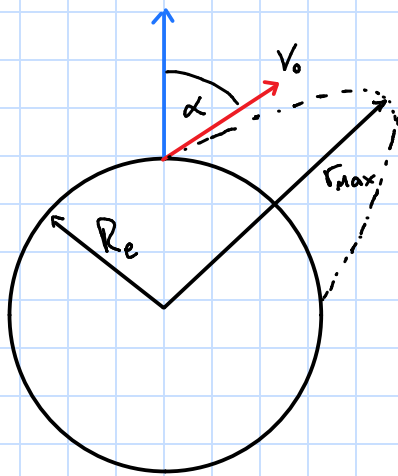




Se tiene que

$$\vec{r} = r\hat{r} \quad \text{y} \quad \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\text{En } t=0 : \dot{\vec{r}}(t=0) = \vec{V}_0$$

$$\vec{V}_0 = V_0 \cos(\alpha)\hat{r} + \underbrace{V_0 \sin(\alpha)}_{r\dot{\theta}}\hat{\theta}$$

Luego el momento angular es constante

$$\Rightarrow m r^2 \dot{\theta} = m r^2(0) \dot{\theta}(0) = m R_e \cdot V_0 \sin(\alpha)$$

Por conservación de la energía:

$$\frac{1}{2} m \dot{\vec{r}}^2 - \frac{GMm}{r} = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - \frac{GMm}{r} = E$$

Evalando en $r(t_{max}) = r_{max}$ y $r(0)$ ($\dot{r}(t_{max}) = 0$)

$$\frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - \frac{GMm}{r} = \frac{1}{2} m (\dot{r}_0^2 + r_0^2 \dot{\theta}_0^2) - \frac{GMm}{r_0}$$

$$\text{y } r_{max}^2 \dot{\theta}(t_{max}) = R_e \cdot V_0 \sin(\alpha) , \quad \dot{r}(t_{max}) = 0$$

$$\Rightarrow \frac{1}{2} (R_e V_0 \sin(\alpha))^2 - \frac{GM}{r_{max}} = \frac{1}{2} (V_0^2 \cos^2(\alpha) + V_0^2 \sin^2(\alpha)) - \frac{GM}{R_e}$$

$$\frac{R_e V_0^2 \sin^2(\alpha)}{2} - \frac{V_0^2}{2} + \frac{GM}{R_e} = \frac{GM}{r_{max}}$$

$$r_{max} = \frac{GM}{\left(\frac{R_e V_0^2 \sin^2(\alpha) - V_0^2}{2} + \frac{GM}{R_e} \right)}$$