1) $F = -Kx + a/x^3$

a) $V(x)$? $\quad V(x) = -\int F\,dx$

$$\int F\,dx = \int Kx\,dx - \int a/x^3\,dx = \frac{Kx^2}{2} + \frac{a}{2x^2} + C = V(x)$$

Si $x \to 0 \Rightarrow V(x) \to \infty$

Si $x \to \infty \Rightarrow V(x) \to \frac{Kx^2}{2}$

$$\frac{d(V(x))}{dx} = Kx - \frac{a}{x^3} = 0$$

$$Kx = \frac{a}{x^3} \Rightarrow x^4 = \frac{a}{K} \Rightarrow x_{eq} = \pm\sqrt[4]{\frac{a}{K}}$$

$$\frac{d(\dot{V}(x))}{dx} = K + \frac{3a}{x^4} > 0 \quad \therefore \text{ estable}$$

Luego $\quad m\ddot{x} = -Kx + \frac{a}{x^3}$

Utilizando pequeñas oscilaciones en torno a x_0. Queda taylor

$x = x_0 + \delta$, con $\delta \ll x_0$

$$\Rightarrow \ddot{\delta} + w^2\delta = 0 \quad \text{con } w^2 = \frac{1}{m}\frac{\partial^2(V(x))}{\partial x^2}$$

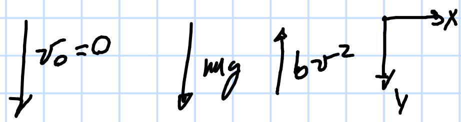
$$w^2 = \left(K + \frac{3a}{x_0^4}\right)\cdot\frac{1}{m} = (K + 3K)\cdot\frac{1}{m} = \frac{4K}{m}$$

$$w = \sqrt{\frac{4K}{m}}, \quad x(t) = x_0 + A\cos(wt + \phi)$$

b) Si $E^2 \gg Ka \Rightarrow E^2 \gg V(x)$

Luego, la velocidad de la partícula es muy grande $\Rightarrow \frac{a}{x^3}$ se vuelve irrelevante $\Rightarrow$ X actúa como MAS ya que $F = -Kx$

2)



Cambio de enunciado: caida libre.

$$\Rightarrow \quad m\ddot{y} = mg - bv^2$$

$$m\frac{dv}{dt} = mg - bv^2$$

$$\frac{dv}{dt} = g - \frac{b}{m}v^2$$

$$\frac{dv}{\left(g - \frac{b}{m}v^2\right)} = dt \qquad \Big/ \int_0^t$$

$$\int_{v_0}^{v} \frac{dv}{g - \frac{b}{m}v^2} = \int_0^t dt$$

Con integral conocida

$$\sqrt{\frac{m}{bg}} \cdot \operatorname{arctanh}\left(\sqrt{\frac{b}{mg}} \cdot v(t)\right)\Bigg|_{v_0=0}^{v} = t$$

$$\sqrt{\frac{m}{bg}} \cdot \operatorname{arctanh}\left(\sqrt{\frac{b}{mg}} \cdot v(t)\right) = t$$

$$v(t) = \sqrt{\frac{mg}{b}} \tanh\left(\sqrt{\frac{bg}{m}} \cdot t\right)$$

$$\frac{\partial y}{\partial t} = \sqrt{\frac{mg}{b}} \tanh\left(\sqrt{\frac{b}{mg}} \cdot t\right)$$

$$\int_{y_0}^{y} \partial y = \sqrt{\frac{mg}{b}} \int_0^t \tanh\left(\sqrt{\frac{b}{mg}} \cdot t\right) dt = \sqrt{\frac{mg}{b}} \cdot \sqrt{\frac{m}{bg}} \ln\left(\cosh\left(\sqrt{\frac{bg}{m}}\, t\right)\right)\Bigg|_0^t \quad \Bigg/ \begin{array}{l} \cosh(0) = 1 \\ \ln(1) = 0 \end{array}$$

$$y(t) - y_0 = \frac{m}{b} \ln\left(\cosh\left(\sqrt{\frac{bg}{m}}\, t\right)\right) \,//$$

3)

$$m\dot{v} = -mg - be^{\alpha|v|}$$

a) $v(t)$?

$$m\frac{dv}{dt} = -mg - be^{\alpha|v|}$$

si cae del reposo
$v>0 \Rightarrow |v| = v$

$$\frac{dv}{g+\frac{b}{m}e^{\alpha v}} = -dt$$

C.V. $e^{\alpha v} = u \Rightarrow \alpha e^{\alpha v} dv = du \Rightarrow \alpha u\, dv = du \Rightarrow dv = \frac{du}{\alpha u}$

$$\Rightarrow \frac{1}{g+\frac{b}{m}u} \cdot \frac{du}{\alpha u} = -dt \quad \Big/ \int_0^t$$

$$\underbrace{\int \frac{1}{g+\frac{b}{m}u} \cdot \frac{du}{\alpha u}} = \int -dt$$

$$\frac{1}{\alpha}\left(\frac{A}{g+\frac{b}{m}u} + \frac{B}{u}\right) = \frac{1}{\alpha}\left(\frac{uA + B(g+b/m\cdot u)}{u(g+\frac{b}{m}u)}\right)$$

$$uA + B(g + b/m \cdot u) = 1$$

$$u\left(A + B\frac{b}{m}\right) + Bg = 1$$

$$B = \frac{1}{g} \quad y \quad A = -\frac{bB}{m} = \frac{-b}{mg}$$

$$\Rightarrow -\frac{b}{mg\alpha}\int \frac{1}{g+\frac{b}{m}u}du + \frac{1}{g\alpha}\int\frac{du}{u} = -\int dt$$

$$\frac{1}{\alpha g}\left(\ln(u)\right) - \frac{b}{mg\alpha}\ln\left(g+\frac{b}{m}u\right) = -t \quad \Big/ \cdot\alpha g, \ \exp(\cdot)$$

$$u - e^{b/m}\cdot\left(g+\frac{b}{m}u\right) = e^{-t\alpha g} \quad \Big| \ u = e^{\alpha v}$$

$$e^{\alpha v} - \frac{b}{m}e^{b/m}\cdot e^{\alpha v} - e^{b/m}\cdot g = e^{-\alpha g t}$$

$$e^{\alpha v}\left(1 - \frac{b}{m}e^{b/m}\right) = e^{-\alpha g t} + e^{b/m}\cdot g$$

$$e^{\alpha v}\left(1-\frac{b}{m}e^{b/m}\right) = e^{-\alpha g t} + e^{b/m}\cdot g$$

$$e^{\alpha v} = \frac{e^{-\alpha g t} + e^{b/m}\cdot g}{\left(1-\frac{b}{m}e^{b/m}\right)} \Bigg/ \Bigg|_{t_0}^{t}$$

$$v(t) - v_0 = \frac{1}{\alpha}\left(\frac{e^{-\alpha g t} + e^{b/m}\cdot g}{\left(1-\frac{b}{m}e^{b/m}\right)} - \frac{e^{b/m}\cdot g}{\left(1-\frac{b}{m}e^{b/m}\right)}\right)$$

$$v(t) = \frac{1}{\alpha}\left(\frac{e^{-\alpha g t}}{\left(1-\frac{b}{m}e^{b/m}\right)}\right)$$

c) Utilizando expansión de Taylor al rededor de $v=0$.

$$e^{\alpha v} = 1 + \alpha v + \frac{(\alpha v)^2}{2!} + \frac{(\alpha v)^3}{3!} + \dots$$

si αv pequeño $\Rightarrow$ $e^{\alpha v} \approx 1 + \alpha v$

$$\frac{\partial v}{\partial t} = g - \frac{b}{m}e^{\alpha v} = g - \frac{b}{m}(1+\alpha v) = g - \frac{b}{m} - \frac{b\alpha v}{m}$$

$$\Rightarrow \frac{\partial v}{\partial t} = \left(g - \frac{b}{m}\right) - \frac{\alpha b}{m}\cdot v$$

$$v_h = A\cdot \exp(-\alpha b t/m)$$

$$v = v_h + v_p \quad , \text{ si } v_p = cte \Rightarrow \dot{v}_p = 0$$

$$\Rightarrow \dot{v} = \left(g - \frac{b}{m}\right) - \frac{\alpha b}{m}v$$

$$\dot{v}_h = \left(g - \frac{b}{m}\right) - \frac{\alpha b}{m}(v_h + v_p)$$

$$g - \frac{b}{m} = \frac{\alpha b}{m}v_p$$

$$v_p = (g - b/m)\frac{m}{\alpha b} \quad , \quad v(0) = 0 \Rightarrow A = -v_p$$

Expandiendo $v = v_p\left(1 - \left(1 - \frac{\alpha b}{m}t + \frac{(\alpha b/m)^2 t^2}{2}\right)\right) = v_p\left(\frac{\alpha b}{m}t - \left(\frac{\alpha b}{m}\right)^2\frac{t^2}{2}\right)$

Entonces $v(t) = \left(g - \frac{b}{m}\right)t - \frac{1}{2}\left(g - \frac{b}{m}\right)\frac{b\alpha}{m}t^2$

d) si t muy pequeño, cumple la expansión de taylor

Luego

$$\left.\frac{\partial v}{\partial t}\right|_{t=0} = \left(g - \frac{b}{m}\right) - \frac{1}{2}\left(g - \frac{b}{m}\right)\cancel{\frac{b\alpha}{m}t \cdot 2}^{\to 0}$$

$$\left.\frac{\partial v}{\partial t}\right|_{t=0} = g - \frac{b}{m} \Rightarrow \dot{v} \neq g$$ por lo que la aceleración no solo depende de g.