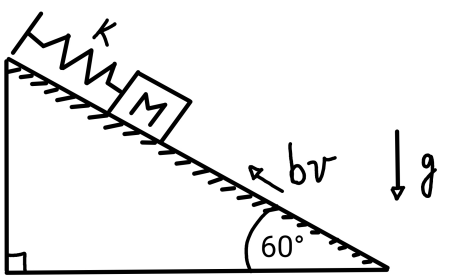
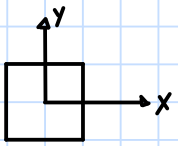
a)

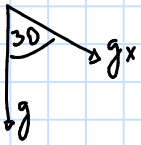


Tomando la masa como punto de referencia



$$F_x = M\ddot{x} = -Kx - b\dot{x} + M g_x$$

g_x?



$$\Rightarrow g_x = g\cos(30) = F$$

$$M\ddot{x} + b\dot{x} + Kx = F, \quad F \text{ cte} \Rightarrow \text{C.V.} \quad z = x - F/K, \ \dot{z} = \dot{x}, \ \ddot{z} = \ddot{x}$$

$$M\ddot{z} + b\dot{z} + Kz = 0 \Rightarrow \ddot{z} + \frac{b}{M}\dot{z} + \frac{K}{M}z = 0$$

Oscilación? No sabemos

Si $z = C\cdot e^{\alpha t} \Rightarrow \ddot{z} + \frac{b}{M}\dot{z} + \frac{K}{M}z = \alpha^2 \cdot C e^{\alpha t} + \alpha\frac{b}{M}\cdot C e^{\alpha t} + \frac{K}{M}\cdot C e^{\alpha t} = 0$

Dividiendo por $Ce^{\alpha t}$

$$\Rightarrow \alpha^2 + \frac{b}{M}\alpha + \frac{K}{M} = 0$$

Resolviendo

$$\Rightarrow \alpha^2 + \frac{b}{M}\alpha + \frac{K}{M} = 0 \quad \text{reescribimos} \quad \frac{b}{2M} = \gamma \quad \text{y} \quad \sqrt{\frac{K}{M}} = w_0$$

$$\alpha^2 + 2\gamma\alpha + w_0^2 = 0 \Rightarrow \alpha = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4w_0^2}}{2} = -\gamma \pm \underbrace{\sqrt{\gamma^2 - w_0^2}}_{w_1}$$

Entonces $z = Ce^{\alpha t} = e^{-\gamma t}\left(A\cdot e^{w_1 t} + B\cdot e^{-w_1 t}\right)$

Para que oscile, debe haber un seno o coseno, entonces w_1 debe ser imaginario ya que

$$e^{i\theta} = \cos(\theta) + i\,\text{sen}(\theta) \Rightarrow \gamma^2 < w_0^2 \Rightarrow \frac{b^2}{4M^2} < \frac{K}{M} \Rightarrow \text{Oscila}$$

b) $U(x) = \frac{1}{2}Kx^2 + Mg_x x$

$\frac{\partial U(x)}{\partial x} = Kx + Mg_x = 0$, equilibrio en $x = \frac{Mg_x}{K}$

$\frac{\partial^2 U(x)}{\partial x^2} = K \Rightarrow w_0 = \sqrt{\frac{K}{M}} \Rightarrow Mg$ no afecta a w_0 ni a w_1.

c) $\gamma^2 = w_0^2 \Rightarrow z = C \cdot e^{-\gamma t} \Rightarrow$

con $C > 0$

$\gamma^2 > w_0^2 \Rightarrow z = e^{-\gamma t}\left(e^{w_1 t} + e^{-w_1 t}\right) = e^{-\gamma t}\left(e^{\sqrt{\gamma^2 - w_0^2}\,t} + e^{-\sqrt{\gamma^2 - w_0^2}\,t}\right)$

Además $\gamma \geq \sqrt{\gamma^2 - w_0^2} \Rightarrow$