$$F(r) = -Kr^{-4}\hat{r}$$

a) Integrando $F(r)$

$$U(r) = \int F(r)dr = -\int -Kr^4 = \frac{Kr^5}{5} \quad (1)$$

Movimiento:

$$m\ddot{r} - mr\dot{\theta}^2 = F(r) \quad (2)$$

$$mr\ddot{\theta} + 2m\dot{r}\dot{\theta} = 0 \quad (3)$$

de (3), multiplicando por r

$$mr^2\ddot{\theta} + 2mr\dot{r}\dot{\theta} = \frac{d(mr^2\dot{\theta})}{dt} = 0$$

$$mr^2\dot{\theta} = L \ (\text{cte}) \Rightarrow \dot{\theta} = \frac{L}{mr^2}$$

Luego $E = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + U$

$$E = \frac{1}{2}m\left(\dot{r}^2 + \frac{L^2}{m^2r^2}\right) + U = \frac{m\dot{r}^2}{2} + \frac{L^2}{2mr^2} + \frac{Kr^5}{5}$$

Derivando $U_{eff} = \frac{L^2}{2mr^2} + \frac{Kr^5}{5}$

$$\frac{\partial U_{eff}}{\partial r} = -\frac{L^2}{mr^3} + Kr^4 = 0$$

$$r_0 = \left(\frac{L^2}{mK}\right)^{1/7}$$

$$\Rightarrow U_{eff} = \frac{L^2}{m}\left(\frac{mK}{L^2}\right)^{1/7} + \frac{K}{5}\left(\frac{L^2}{mK}\right)^{1/7}$$

Luego $\dot{r} = 0 \Rightarrow E = U_{eff}$

b) $$W^2 = \frac{1}{m}\frac{\partial^2 U_{eff}}{\partial r^2}\bigg|_{r_0} = \frac{1}{m}\left(\frac{3L^2}{mr_0^4} + 4Kr_0^3\right)$$

$$W = \sqrt{\frac{1}{m}\left(\frac{3L^2}{mr_0^4} + 4Kr_0^3\right)}$$