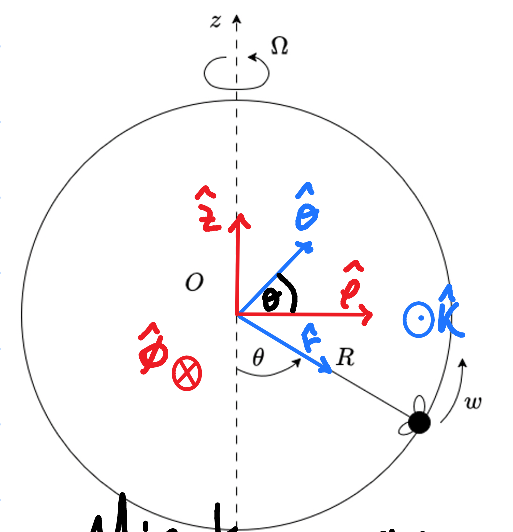
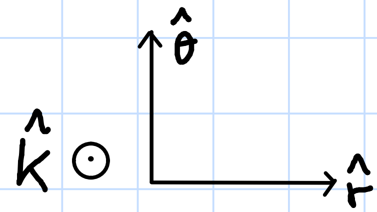
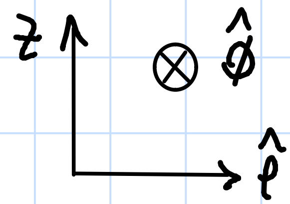



El sistema de coordenadas cilíndricas es tal que




Mientras que en otro plano




Donde $\dot\theta = w$ y $\dot\phi = \Omega$

Luego, por trigonometría se tiene que

$$\hat r = \text{sen}(\theta)\,\hat\rho - \cos(\theta)\,\hat z$$

$$\hat\theta = \cos(\theta)\,\hat\rho + \text{sen}(\theta)\,\hat z$$

$$\hat k = -\hat\phi$$

Luego $\vec r = R\hat r + \cancel{z\hat k}^{\,0} = R(\text{sen}(\theta)\hat\rho - \cos(\theta)\hat z)$

$$\vec v = \cancel{\dot r \hat r}^{\,0} + r\dot\theta\,\hat\theta + \cancel{\dot z \hat k}^{\,0} = Rw(\cos(\theta)\hat\rho + \text{sen}(\theta)\hat z)$$

$$\vec a = (\cancel{\ddot r}^{\,0} - r\dot\theta^2)\hat r + (2\cancel{\dot r}^{\,0}\dot\theta + \cancel{r\ddot\theta}^{\,0})\hat\theta + \cancel{\ddot z \hat k}^{\,0}$$

$$\vec a = -Rw^2(\text{sen}(\theta)\hat\rho - \cos(\theta)\hat z)$$

Entonces las fuerzas ficticias son tal que

$$\vec{F}_{tras} = -m\frac{d^2r}{dt^2}\hat{r} = 0$$

$$\vec{F}_{cor} = -2m\vec{\Omega}\times\vec{v} = -2m\left(\Omega\hat{z}\times R\omega\left(\cos(\theta)\hat{\rho} + \cancel{\operatorname{sen}(\theta)\hat{z}}^{\,0}\right)\right)$$

$$\vec{F}_{cor} = -2m\Omega R\omega\cos(\theta)\hat{\phi} \quad \left(\hat{\phi} \text{ apunta en dir opuesta}\right)$$

$$\vec{F}_{cf} = -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}) = -m\Omega\hat{z}\times R\left(\Omega\hat{z}\times\left(\operatorname{sen}(\theta)\hat{\rho} - \cancel{\cos(\theta)\hat{z}}^{\,0}\right)\right)$$

$$\vec{F}_{cf} = -mR\Omega\hat{z}\times\left(\Omega\operatorname{sen}(\theta)\hat{\phi}\right)$$

$$\vec{F}_{cf} = mR\Omega^2\operatorname{sen}(\theta)\hat{\rho}$$

Además, las fuerzas sobre la mosca son

$$F = -N_k\hat{k} - N_r\hat{r}$$

$$F = N_k\hat{\phi} - N_r\left(\operatorname{sen}(\theta)\hat{\rho} - \cos(\theta)\hat{z}\right)$$

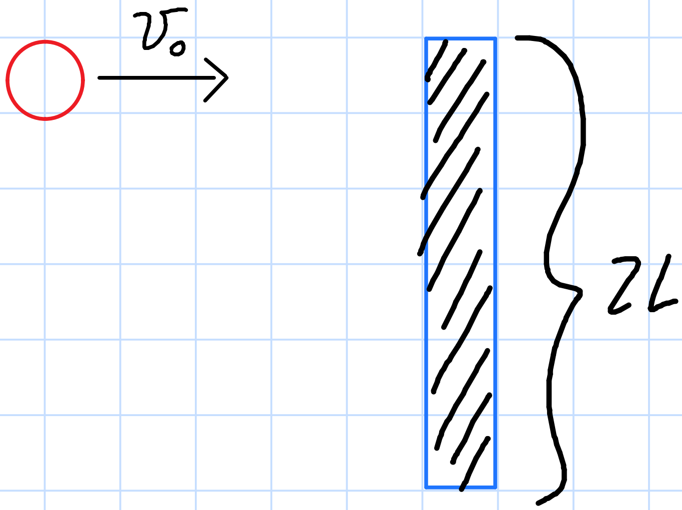
Entonces la ecuación de movimiento resulta

$$m\vec{a} = -N_r\left(\mathrm{sen}(\theta)\hat{\rho} - \cos(\theta)\hat{z}\right) + N_k\hat{\phi} + mR\Omega^2\,\mathrm{sen}(\theta)\hat{\rho}$$
$$-2m\Omega R w\cos(\theta)\hat{\phi}$$

b) Sabemos que $\vec{a}$ no tiene componentes en $\hat{\phi}$, entonces podemos igualar

perpendicular

$$N_k\hat{\phi} = 2m\Omega R w\cos(\theta)\hat{\phi}$$
$$\hookrightarrow F_\perp = 2m\Omega R w\cos(\theta)$$




1) Conservación de TODO

Energia Cinetica:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}Mv_b^2 + \frac{1}{2}I\omega^2 \quad (1)$$

Momento Lineal:

$$mv_0 = Mv_b + mv_f \quad (2)$$

Momento angular:

$$\mathcal{L}_i = (L\hat{y} + a\hat{x}) \times (-mv_0\hat{x}) = mv_0 L\hat{z}$$

$$\mathcal{L}_f = mv_f L\hat{z} + I\omega\hat{z}$$

$$\Rightarrow \quad mv_0 L = mv_f L + I\omega \quad (3)$$

$$I = \int r\,dm = \frac{M}{2L}\int_{-L}^{L} r^2 dr = \frac{Mr^3}{6L}\Bigg|_{-L}^{L} = \frac{ML^2}{3}$$

De (2) $$v_f = \frac{mv_0 - Mv_b}{m}$$

Reemplazando en (3)

$$mv_0 L = m\left(\frac{mv_0 - Mv_b}{m}\right)L + I\omega$$

$$Mv_b L = I\omega$$

$$v_b = \frac{I\omega}{ML} = \frac{ML^2\omega}{3ML} = \frac{L\omega}{3}$$

Ahora en (1)

$$\frac{mv_0^2}{2} = \frac{m}{2}\left(v_0^2 - \frac{2v_0 Mv_b}{m} + \frac{M^2 v_b^2}{m^2}\right) + \frac{ML^2\omega^2}{18} + \frac{ML^2\omega^2}{6}$$

$$\frac{v_0 ML\omega}{3} = \frac{M^2L^2\omega^2}{18m} + \frac{2ML^2\omega^2}{9} \Rightarrow \frac{v_0}{3} = \frac{ML\omega}{18m} + \frac{2L\omega}{9}$$

$$\frac{v_0}{3} = \frac{MLw}{18m} + \frac{2Lw}{9} \Rightarrow 6v_0 m = MLw + 4Lwm$$

$$w = \frac{6v_0 m}{ML + 4Lm}$$

$$wL = \frac{6v_0 m}{M + 4m}$$

Finalmente:

$$v_f = v_0 - \frac{M}{m} v_b = v_0 - \frac{2v_0 M}{M+4m}$$

$$v_f = v_0 \left( \frac{M + 4m - 2M}{M + 4m} \right) = v_0 \left( \frac{4m - M}{4m + M} \right)$$