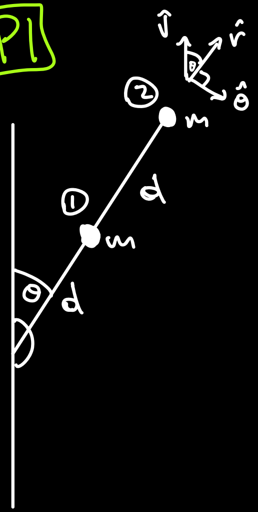
P1



a) Como es una rotación usamos $\vec{L}_0, \vec{\tau}_0$:

$$\vec{L}_0 = \sum_i \vec{L}_{0,i} = \vec{L}_{0,1} + \vec{L}_{0,2}$$

Donde:

$$\vec{L}_{0,1} = m\vec{r}_1 \times \vec{v}_1 = m(d\hat{r}) \times (d\dot{\theta}\hat{\theta}) = md^2\dot{\theta}\hat{k}$$

$$\vec{L}_{0,2} = m\vec{r}_2 \times \vec{v}_2 = m(2d\hat{r}) \times (2d\dot{\theta}\hat{\theta}) = 4md^2\dot{\theta}\hat{k}$$

$$\Rightarrow \vec{L}_0 = 5md^2\dot{\theta}\hat{k}$$

$\vec{\tau}_0 = \sum \vec{r}_i \times \vec{F}$; No es necesario considerar las tensiones pues

$\vec{T} = T\hat{r}$ y $\vec{r} = r\hat{r} \Rightarrow \vec{r} \times \vec{T} = 0$, por ende solo tomamos el peso.

$$m\vec{g} = -mg\hat{\jmath} = -mg(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}) = mg(-\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$\Rightarrow \vec{\tau}_0 = \vec{r}_1 \times m\vec{g} + \vec{r}_2 \times m\vec{g}$$

$$= d\hat{r} \times mg(-\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) + 2d\hat{r} \times mg(-\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$$

$$= mgd\sin\theta\,\hat{k} + 2mgd\sin\theta\,\hat{k} = 3mgd\sin\theta\,\hat{k}$$

Rel. $\vec{\tau}_0$ y $\vec{L}_0$: $\dot{\vec{L}}_0 = \vec{\tau}_0$

$$\therefore 5\cancel{m}d^{\cancel{2}}\ddot{\theta}\,\cancel{\hat{k}} = 3\cancel{m}g\cancel{d}\sin\theta\,\cancel{\hat{k}} \Rightarrow \boxed{\ddot{\theta} = \frac{3g\sin\theta}{5d}}$$

Queremos la velocidad angular ($\dot{\theta}$) $\Rightarrow$ integramos

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\frac{d\theta}{dt} = \dot{\theta}\frac{d\dot{\theta}}{dt} = \frac{3g\sin\theta}{5d} \Rightarrow \dot{\theta}\,d\dot{\theta} = \frac{3g\sin\theta}{5d}\,d\theta \Bigg/ \int_{\cancel{\dot{\theta}(0)}\to 0}^{\dot{\theta}(\theta)}, \int_{\cancel{\theta_0}\to 0}^{\theta}$$

soltado — parte vertical

$$\Rightarrow \int_0^{\dot{\theta}(\theta)} \dot{\theta}\,d\dot{\theta} = \frac{3g}{5d}\int_0^{\theta} \sin\theta \Rightarrow \frac{\dot{\theta}^2(\theta)}{2} = \frac{3g}{5d}(-\cos\theta)\Big|_0^{\theta}$$

$$\Rightarrow \boxed{\dot{\theta}^2(\theta) = \frac{6g}{5d}(1-\cos\theta)}$$

b) Como hay 2 masas usamos cm:

Queremos la fza. sobre el eje $\Rightarrow$ 2da Ley de Newton

$M\vec{A}_{cm} = \sum \vec{F}_{ext}$; $M = m+m = 2m$; $\vec{A}_{cm} = \dot{\vec{V}}_{cm} = \ddot{\vec{R}}_{cm}$

$$\vec{R}_{cm} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{m d\hat{r} + m 2d\hat{r}}{2m} = \frac{3\cancel{m}d\hat{r}}{2\cancel{m}} = \frac{3d}{2}\hat{r}$$

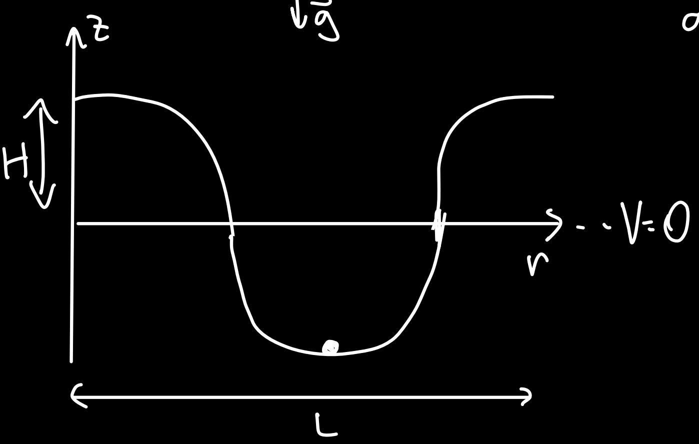
$$\Rightarrow \vec{V}_{cm} = \frac{3d}{2}\dot{\theta}\hat{\theta} \quad \Rightarrow \vec{A}_{cm} = \frac{3d}{2}\ddot{\theta}\hat{\theta} - \frac{3d}{2}\dot{\theta}\hat{r}$$

Luego:

$$\cancel{2}m \cdot \frac{3d}{\cancel{2}}\left(\ddot{\theta}\hat{\theta} - \dot{\theta}\hat{r}\right) = 2m\vec{g} + \vec{F}_{eje} \quad (\vec{T} \text{ es interna})$$

$$\Rightarrow \boxed{\vec{F}_{eje} = -2m\vec{g} + 3d\left(\ddot{\theta}\hat{\theta} - \dot{\theta}\hat{r}\right)} \quad [\text{Todo conocido en polares}]$$

P2 $z = H\cos\left(\frac{2\pi}{L} r\right)$



a) La velocidad en cilíndricas es:

$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{k}$

Imponemos velocidad inicial radial:

$\vec{v}_0 = v_0\hat{r} \quad (\dot{r}(0) = v_0)$

Analisis velocidad final: $\vec{v}_f = \dot{r}_f\hat{r} + r_f\dot{\theta}_f\hat{\theta} + \dot{z}_f\hat{k}$

En la cima, el z es máximo $\Rightarrow \dot{z}_f = 0$

Además, no hay fzas. en $\hat{\theta} \Rightarrow F_\theta = 0 \Rightarrow a_\theta = 0$

$\Rightarrow \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = 0 \Rightarrow r^2\dot{\theta} = \text{cte.}$

Inicialmente: $r(0) = L/2$, $\dot{\theta}(0) = 0 \Rightarrow r^2\dot{\theta} = 0$ donde $r \neq 0$ en gran parte $\Rightarrow \dot{\theta} = 0 \Rightarrow \dot{\theta}_f = 0$

Como llegamos justo a la cima queremos que $\dot{r}_f = 0$ para que no avance y estemos en el caso borde de la mín energía necesaria.

$\Rightarrow V_0 = 0$

La energía se conserva: (f: cima)

$$E_{MT_i} = E_{MT_f} \Rightarrow K_i + V_{mg,i} = \cancel{K_f}^{\,0} + V_{mg,f}$$

$$\Rightarrow \frac{1}{2}\cancel{m}v_0^2 - \cancel{m}gH = \cancel{m}gH \Rightarrow \boxed{v_0^2 = 4gH}$$

b) Ahora la velocidad inicial es: $\vec{v}_0 = v_0\hat{\theta}$

Para la velocidad final: $\vec{v}_f = \dot{r}_f\hat{r} + r_f\dot{\theta}_f\hat{\theta} + \dot{z}_f\hat{k}$

En la cima z es máximo $\Rightarrow \dot{z}_f = 0$

Analogo a a), $\dot{r}_f = 0 \to$ de $F_\theta = 0$

Sin embargo, $r^2\dot{\theta} = cte. \not\Rightarrow \dot{\theta} = 0$ pues $\dot{\theta}(0) = \frac{v_0}{r_0}$ $\quad (r_0 = L/2,\ r_f = L)$

$\Rightarrow$ Se cumple $r_f^2\dot{\theta}_f = r_0^2\dot{\theta}_0 \Leftrightarrow L^2\dot{\theta}_f = v_0 r_0 = v_0\frac{L}{2} \Rightarrow \dot{\theta}_f = \frac{v_0}{2L}$

$\therefore \vec{v}_f = r_f\dot{\theta}_f\hat{\theta} = L\cdot\frac{v_0}{2L}\hat{\theta} = \frac{v_0}{2}\hat{\theta}$

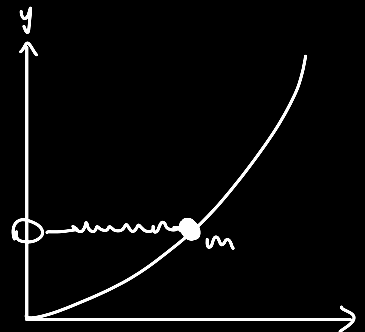
La energía se conserva:

$EMT_f = EMT_i \Rightarrow K_f + V_{mg,f} = K_i + V_{mg,i}$

$\Rightarrow \frac{1}{2}m\frac{v_0^2}{4} + mgH = \frac{1}{2}mv_0^2 - mgH$

$\Rightarrow \frac{3}{8}\cancel{m}v_0^2 = 2\cancel{m}gH \Rightarrow \boxed{v_0^2 = \frac{16}{3}gH}$

P3 $y=\frac{x^2}{2L}$ $\vec{F}(x,y)=Ay\hat{\imath}+Bx\hat{\jmath}=\begin{pmatrix}F_x\\F_y\end{pmatrix}$



a) $\vec{F}$ conservativo $\Leftrightarrow \nabla\times\vec{F}=0$

$$\nabla\times\vec{F}=\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)\hat{k}$$

$$=(B-A)\hat{k}$$

$\therefore$ $\boxed{\vec{F} \text{ conservativo} \Leftrightarrow B=A}$

Luego para $B\neq A$ es no conservativo.

Caso $A=B$: $\vec{F}=A(y\hat{\imath}+x\hat{\jmath})$

El potencial es tq $\vec{F}=-\nabla V_F$

$$\Rightarrow A(y\hat{\imath}+x\hat{\jmath})=-\frac{\partial V_F}{\partial x}\hat{\imath}-\frac{\partial V_F}{\partial y}\hat{\jmath} \Rightarrow Ay=-\frac{\partial V_F}{\partial x} \wedge Ax=-\frac{\partial V_F}{\partial y}$$

$$\Rightarrow -Axy+\text{cte.}=V_F \wedge -Axy+\text{cte.}=V_F$$

$\therefore$ $\boxed{V_F(x,y)=-Axy+\text{cte.}}$ (Se puede tomar $x=0, y=0$ de ref. $\Rightarrow V_{ref}=0$ y $V_F=-Axy$)

b) $W_F=\int\vec{F}\cdot d\vec{r}=\int(Ay\hat{\imath}+Bx\hat{\jmath})\cdot(dx\,\hat{\imath}+dy\,\hat{\jmath})=\int_{x_i}^{x_f}Ay\,dx+\int_{y_i}^{y_f}B\boxed{x\,dy}$

$x\,dy=x\,dy\,\frac{dx}{dx}=x\frac{dy}{dx}dx$
$=x\left(\frac{2x}{2L}\right)dx$
$=\frac{x^2}{L}dx$

$$=\frac{A}{2L}\int_{x_i}^{x_f}x^2dx+\frac{B}{L}\int_{x_i}^{x_f}x^2dx=\left(\frac{A}{2L}+\frac{B}{L}\right)\frac{x^3}{3}\bigg|_{x_i}^{x_f}$$

$$=\frac{1}{3L}\left(\frac{A}{2}+B\right)(x_f^3-x_i^3)$$

c) El potencial total es la suma de potenciales de las fzas. conservativas.

$V_{mg}=mgy$ ($y=0$ de ref.)

$V_{resorte}=\frac{1}{2}k(x-L)^2$

$$\Rightarrow V(x)=\frac{mgx^2}{2L}+\frac{1}{2}k(x-L)^2$$

d) Es un sistema no conservativo:

$$EMT_f - EMT_i = W_{FNC} \quad (*)$$

Partimos a distancia L del eje vertical $\Rightarrow y_i = L$ $(\Rightarrow x_i = \sqrt{2}L)$

$$\Rightarrow EMT_i = K_i + V(x_i) = \frac{1}{2}mv_0^2 + \frac{mg\,\cancel{2}L^{\cancel{2}}}{\cancel{2L}} + \frac{1}{2}k(\sqrt{2}L - L)^2 = \frac{1}{2}mv_0^2 + mgL + \frac{1}{2}kL^2(\sqrt{2}-1)^2$$

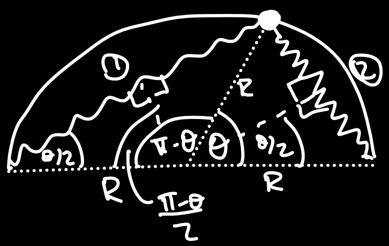
$$EMT_f = \cancelto{0}{K_f} + V(x_f) = \frac{mgx_f^2}{2L} + \frac{1}{2}k(x_f - L)^2$$

$$W_{FNC} = W_F = \frac{1}{3L}\left(\frac{A}{2} - B\right)\left(x_f^3 - \sqrt{2}^3L^3\right)$$

Reemplazando en (*) se despeja x_f

P4

a) 2 Δ isosceles de lado R.



$$V(\theta) = V_{resorte,1} + V_{resorte,2}$$
$$= \frac{1}{2}k(l_1 - R/2)^2 + \frac{1}{2}k(l_2 - R/2)^2$$

donde: $l_1 = 2R\cos(\theta/2)$ $\quad l_2 = 2R\sin(\theta/2)$

$$\Rightarrow V(\theta) = \frac{1}{2}KR^2(2\cos(\theta/2) - 1/2)^2 + \frac{1}{2}KR^2(2\sin(\theta/2) - 1/2)$$
$$= \frac{1}{2}KR^2\left[4\cos^2(\theta/2) - 2\cos(\theta/2) + 1/4 + 4\sin^2(\theta/2) - 2\sin(\theta/2) + 1/4\right]$$
$$= \frac{1}{2}KR^2\left[4 + 1/2 - 2\cos(\theta/2) - 2\sin(\theta/2)\right]$$
$$= \frac{1}{2}KR^2\left[9/2 - 2\cos(\theta/2) - 2\sin(\theta/2)\right]$$

Puntos de equilibrio: $V'(\theta_{eq}) = 0$

$$V'(\theta) = \frac{1}{2}KR^2\left[\cancel{+2}\cdot + \sin(\theta/2)\cdot\cancel{1/2} - \cancel{2}\cos(\theta/2)\cdot\cancel{1/2}\right]$$
$$= \frac{KR^2}{2}\left[\sin(\theta/2) - \cos(\theta/2)\right]$$

Imponiendo: $\sin(\theta_{eq}/2) = \cos(\theta_{eq}/2)$

$$\Rightarrow tg(\theta_{eq}/2) = 1 \Rightarrow \frac{\theta_{eq}}{2} = \frac{\pi}{4} \Rightarrow \boxed{\theta_{eq} = \frac{\pi}{2}}$$

b) La cinetica es: $K = \frac{1}{2}mv^2$, $\vec{r} = R\hat{r} \Rightarrow \vec{v} = R\dot{\theta}\hat{\theta}$

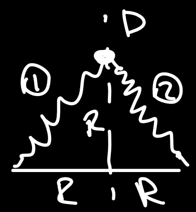
$$\Rightarrow K = \frac{1}{2}\overbrace{mR^2}^{\alpha}\dot{\theta}^2 \quad y \quad \omega_0^2 = \frac{V''(\pi/2)}{\alpha}$$

donde $V''(\theta) = \frac{KR^2}{2}\left[\frac{1}{2}\cos\left(\frac{\theta}{2}\right) + \frac{1}{2}\sin\left(\frac{\theta}{2}\right)\right]$

$$\Rightarrow V''\left(\frac{\pi}{2}\right) = \frac{KR^2}{2}\left[\frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{1}{2}\cdot\frac{1}{\sqrt{2}}\right] = \frac{KR^2}{2\sqrt{2}} > 0 \Rightarrow \text{estable}$$

$$\therefore \omega_0^2 = \frac{K\cancel{R^2}}{2\sqrt{2}}\,\frac{1}{m\cancel{R^2}} = \frac{1}{2\sqrt{2}}\,\frac{K}{m}$$

c) Sist. conservativo $\Rightarrow$ EMT = cte.

D) 

La ext. de los resortes es $\sqrt{R^2+R^2} = \sqrt{2}R$

$$\Rightarrow EMT_D = K_D + V_{1,\text{resorte}} + V_{2,\text{resorte}} = \frac{1}{2}mv_0^2 + 2\cdot\cancel{\tfrac{1}{2}}\cdot K\left(\sqrt{2}R - R/2\right)^2$$

$$= \frac{1}{2}mv_0^2 + KR^2\underbrace{\left(\sqrt{2}-1/2\right)^2}_{(2-\sqrt{2}+1/4)=(9/4-\sqrt{2})}$$

B) 

En B el resorte 2 tiene extensión nula:

$$\Rightarrow EMT_B = \frac{1}{2}m\left(\frac{v_0}{2}\right)^2 + \frac{K}{2}\left(2R-R/2\right)^2 + \frac{K}{2}\left(-R/2\right)^2$$

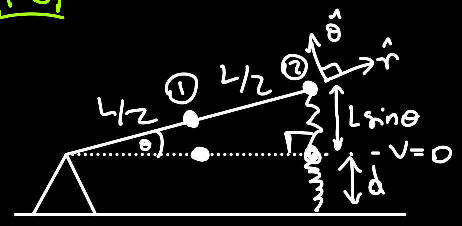
$$= \frac{1}{8}mv_0^2 + \frac{9KR^2}{8} + \frac{KR^2}{8} = \frac{1}{8}mv_0^2 + \frac{5KR^2}{4}$$

Como $EMT_D = EMT_B$

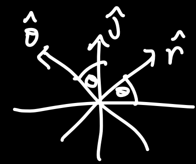
$$\frac{1}{2}mv_0^2 + KR^2\left(9/4-\sqrt{2}\right) = \frac{1}{8}mv_0^2 + \frac{5KR^2}{4}$$

$$\Rightarrow \frac{3}{8}mv_0^2 = KR^2\left(\sqrt{2}-1\right) \Rightarrow \boxed{v_0^2 = \frac{8KR^2(\sqrt{2}-1)}{3m}}$$

P5



$|\theta| << 1$, resorte vertical



a) $\vec{L}_0 = m\frac{L^2}{4}\dot{\theta}\hat{k} + mL^2\dot{\theta}\hat{k} = \frac{5}{4}mL^2\dot{\theta}\hat{k}$

$\vec{mg} = -mg\hat{\jmath} = mg(-\sin\theta\,\hat{r} - \cos\theta\,\hat{\theta})$

$\Rightarrow \vec{\tau}_{0,mg} = \frac{L}{2}\hat{r}\times mg(-\sin\theta\,\hat{r} - \cos\theta\,\hat{\theta}) + L\hat{r}\times mg(-\sin\theta\,\hat{r} - \cos\theta\,\hat{\theta})$

$= -\frac{mgL}{2}\cos\theta\,\hat{k} - mgL\cos\theta\,\hat{k} = -\frac{3mgL}{2}\cos\theta\,\hat{k}$

$\vec{F}_e = K(l_0 - L\sin\theta - d)\hat{\jmath}$ dado que el resorte se comprime al haber masa sobre el resorte $\Rightarrow$ largo $= d + L\sin\theta$

$\Rightarrow \vec{\tau}_{0,Fe} = L\hat{r}\times K(l_0 - L\sin\theta - d)(\sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}) = KL(l_0 - L\sin\theta - d)\cos\theta\,\hat{k}$

Como: $\dot{\vec{L}}_0 = \vec{\tau}_0 \Leftrightarrow \frac{5}{4}mL^{\cancel{2}}\ddot{\theta}\cancel{\hat{k}} = -\frac{3mg\cancel{L}}{2}\cos\theta\cancel{\hat{k}} + K\cancel{L}(l_0 - L\sin\theta - d)\cos\theta\cancel{\hat{k}}$

$\Leftrightarrow \frac{5}{4}mL\ddot{\theta} = -\frac{3mg}{2}\cos\theta + K(l_0 - L\sin\theta - d)\cos\theta$

En el equilibrio: $\ddot{\theta} = 0$

$\Rightarrow K\cos\theta(l_0 - L\sin\theta - d) = \frac{3mg}{2}\cos\theta$

En el equilibrio: $\theta = 0$

$\Rightarrow K(l_0 - d) = \frac{3mg}{2} \Rightarrow \boxed{d = l_0 - \frac{3mg}{2K}}$

b) $d = \frac{l_0}{2} \Rightarrow l_0 = \frac{3mg}{K}$

$EMT = K_1 + K_2 + V_{mg,1} + V_{mg,2} + V_{resorte}$

$K_1 = \frac{1}{2}mL^2\dot{\theta}^2 \qquad K_2 = \frac{1}{2}m\frac{L^2}{4}\dot{\theta}^2 = \frac{1}{8}mL^2\dot{\theta}^2$

$V_{mg,1} = \frac{mgL\sin\theta}{2}$

$V_{mg,2} = mgL\sin\theta$

$V_{resorte} = \frac{1}{2}k\left(d + L\sin\theta - l_0\right)^2 = \frac{1}{2}k\left(L\sin\theta - \frac{l_0}{2}\right)^2$

$\Rightarrow EMT = \frac{5}{8}mL^2\dot{\theta}^2 + \frac{3mgL\sin\theta}{2} + \frac{1}{2}k\left(L\sin\theta - \frac{l_0}{2}\right)^2$

c) $EMT = cte. \;\Big/ \frac{d}{dt}$

$\Rightarrow 0 = \frac{5}{4}mL^2\cancel{\dot{\theta}}\ddot{\theta} + \frac{3mgL\cos\theta\cancel{\dot{\theta}}}{2} + k\left(L\sin\theta - \frac{l_0}{2}\right)L\cos\theta\cancel{\dot{\theta}}$

Como $|\theta| \ll 1 \Rightarrow \sin\theta = \theta \;;\; \cos\theta = 1$

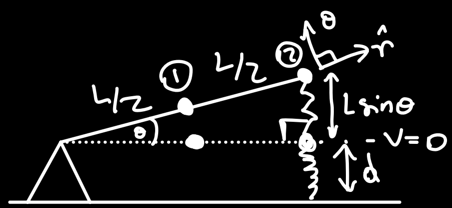
$\Rightarrow 0 = \frac{5}{4}mL^2\ddot{\theta} + \frac{3mgL}{2} + k\left(L\theta - \frac{l_0}{2}\right)L$

$= \frac{5}{4}mL^2\ddot{\theta} + \cancel{\frac{3mgL}{2}} + kL^2\theta - \cancel{\frac{kL}{2}\frac{3mg}{k}}$

$= \frac{5}{4}mL^2\ddot{\theta} + kL^2\theta$

$\Rightarrow \ddot{\theta} + \underbrace{\boxed{\frac{4k}{5m}}}_{= \omega_0^2}\theta = 0$ (oscilador armonico)

Alternativo a):



$V = \frac{3mgL}{2}\sin\theta + \frac{1}{2}k\left(d + L\sin\theta - l_0\right)^2$

$V' = \frac{3mgL}{2}\cos\theta + k\left(d + L\sin\theta - l_0\right)L\cos\theta$

$V' = L\cos\theta\left(\frac{3mg}{2} + k\left(d + L\sin\theta - l_0\right)\right)$

Queremos el eq. en $\theta = 0$: (Cuando esta horizontal)

$$V' = L\left(\frac{3mg}{2} + K(d - l_0)\right)$$

Eq.: $V' = 0 \iff \frac{3mg}{2} = K(l_0 - d)$

$$\iff \boxed{d = l_0 - \frac{3mg}{2K}}$$

∴ Los approach dinámica ($\vec{\tau}_0$ y $\vec{L}_0$) y energía son equivalente