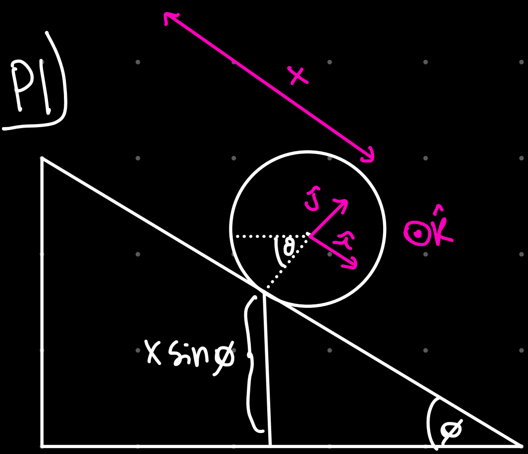
P1)



Una coord. generalizada: x

Partimos definiendo el Lagrangiano:

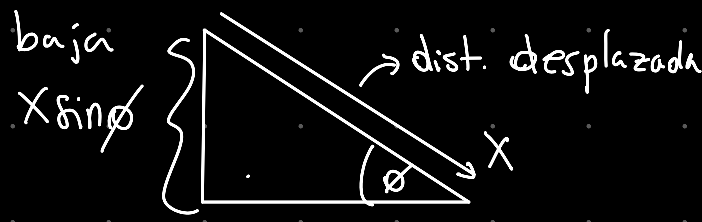
$$\mathcal{L} = K - V$$

$$K = K_{cm} + \underbrace{K_{REL}}_{\text{rotación}}$$

$$K_{cm} = \frac{1}{2} m v_{cm}^2 \ , \ \vec{R}_{cm} = x\,\hat{\imath} \Rightarrow \vec{V}_{cm} = \dot{x}\,\hat{\imath}$$

$$K_{REL} = \frac{1}{2}\vec{\Omega}\cdot(I_{cm}\vec{\Omega}) \ , \ \vec{\Omega} = -\dot{\theta}\hat{k}$$

$$= \frac{1}{2} I \dot{\theta}^2$$



$$\Rightarrow K = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I \dot{\theta}^2$$

Luego $V = V_{mg} = mgh = -mgx\sin\phi$

$$\therefore \mathcal{L} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I\dot{\theta}^2 + mgx\sin\phi$$

Para $\dot{\theta}$; Cond. no resbalamiento

Para cualquier punto p: $\vec{V}_p = \vec{V}_{cm} + \vec{\Omega}\times\vec{CP}$ vetor del centro a $\vec{P}$

Tomando P como el punto de contacto con la sup.

$$\vec{V}_p = 0 = \dot{x}\,\hat{\imath} + (-\dot{\theta}\,\hat{k})\times(-R\,\hat{j}) \Rightarrow \dot{x} = R\dot{\theta}$$

$$\Rightarrow \mathcal{L} = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I\frac{\dot{x}^2}{R^2} + mgx\sin\phi$$

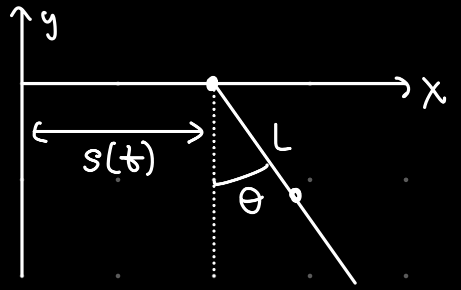
Euler-Lagrange: $\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right) - \frac{\partial\mathcal{L}}{\partial x} = 0$

$$\frac{\partial\mathcal{L}}{\partial\dot{x}} = m\dot{x} + \frac{I^2}{R^2}\dot{x} \Rightarrow \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right) = \left(m + \frac{I}{R^2}\right)\ddot{x}$$

$$\frac{\partial\mathcal{L}}{\partial x} = mg\sin\phi$$

$$\Rightarrow \left(m + \frac{I}{R^2}\right)\ddot{x} - mg\sin\phi = 0 \quad \Rightarrow \ddot{x} = \frac{mg\sin\phi}{m + I/R^2} = g\sin\phi\left(\frac{1}{1 + I/mR^2}\right)$$

P2)



Una coord. generalizada: θ

$$\mathcal{L} = K - V$$

$$K = K_{CM} + K_{REL}$$

$K_{cm} = \frac{1}{2} m v_{cm}^2$, $\vec{r}_{cm} = (s(t) + L\sin\theta)\hat{\imath} + L\cos\theta\,\hat{\jmath}$

$$\vec{v}_{cm} = (\dot{s} + L\dot{\theta}\cos\theta)\hat{\imath} - L\dot{\theta}\sin\theta\,\hat{\jmath}$$

$$\Rightarrow K_{cm} = \frac{1}{2} m \left[\dot{s}^2 + L^2\dot{\theta}^2\cos^2\theta + 2\dot{s}\dot{\theta}L\cos\theta + L^2\dot{\theta}^2\sin^2\theta \right]$$

$$= \frac{1}{2} m \left[\dot{s}^2 + L^2\dot{\theta}^2 + 2\dot{s}\dot{\theta}L\cos\theta \right]$$

$K_{REL} = \frac{1}{2} \vec{\Omega}\cdot(I_{cm}\vec{\Omega})$, $\vec{\Omega} = \dot{\theta}\hat{k}$

$$= \frac{1}{2} I \dot{\theta}^2$$

$$V = V_{mg} = -mgL\cos\theta$$

$$\therefore \mathcal{L} = \frac{1}{2} m \left[\dot{s}^2 + 2\dot{s}\dot{\theta}L\cos\theta \right] + \frac{1}{2}(mL^2 + I)\dot{\theta}^2 + mgL\cos\theta$$

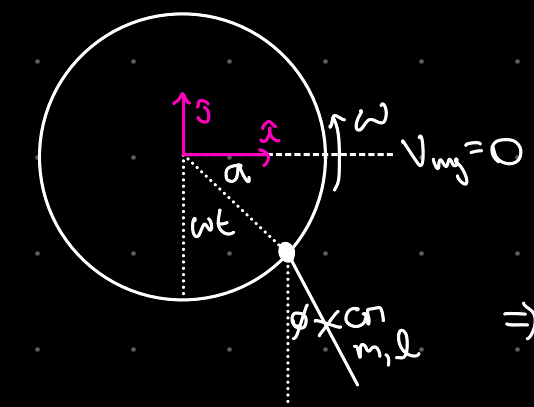
$$\Rightarrow \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = (mL^2 + I)\dot{\theta} + m\dot{s}L\cos\theta$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = (mL^2 + I)\ddot{\theta} + mL\ddot{s}\cos\theta - mL\dot{s}\dot{\theta}\sin\theta$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = -m\dot{s}\dot{\theta}L\sin\theta - mgL\sin\theta$$

Euler Lagrange: $(mL^2 + I)\ddot{\theta} + mL\ddot{s}\cos\theta + mgL\sin\theta = 0$

P3)



Una coordenada generalizada: ϕ

$$K = K_{cm} + K_{REL}$$

$$K_{cm} = \frac{1}{2} m v_{cm}^2 \qquad K_{REL} = \frac{1}{2} \vec{\Omega}\cdot(I\vec{\Omega})$$

$$\vec{R}_{cm} = \left(a\sin(\omega t) + \frac{l}{2}\sin\phi\right)\hat{x} + \left(a\cos(\omega t) + \frac{l}{2}\cos\phi\right)\hat{y}$$

$$\Rightarrow \vec{V}_{cm} = \left(a\omega\cos(\omega t) + \frac{l}{2}\dot{\phi}\cos\phi\right)\hat{x} + \left(-a\omega\sin(\omega t) - \frac{l}{2}\dot{\phi}\sin\phi\right)\hat{y}$$

$$\Rightarrow v_{cm}^2 = a^2\omega^2\cos^2(\omega t) + a\omega l\dot{\phi}\cos(\omega t)\cos\phi + \frac{l^2}{4}\dot{\phi}^2\cos^2\phi + a^2\omega^2\sin^2(\omega t)$$

$$+ a\omega l\dot{\phi}\sin(\omega t)\sin\phi + \frac{l^2}{4}\dot{\phi}^2\sin^2\phi$$

$$= a^2\omega^2 + a\omega l\dot{\phi}\left(\cos(\omega t)\cos\phi + \sin(\omega t)\sin\phi\right) + \frac{l^2}{4}\dot{\phi}^2$$

$$= a^2\omega^2 + a\omega l\dot{\phi}\cos(\omega t - \phi) + \frac{l^2}{4}\dot{\phi}^2$$

$$\vec{\Omega} = \dot{\phi}\hat{k} \Rightarrow K_{REL} = \frac{1}{2} I \dot{\phi}^2$$

$$V = -mg\left(a\cos(\omega t) + \frac{l}{2}\cos\phi\right)$$

Luego: $\mathcal{L} = K - V = \frac{1}{2}m\left[a^2\omega^2 + a\omega l\dot{\phi}\cos(\omega t+\phi) + \frac{l^2\dot{\phi}^2}{4}\right] + \frac{1}{2}I\dot{\phi}^2 + mg\left(a\cos(\omega t) + \frac{l}{2}\cos\phi\right)$

$$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{1}{2}m\left[a\omega l\cos(\omega t+\phi) + \frac{l^2}{2}\dot{\phi}\right] + I\dot{\phi}$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) = \left(\frac{ml^2}{4} + I\right)\ddot{\phi}$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -\frac{1}{2}ma\omega l\dot{\phi}\sin(\omega t+\phi) - \frac{1}{2}mgl\sin\phi$$

$$\Rightarrow \left(\frac{ml^2}{4} + I\right)\ddot{\phi} + \frac{1}{2}ml\left[a\omega\dot{\phi}\sin(\omega t+\phi) + g\sin\phi\right] = 0$$