Partícula m

$$F = -a v^{1/2} \qquad a > 0 \,,\ v = \dot{x}$$

$$\vec{v}(0) = v_0 \hat{x} \,,\ v_0 > 0$$

i) Encuentre t_1 al cual la partícula se detiene

$$m\ddot{x} = -a\dot{x}^{1/2}$$

$$m\frac{dv}{dt} = -a v^{1/2}$$

$$v^{-1/2} dv = \frac{-a}{m} dt \quad / \int_0^{t_1}$$

$$\int_{v_0}^{0} v^{-1/2} dv = \frac{-a}{m}\int_0^{t_1} dt$$

$$\frac{d(2v^{1/2})}{dv} = 2\cdot\frac{1}{2}\cdot v^{-1/2} = v^{-1/2}$$

$$2v^{1/2}\Big|_{v_0}^{0} = \frac{-a}{m}(t_1 - 0)$$

$$-2v_0^{1/2} = \frac{-a}{m} t_1 \Rightarrow t_1 = \frac{2v_0^{1/2} m}{a}$$

ii) Distancia

Para un t general

$$2v^{1/2}\Big|_{v_0}^{v} = \frac{-a}{m}(t - 0)$$

$$2\left(v^{1/2} - v_0^{1/2}\right) = -\frac{at}{m} \quad \Rightarrow \quad v = \left(v_0^{1/2} - \frac{at}{2m}\right)^2$$

Luego $$x(t_1) = \int_0^{t_1} v\,dt = \int_0^{t_1}\left(v_0 - \frac{a t v_0^{1/2}}{m} + \frac{a^2 t^2}{4m^2}\right)dt$$

$$\Rightarrow x(t_1) = v_0 t_1 - \frac{t_1^2}{2}\cdot\frac{a v_0^{1/2}}{m} + \frac{t_1^3}{3}\cdot\frac{a^2}{4m^2}$$

Reemplazando $t_1 = \dfrac{2v_0^{1/2} m}{a}$

$$x(t_1) = \frac{2v_0^{3/2} m}{a} - \frac{4m^2 v_0}{a^2\cdot 2}\cdot\frac{a v_0^{1/2}}{m} + \frac{8 v_0^{3/2} m^3}{a^3\cdot 3}\cdot\frac{a^2}{4m^2} = \frac{2v_0^{3/2} m}{a} - \frac{2v_0^{3/2} m}{a} + \frac{2v_0^{3/2} m}{a\cdot 3}$$

$$x(t_1) = \frac{2v_0^{3/2} m}{3a}$$

iii) Potencia disipada en punto medio

(otra forma es usando $\frac{\partial v}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial v}{\partial t}$)

$$X = \frac{X(t_1)}{2} = \frac{v_0^{3/2} m}{3a}$$

$$X(t) = \int_0^t v\, dt'$$

$$m\frac{dv}{dt} = -a v^{1/2} \Rightarrow dt = \frac{m\, dv}{-a\sqrt{v}}$$

$$\Rightarrow X(v') = -\int_{v_0}^{v'} \frac{m v\, dv}{a\sqrt{v}} = -\frac{m}{a}\int_{v_0}^{v'} v^{1/2} dv = -\frac{2m}{3a} v^{3/2}\Big|_{v_0}^{v'}$$

$$X(v') = \frac{2m}{3a}\left(v_0^{3/2} - v'^{3/2}\right)$$

$$X(v') = \frac{X(t_1)}{2} = \frac{v_0^{3/2} \cancel{m}}{\cancel{3a}} = \frac{2\cancel{m}}{\cancel{3a}}\left(v_0^{3/2} - v'^{3/2}\right)$$

$$\frac{v_0^{3/2}}{2} = v'^{3/2}$$

$$v' = \frac{v_0}{\sqrt[3]{2^2}}$$

$$P = F \cdot v = -a v^{1/2} \cdot v = -a v^{3/2} = -\frac{a v_0^{3/2}}{2}$$