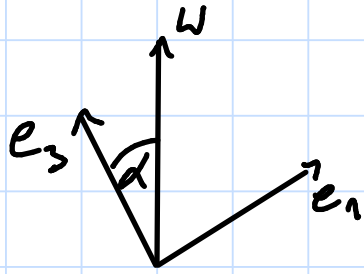


1)



$$\vec{w}_e = (w \sin(\alpha), 0, w \cos(\alpha))$$

$$\dot{\vec{w}}_e = 0$$

$$I_1 = I_2 = \frac{1}{4} MR^2, \quad I_3 = \frac{1}{2} MR^2$$

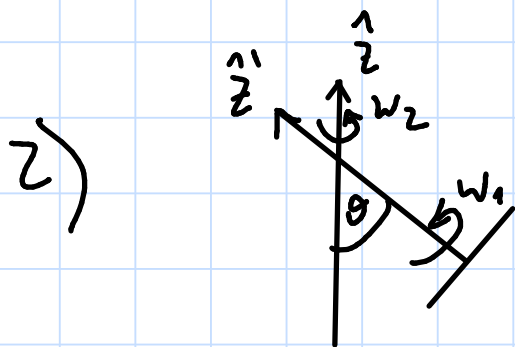
$$\Rightarrow \tau_1 = \tau_3 = 0 \quad \xrightarrow{w_2=0} \quad \tau_2 = w_{e1} w_{e2} (I_3 - I_1)$$

$$\tau_2 = w^2 \sin(\alpha) \cos(\alpha) \frac{MR^2}{4}$$

$$\vec{\tau} = \vec{r} \times \vec{F} = d \hat{y} \times -F \hat{x} + -d \hat{y} \times F \hat{x} = 2dF \hat{z}$$

$$|\vec{\tau}| = |\tau_2| \Rightarrow 2dF = w^2 \sin(\alpha) \cos(\alpha) \frac{MR^2}{4}$$

$$\Rightarrow F = \frac{w^2 \sin(\alpha) \cos(\alpha) MR^2}{8d} = \frac{MR^2 w^2 \sin(2\alpha)}{16d}$$



$$\vec{w}_1 = w_1 \hat{z}$$

$$\vec{w}_2 = w_2 \sin(\theta) \hat{y}' + w_2 \cos(\theta) \hat{z}'$$

$$\vec{\Omega} = w_2 \sin(\theta) \hat{y}' + (w_1 + w_2 \cos(\theta)) \hat{z}'$$

$$I_{cm} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \frac{MR^2}{4}$$

Usando Steiner $I_P = \begin{pmatrix} 4L^2 + R^2 & 0 & 0 \\ 0 & 4L^2 + R^2 & 0 \\ 0 & 0 & 2R^2 \end{pmatrix} \frac{M}{4}$

Luego $\vec{L}_P = \vec{I}_P \vec{\Omega} =$

$$\vec{L}_P = \frac{M}{4} \left((4L^2 + R^2) w_2 \sin(\theta) \hat{y}' + 2R^2 (w_1 + w_2 \cos(\theta)) \hat{z}' \right)$$

Dado que el sistema solo rota podemos definir la derivada del momento angular como

$$\dot{\vec{L}}_P = \vec{\omega} \times \vec{L}_P$$

$$\Rightarrow \dot{\vec{l}}_p = \vec{\omega}_2 \times \frac{M}{4} \left((4L^2 + R^2) \omega_2 \sin(\theta) \hat{y}' + 2R^2 (\omega_1 + \omega_2 \cos(\theta)) \hat{z}' \right)$$

$$\dot{\vec{l}}_p = - \left(ML^2 + \frac{MR^2}{4} \right) \omega_2^2 \sin(\theta) \cos(\theta) \hat{x} + 2R^2 (\omega_1 + \omega_2 \cos(\theta)) \omega_2 \sin(\theta) \hat{x}$$

Luego, el torque se produce únicamente por el peso

$$\begin{aligned} \vec{\tau} &= M(-L\hat{z}') \times (g \cos(\theta) \hat{z}' + g \sin(\theta) \hat{y}') \\ \vec{\tau} &= -MLg \sin(\theta) \hat{x} \end{aligned}$$

$$\text{Luego } \dot{\vec{l}}_p = \vec{\tau} \quad \left(\text{ver diferentes valores de } \theta \text{ y } \omega \right)$$

Ver que este método es equivalente a haber usado Euler ($\dot{\vec{l}} = \vec{\tau} = \text{Euler}$)