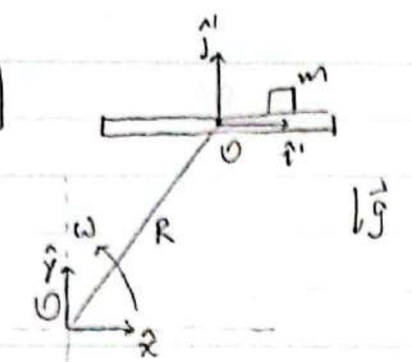
P1



a) Calculamos los términos

① Posición $\vec{r}'$ (Asumimos que la masa no se despega)

$$\vec{r} = x\hat{i}' \Rightarrow \dot{\vec{r}} = \dot{x}\hat{i}'$$
$$\Rightarrow \ddot{\vec{r}} = \ddot{x}\hat{i}'$$

② $\vec{F}_{TOT} = \vec{N} + m\vec{g} = N\hat{j}' - mg\hat{j}'$

③ Posición $\vec{R}$, como el brazo tiene vel. angular cte ω, el ángulo entre $\hat{x}$ y el brazo es ωt

$$\Rightarrow \vec{R} = R(\cos\omega t\,\hat{i}' + \sin\omega t\,\hat{j}')$$
$$\Rightarrow \ddot{\vec{R}} = -R\omega^2(\cos\omega t\,\hat{i}' + \sin\omega t\,\hat{j}')$$

④ Como la plataforma se mantiene horizontal $\Rightarrow$ no rota

$$\Rightarrow \vec{\Omega} = \dot{\vec{\Omega}} = 0$$

Reemplazando en la ec de $m\ddot{\vec{r}}'$

$$\underbrace{m\ddot{x}\hat{i}'}_{m\ddot{\vec{r}}'} = \underbrace{N\hat{j}' - mg\hat{j}'}_{F_{real}} + \underbrace{R\omega^2 m(\cos\omega t\,\hat{i}' + \sin\omega t\,\hat{j}')}_{-m\ddot{\vec{R}}}$$

Por componentes

$\hat{i}'$] $m\ddot{x} = mR\omega^2\cos\omega t \quad \leftarrow$ EOM (1)

$\hat{j}'$] $0 = N - mg + mR\omega^2\sin\omega t \quad$ (2)

b) La distancia max lo obtenemos de $\int(1)dt$

$$\cancel{m}\int_0^t \ddot{x}\,dt = \cancel{m}R\omega^2\int_0^t \cos\omega t\,dt$$

$$\dot{x}(t) - \cancel{\dot{x}(0)} = R\omega^2\sin\omega t\Big|_0^t$$

parte del reposo $\Rightarrow \dot{x}(0)=0$

$$\dot{x}(t) = R\omega\sin\omega t \qquad /\int dt$$

$$x(t) - \cancel{x(0)} = R\cos\omega t\Big|_0^t$$

parte en origen O' $\Rightarrow x(0)=0$

$$x(t) = -R\cos\omega t + R$$
$$= R(1-\cos\omega t)$$

el máximo se tiene en $\cos\omega t = -1$

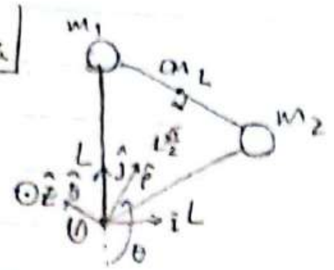
$$\Rightarrow \boxed{x_{max} = 2R}$$

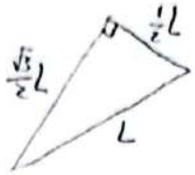
c) ω_{max} para que no despegue $\Rightarrow N>0$

De (2)

$$N = mg - mR\omega^2\sin\omega t > 0$$
$$\Rightarrow g > R\omega^2\sin\omega t \qquad \text{sin() es max en 1}$$
$$\Rightarrow g > R\omega^2$$
$$\Rightarrow \boxed{\omega < \sqrt{\frac{g}{R}}}$$

P2

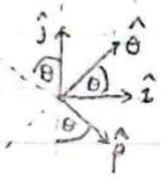




a) Calculando torque c/r a O con CM el CM está entre las masas y la única fuerza externa es $m\vec{g}$.

$$\Rightarrow \vec{r}_{cm} = L\frac{\sqrt{3}}{2}\hat{\rho} \quad \wedge \quad \vec{F}_{TOT} = -2mg\,\hat{j}$$

donde



$$\Rightarrow \hat{j} = (-\cos\theta\,\hat{\rho} + \sin\theta\,\hat{\theta})$$

Tomemos el sistema O–O como si fuese una masa sola

$$\Rightarrow \vec{\tau}_O = \frac{\sqrt{3}}{2}L\,\hat{\rho} \times \overbrace{-2mg(-\cos\theta\,\hat{\rho} + \sin\theta\,\hat{\theta})}$$
$$= -\sqrt{3}\,L\,mg\sin\theta\,\hat{k}$$

Luego, momentum de c/masa

$$\vec{L}_O = \sum_i mL\hat{\rho} \times L\dot{\theta}\hat{\theta} = 2mL^2\dot{\theta}\,\hat{k}$$

$$\Rightarrow \frac{d}{dt}\vec{L}_O = 2L^2\ddot{\theta}\,\hat{k}$$

Igualando $\frac{d}{dt}\vec{L}_O = \vec{\tau}_O$

$$2L^2\ddot{\theta}\,\hat{k} = -\sqrt{3}\,L\,mg\sin\theta\,\hat{k}$$

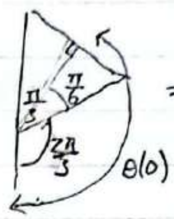
$$\Rightarrow \boxed{\ddot{\theta} = -\frac{\sqrt{3}g}{2L}\sin\theta} \quad / \,\dot{\theta}$$

integrando para tener $\dot{\theta}$

$$\ddot{\theta}\dot{\theta} = -\frac{\sqrt{3}g}{2L}\sin\theta\,\dot{\theta}$$

$$\Leftrightarrow \frac{d}{dt}\left(\frac{1}{2}\dot{\theta}^2\right) = \frac{d}{dt}\left(\frac{\sqrt{3}g}{2L}\cos\theta\right) \Big/ \int dt$$

Como $\dot{\theta}(0) = 0$ y



$$\Rightarrow \theta(0) = \frac{\pi}{6} + \frac{2\pi}{3} = \frac{5\pi}{6}$$

$$\Rightarrow \cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \frac{1}{2}\dot{\theta}^2 = \frac{\sqrt{3}g}{2L}\left(\cos\theta - \left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$\Rightarrow \boxed{\dot{\theta}^2 = \frac{\sqrt{3}g}{L}\left(\cos\theta + \frac{\sqrt{3}}{2}\right)}$$

b) Ahora, $M_{TOT}\ddot{\vec{r}}_{cm} = \vec{F}_{ext}$

$$\ddot{\vec{r}}_{cm} = L\frac{\sqrt{3}}{2}\left(\ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{\rho}\right)$$

$$\vec{F}_{ext} = \vec{F}_{eje} - 2mg\hat{j}$$
$$= \vec{F}_{eje} + 2mg(\cos\theta\,\hat{\rho} - \sin\theta\,\hat{\theta})$$

$$\Rightarrow \cancel{2}m\,L\frac{\sqrt{3}}{\cancel{2}}\left(\ddot{\theta}\hat{\theta} - \dot{\theta}^2\hat{\rho}\right) = \underbrace{\vec{F}_{eje}}_{F_\rho\hat{\rho} + F_\theta\hat{\theta}} + 2mg(\cos\theta\hat{\rho} - \sin\theta\hat{\theta})$$

Por componentes

$$\hat{\rho}]\quad -mL\sqrt{3}\,\dot{\theta}^2 = F_\rho + 2mg\cos\theta \quad (1)$$
$$\hat{\theta}]\quad mL\sqrt{3}\,\ddot{\theta} = F_\theta - 2mg\sin\theta \quad (2)$$

De (1), reemplazamos $\dot{\theta}^2$

$$F_\rho = -mL\sqrt{3}\,\dot{\theta}^2 - 2mg\cos\theta$$
$$= -m\cancel{L}\sqrt{3}\,\frac{\sqrt{3}}{\cancel{L}}g\left(\cos\theta + \frac{\sqrt{3}}{2}\right) - 2mg\cos\theta$$
$$= mg\left(-5\cos\theta - \frac{3\sqrt{3}}{2}\right)$$

P2] Continuamos con (2)

$$F_\theta = mL\sqrt{3}\,\ddot{\theta} + 2mg\sin\theta$$

ya calculamos $\ddot{\theta}$

$$F_\theta = m\cancel{L}\sqrt{3}\left(-\frac{\sqrt{3}g}{2\cancel{L}}\sin\theta\right) + 2mg\sin\theta$$
$$= \frac{1}{2}mg\sin\theta$$

Finalmente

$$\vec{F}_{eje} = F_\rho\,\hat{\rho} + F_\theta\,\hat{\theta}$$
$$\boxed{\vec{F}_{eje} = mg\left[-\left(5\cos\theta + \frac{3\sqrt{3}}{2}\right)\hat{\rho} + \frac{1}{2}\sin\theta\,\hat{\theta}\right]}$$

c) Tenemos que en pto eq. $\ddot{\theta} = 0$, esto en

$$\cancel{\ddot{\theta}}^{\,0} = -\frac{\sqrt{3}g}{2L}\sin\theta \Rightarrow \sin\theta = 0$$
$$\Rightarrow \theta = k\pi,\ k \in \mathbb{Z}$$

El caso estable se ve con $\frac{d}{d\theta}\ddot{\theta}\Big|_{q}$ <0 Est. / >0 No est.

$$\frac{d\ddot{\theta}}{d\theta} = -\frac{\sqrt{3}g}{2L}\cos\theta$$

Consideramos los casos $\theta_{eq} = 0, \pi$

$\underline{\theta_{eq} = 0}$ $\quad \frac{d\ddot{\theta}}{d\theta}(0) = -\frac{\sqrt{3}g}{2L} < 0 \Rightarrow$ estable!

$\underline{\theta_{eq} = \pi}$ $\quad \frac{d\ddot{\theta}}{d\theta}(\pi) = \frac{\sqrt{3}g}{2L} > 0 \Rightarrow$ inestable!

Luego, como $\theta_{eq\,est} = 0$, usamos $\sin\theta \approx \theta$

$$\Rightarrow \ddot{\theta} + \underbrace{\frac{\sqrt{3}g}{2L}}_{\omega_0^2}\theta = 0 \quad \Rightarrow \boxed{\omega_0^2 = \frac{\sqrt{3}g}{2L}}$$

Duda aux ??

* Verifiquemos si cumple $\vec{\tau}_0$ calculado de la forma normal

$$\vec{\tau}_0 = \vec{\tau}_1 + \vec{\tau}_2$$
$$= \left(L\frac{\sqrt{3}}{2}\hat{\rho} + \frac{L}{2}\hat{\theta}\right) \times -mg\left(-\cos\theta\,\hat{\rho} + \sin\theta\,\hat{\theta}\right)$$
$$+ \left(L\frac{\sqrt{3}}{2}\hat{\rho} - \frac{L}{2}\hat{\theta}\right) \times -mg\left(\quad " \quad\right)$$

$$= mg\left(-L\frac{\sqrt{3}}{2}\sin\theta\,\hat{k} + \frac{L}{2}\cos\theta\,\hat{k}\right)$$
$$+ mg\left(L\frac{\sqrt{3}}{2}\sin\theta\,\hat{k} - \frac{L}{2}\cos\theta\,\hat{k}\right)$$

$$= -\cancel{2}mg\,L\frac{\sqrt{3}}{\cancel{2}}\sin\theta\,\hat{k}$$

$$\boxed{\vec{\tau}_0 = -mgL\sqrt{3}\sin\theta\,\hat{k}}$$

↳ Nos dio igual.