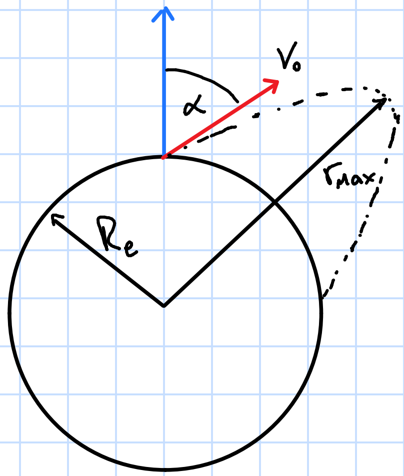

Se tiene que

$$\vec{r} = r\hat{r} \quad \text{y} \quad \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

En $t = 0$: $\dot{\vec{r}}(t=0) = \vec{V_0}$

$$\vec{V_0} = V_0\cos(\alpha)\hat{r} + \underbrace{V_0 sen(\alpha)}_{r\cdot\dot{\theta}}\hat{\theta}$$

Luego el momento angular es constante

$$\Rightarrow m r^2\dot{\theta} = m r^2(0)\,\dot{\theta}(0) = m R_e\cdot V_0 sen(\alpha)$$

Por conservación de la energía:

$$\frac{1}{2}m\dot{\vec{r}}^2 - \frac{GMm}{r} = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{r} = E$$

Evaluando en $r(t_{max}) = r_{max}$ y $r(0)$ $(\dot{r}(t_{max}) = 0)$

$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{r} = \frac{1}{2}m(\dot{r_0}^2 + r_0^2\dot{\theta_0}^2) - \frac{GMm}{r_0}$$

$$\text{y} \quad r_{max}^2\dot{\theta}(t_{max}) = R_e\cdot V_0 sen(\alpha) \quad , \quad \dot{r}(t_{max}) = 0$$

$$\Rightarrow \frac{1}{2}\left(\frac{R_e V_0 sen(\alpha)}{r_{max}}\right)^2 - \frac{GM}{r_{max}} = \frac{1}{2}(V_0^2\cos(\alpha)^2 + V_0^2 sen^2(\alpha)) - \frac{GM}{R_e}$$

$$\frac{R_e^2 V_0^2 sen^2(\alpha)}{2r_{max}^2} - \frac{V_0^2}{2} + \frac{GM}{R_e} = \frac{GM}{r_{max}}$$

$$\left(\frac{V_0^2 - 2GM}{2R_e}\right) r_{max}^2 - GMr_{max} + R_e^2 V_0^2 sen^2(\alpha) = 0$$

$$a = 1 \quad , \quad b = \frac{GM2R_e}{2GM - V_0^2} \quad , \quad c = \frac{2R_e^3 V_0^2 sen(\alpha)}{V_0^2 - 2GM}$$

$$r_{max} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$