \vec{E} = \frac{\sigma_0 z}{2\varepsilon_0}\frac{\delta R_i}{(R_i^2+z^2)^{3/2}}\hat{z}$$

$$[\sigma_0] = \frac{Q}{L^2} \qquad [\delta] = L$$

$$\Longrightarrow \sigma_0\delta = \lambda$$

$$[\sigma_0 \times \delta] = \frac{Q}{L} = [\lambda]$$

$$\therefore \quad \boxed{\vec{E} = \frac{\lambda R_i}{2\varepsilon_0}\frac{z}{(R_i^2+z^2)^{3/2}}\hat{z}}$$

$\boxed{+0.5}$