Formulario de Mecánica

1 Coordenadas Cartesianas: (x, y, z)

1.1 Cinemática

Posición:	$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$
Velocidad:	$\vec{v} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$
Aceleración:	$\vec{a} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$

1.2 Diferenciales

Línea:	$d\vec{l} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$
Superficie:	$d\vec{S} = dydz\hat{\imath} + dxdz\hat{\jmath} + dxdy\hat{k}$
Volumen	$dV = dxdydz$

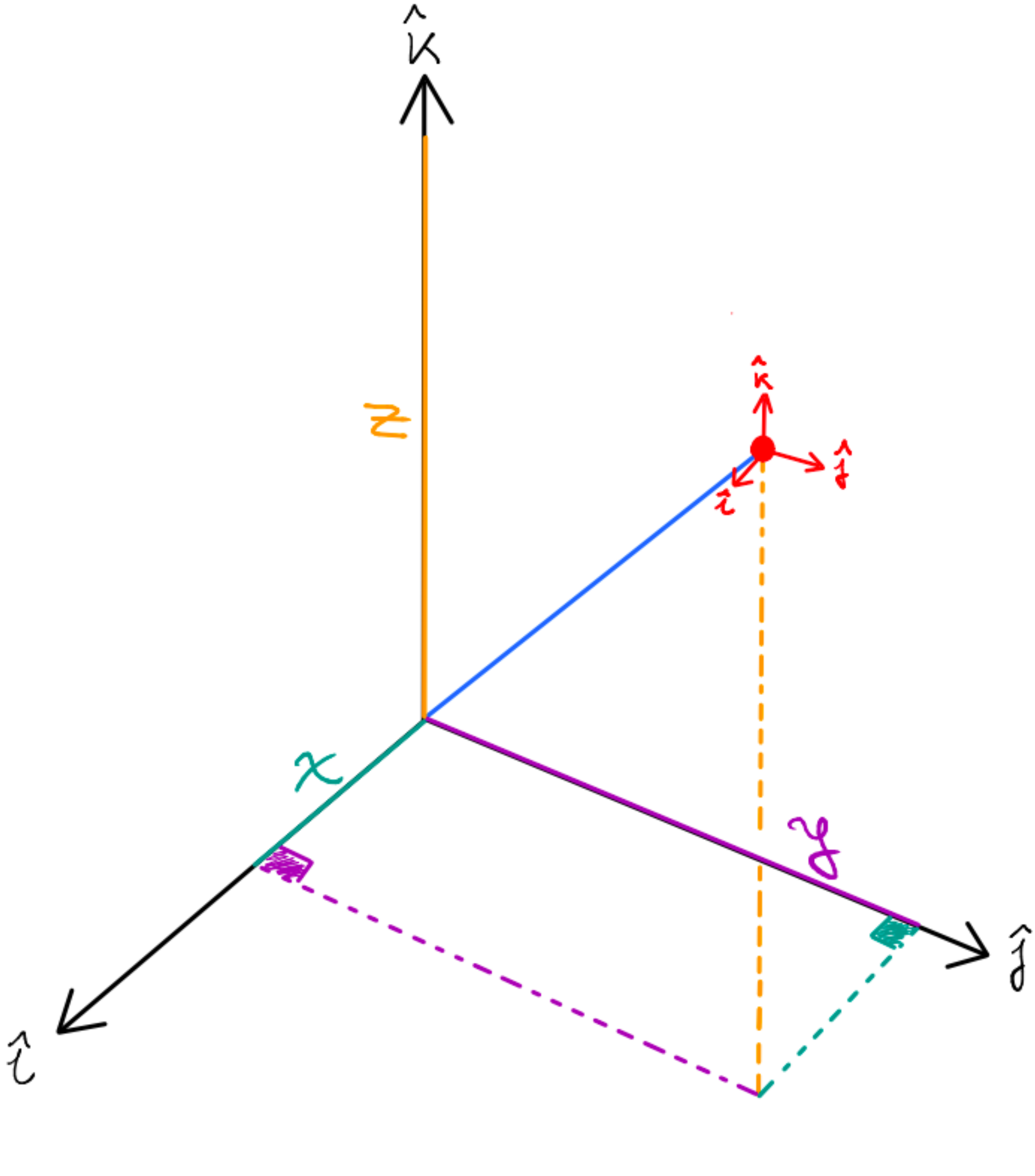
1.3 Operadores

Gradiente:	$\nabla\psi$	$= \frac{\partial\psi}{\partial x}\hat{\imath} + \frac{\partial\psi}{\partial y}\hat{\jmath} + \frac{\partial\psi}{\partial z}\hat{k}$
Divergencia:	$\nabla\cdot\vec{F}$	$= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$
Rotor:	$\nabla\times\vec{F}$	$= \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$
Laplaciano:	$\nabla^2\psi$	$= \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}$

1.4 Transformación de coordenadas

Cilíndricas:	$x = \rho\cos\phi$	$y = \rho\sin\phi$	$z = z$
Esféricas:	$x = r\sin\theta\cos\phi$	$y = r\sin\theta\sin\phi$	$z = r\cos\phi$

1.5 Esquema Coordenadas Cartesianas



2 Coordenadas Cilíndricas: (ρ, ϕ, z)

2.1 Transformación de vectores unitarios

$\hat{\boldsymbol{\rho}} = \cos\phi \hat{\boldsymbol{\imath}} + \sin\phi \hat{\boldsymbol{\jmath}}$	$\hat{\boldsymbol{\phi}} = -\sin\phi \hat{\boldsymbol{\imath}} + \cos\phi \hat{\boldsymbol{\jmath}}$	$\hat{\boldsymbol{k}} = \hat{\boldsymbol{k}}$

2.2 Cinemática

Posición:	$\vec{r} = \rho \hat{\boldsymbol{\rho}} + z \hat{\boldsymbol{k}}$
Velocidad:	$\vec{v} = \dot{\rho} \hat{\boldsymbol{\rho}} + \rho \dot{\phi} \hat{\boldsymbol{\phi}} + \dot{z} \hat{\boldsymbol{k}}$
Aceleración:	$\vec{a} = (\ddot{\rho} - \rho \dot{\phi}^2) \hat{\boldsymbol{\rho}} + (2\dot{\rho}\dot{\phi} + \rho \ddot{\phi}) \hat{\boldsymbol{\phi}} + \ddot{z} \hat{\boldsymbol{k}}$
Aceleración:	$\vec{a} = (\ddot{\rho} - \rho \dot{\phi}^2) \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{d}{dt} \left(\rho^2 \dot{\phi} \right) \hat{\boldsymbol{\phi}} + \ddot{z} \hat{\boldsymbol{k}}$

2.3 Diferenciales

Línea:	$d\vec{l} = d\rho \hat{\boldsymbol{\rho}} + \rho d\phi \hat{\boldsymbol{\phi}} + dz \hat{\boldsymbol{k}}$
Superficie:	$d\vec{S} = \rho d\phi dz \hat{\boldsymbol{\rho}} + d\rho dz \hat{\boldsymbol{\phi}} + \rho d\rho d\phi \hat{\boldsymbol{k}}$
Volumen:	$dV = \rho d\rho d\phi dz$

2.4 Operadores

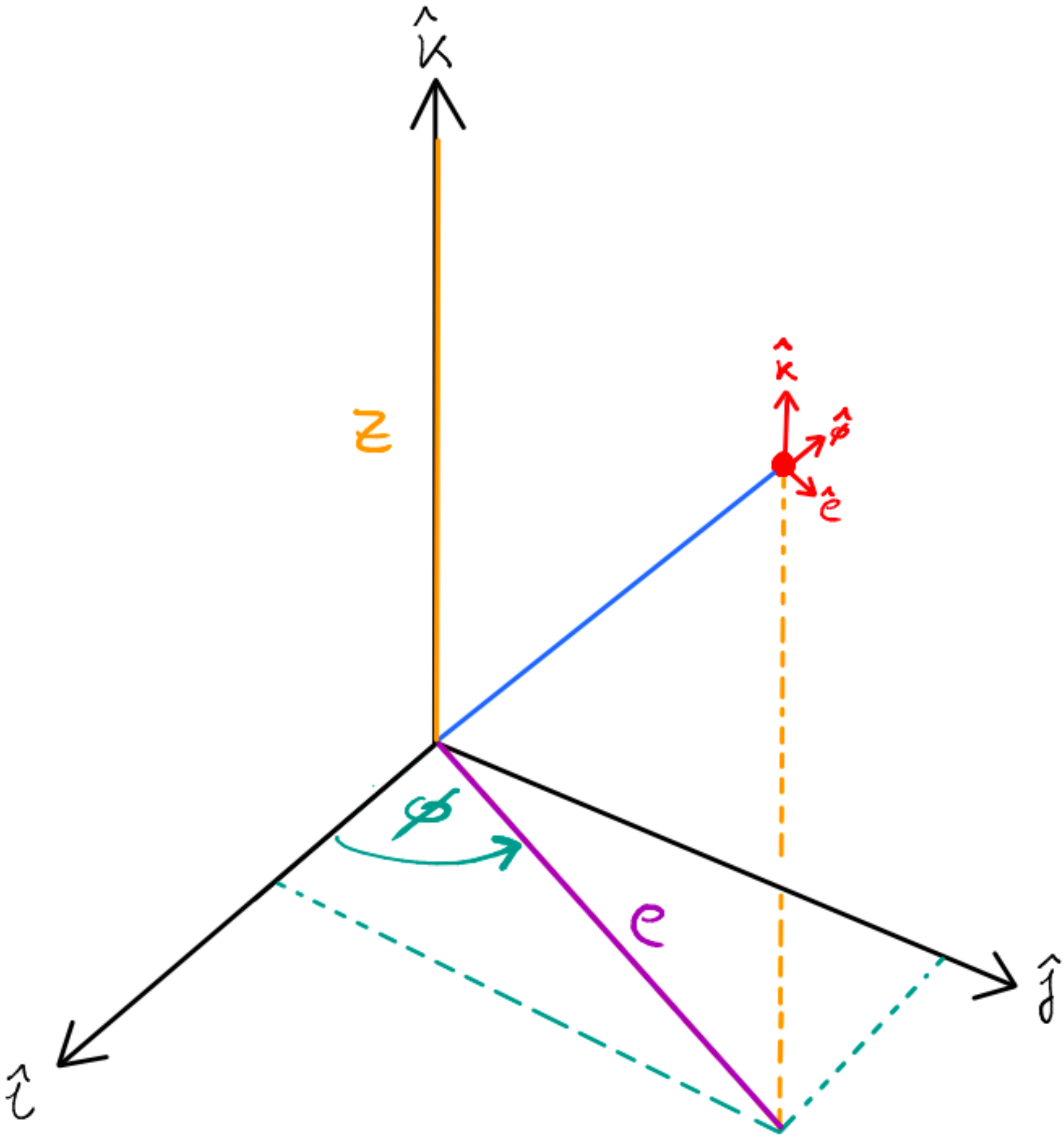
Gradiente:
$$\nabla \psi = \frac{\partial \psi}{\partial \rho} \hat{\boldsymbol{\rho}} + \frac{1}{\rho} \frac{\partial \psi}{\partial \phi} \hat{\boldsymbol{\phi}} + \frac{\partial \psi}{\partial z} \hat{\boldsymbol{k}}$$

Divergencia:
$$\nabla \cdot \vec{F} = \frac{1}{\rho} \frac{\partial (\rho F_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

Rotor:
$$\nabla \times \vec{F} = \left[\frac{1}{\rho} \frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z} \right] \hat{\boldsymbol{\rho}} + \left[\frac{\partial F_\rho}{\partial z} - \frac{\partial F_z}{\partial \rho} \right] \hat{\boldsymbol{\phi}} + \frac{1}{\rho} \left[\frac{\partial (\rho F_\phi)}{\partial \rho} - \frac{\partial F_\rho}{\partial \phi} \right] \hat{\boldsymbol{k}}$$

Laplaciano:
$$\nabla^2 \psi = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2}$$

2.5 Esquema Coordenadas Cilíndricas



3 Coordenadas Esféricas: (r, θ, ϕ)

3.1 Transformación de vectores unitarios

$\hat{\boldsymbol{r}} = \sin\theta\cos\phi\hat{\boldsymbol{\imath}} + \sin\theta\sin\phi\hat{\boldsymbol{\jmath}} + \cos\theta\hat{\boldsymbol{k}}$	$\hat{\boldsymbol{\theta}} = \cos\theta\cos\phi\hat{\boldsymbol{\imath}} + \cos\theta\sin\phi\hat{\boldsymbol{\jmath}} - \sin\theta\hat{\boldsymbol{k}}$	$\hat{\boldsymbol{\phi}} = -\sin\phi\hat{\boldsymbol{\imath}} + \cos\phi\hat{\boldsymbol{\jmath}}$

3.2 Cinemática

Posición:	$\vec{r} = r\hat{\boldsymbol{r}}$
Velocidad:	$\vec{v} = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\phi}\sin\theta\hat{\boldsymbol{\phi}}$
Aceleración:	$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + (r\ddot{\phi}\sin\theta + 2\dot{r}\dot{\phi}\sin\theta + 2r\dot{\phi}\dot{\theta}\cos\theta)\hat{\boldsymbol{\phi}}$
Aceleración:	$\vec{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\hat{\boldsymbol{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right)\hat{\boldsymbol{\phi}}$

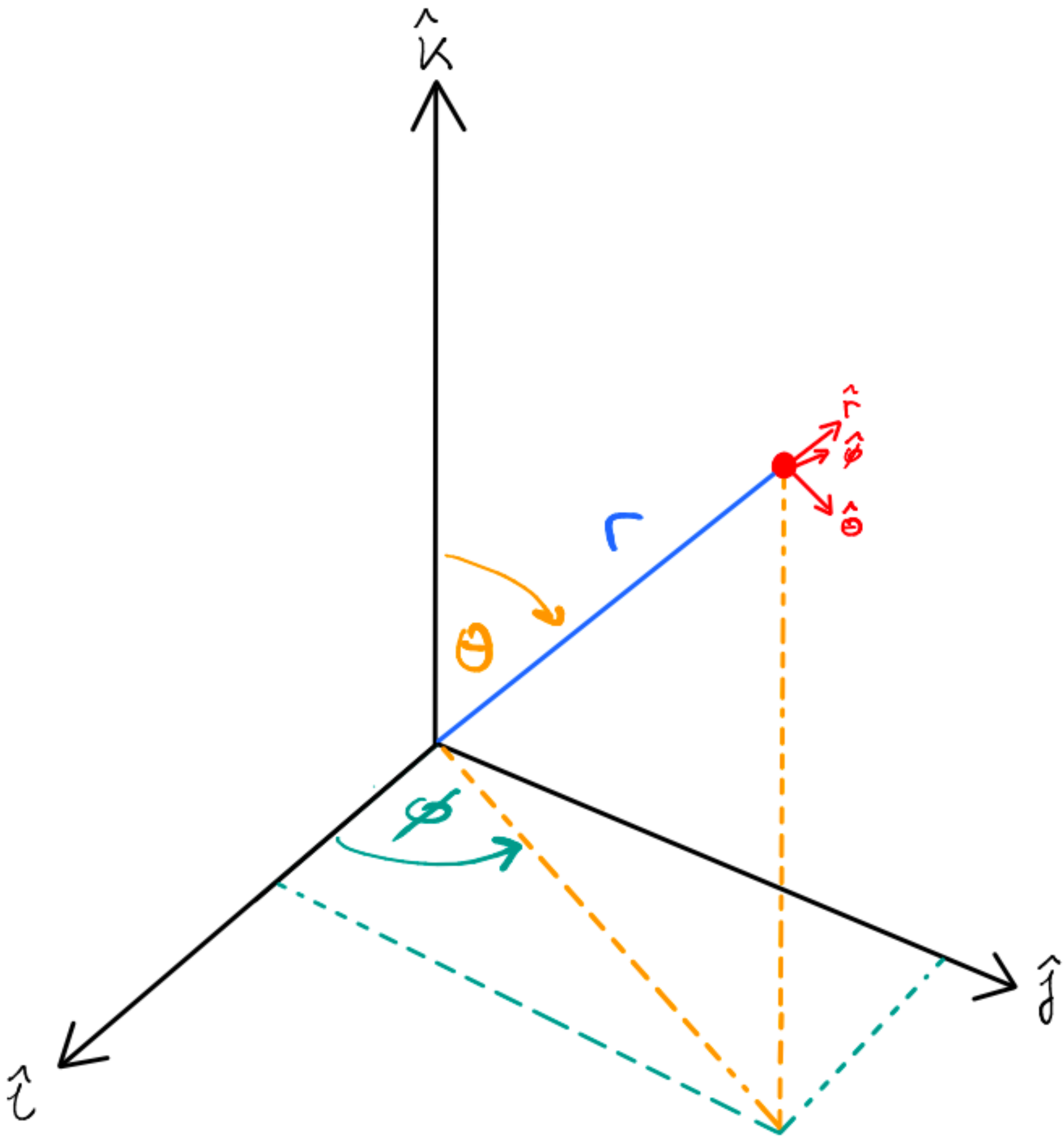
3.3 Diferenciales

Linea	$d\vec{l} = dr\hat{\boldsymbol{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
Superficie	$d\vec{S} = r^2\sin\theta d\theta d\phi\hat{\boldsymbol{r}} + r\sin\theta dr d\phi\hat{\boldsymbol{\theta}} + r dr d\theta\hat{\boldsymbol{\phi}}$
Volumen	$dV = r^2\sin\theta dr d\theta d\phi$

3.4 Operadores

Gradiente:	$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial\psi}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi}\hat{\boldsymbol{\phi}}$
Divergencia:	$\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial(r^2F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta F_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial\phi}$
Rotor:	$\nabla\times\vec{F} = \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta F_\phi)}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right]\hat{\boldsymbol{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial(rF_\phi)}{\partial r}\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial(rF_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$
Laplaciano:	$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$

3.5 Esquema Coordenadas Esféricas



4 Más cálculo vectorial

4.1 Teoremas integrales

Teorema de Gauss:	$\iiint\limits_{\mathcal{V}} \nabla \cdot \vec{F}\, dV = \oiint\limits_{\partial \mathcal{V}} \vec{F} \cdot d\vec{S}$
Teorema de Stokes:	$\iint\limits_{\mathcal{S}} (\nabla \times \vec{F}) \cdot d\vec{S} = \oint\limits_{\partial \mathcal{S}} \vec{F} \cdot d\vec{l}$