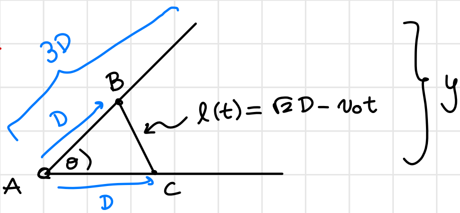
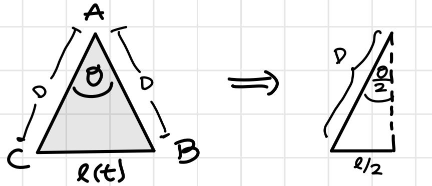
Pauta Aux 4

P1



a) Para esta parte nos piden información sobre $y(t)$ (en qué tiempo es igual a $\frac{3D}{2}$)
Pero $y = 3D \sin\theta$, por lo que, es más cómodo trabajar con $\theta(t)$

Como $\overline{AB} = \overline{AC}$ tenemos un triángulo isósceles



$\Rightarrow$ tenemos $D \sin(\theta/2) = \ell/2$ reemplazo $\ell(t)$

$$2D \sin\left(\frac{\theta(t)}{2}\right) = (\sqrt{2}D - v_0 t) \quad (*)$$

Por otra parte, $y = \frac{3D}{2} \Rightarrow \cancel{3D} \sin\theta = \frac{\cancel{3D}}{2} \Rightarrow \sin\theta = \frac{1}{2}$ y esto se cumple para $\theta = \pi/6$

Luego en el tiempo t^*, $\theta = \pi/6$, reemplazamos en $(*)$

$$\rightarrow 2D \underbrace{\sin\left(\frac{\pi}{6}\frac{1}{2}\right)}_{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \sqrt{2}D - v_0 t^*$$

$$t^* = \frac{1}{v_0}\sqrt{2}D\left(1 - \frac{\cancel{\sqrt{2}}(\sqrt{3}-1)}{2\cancel{\sqrt{2}}}\right)$$

$$= \frac{1}{v_0}\sqrt{2}D\left(1 + \frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\boxed{t^* = \frac{D}{v_0\sqrt{2}}(3 - \sqrt{3})}$$

b) Para esto ocupamos nuevamente $(*)$ y para obtener $\dot{\theta}$ hacemos $\frac{d}{dt}(*)$

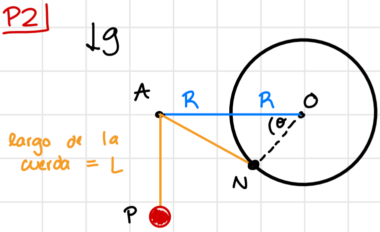
$$\frac{d}{dt} 2D \sin\left(\frac{\theta(t)}{2}\right) = \frac{d}{dt}(\sqrt{2}D - v_0 t)$$

$$\cancel{2}D \cos\left(\frac{\theta(t)}{2}\right)\frac{\dot{\theta}}{\cancel{2}} = -v_0$$

y nos piden la vel. cuando $\theta = \pi/6$

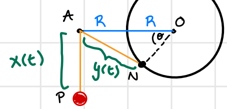
$$D \cos\left(\frac{\pi}{4}\frac{1}{2}\right)\dot{\theta} = -v_0 \Rightarrow \boxed{\dot{\theta} = \frac{-v_0}{D \underbrace{\cos(\pi/12)}_{\frac{1+\sqrt{3}}{2\sqrt{2}}}} = \frac{-v_0 \, 2\sqrt{2}}{D(1+\sqrt{3})}}$$

P2|



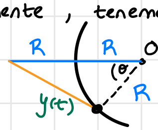
a) $\dot\theta = \omega_0$ constante

nos piden encontrar la rapidez máxima de P (se mueve sólo verticalmente) por lo que, tenemos que parametrizar la posición de P



Sabemos que $x(t)+y(t) = L \quad / \frac{d}{dt}$

$\dot x(t) + \dot y(t) = 0$ $\Longrightarrow$ nos basta encontrar $\dot y(t)$, lo hacemos geométricamente, tenemos un triángulo



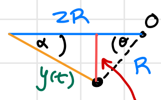
teo coseno

(★) $y^2(t) = (2R)^2 + R^2 - 2(2R)R\cos\theta \quad / \frac{d}{dt}$

$2y\dot y = +4R^2\sin\theta\,\dot\theta$

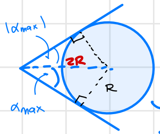
$2y\dot y = 4R^2\sin\theta\,\omega_0$

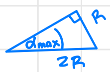
$\dot y = 2R^2\omega_0\frac{\sin\theta}{y}$ → para maximizar $|\dot y|$ hay que max. $\frac{\sin\theta}{y}$ pero notar



$\frac{\boxed{R\sin\theta}}{y} = \frac{\cancel{y}\sin\alpha}{\cancel{y}}$

importante, para maximizar $\sin\alpha$, tenemos que ver cuáles son los α's permitidos y α es máximo cuando la cuerda es tangente al aro





$\alpha_{max} = \sin^{-1}\frac{R}{2R} = \pi/6$

$\Longrightarrow$ $\alpha \in [-\pi/6, \pi/6]$

Por lo que $\max_\theta \frac{\sin\theta}{y} = \frac{1}{R}\max_\alpha \sin\alpha$

$\Longrightarrow$ $\dot y = 2R\omega_0\sin\alpha$ $\Longrightarrow$ $\max|\dot x| = \max|\dot y| = 2R\omega_0 \max_{\alpha\in[-\frac{\pi}{6},\frac{\pi}{6}]}\sin\alpha$

$= 2R\omega_0\sin\pi/6$

$= R\omega_0$

Otra forma: maximicemos $\frac{\sin\theta}{y}$ directamente

$\frac{\sin\theta}{y} \overset{\text{usando } (★)}{=} \frac{\sin\theta}{R\sqrt{5-4\cos\theta}} = f(\theta)$

para encontrar el máximo

$$\frac{d}{d\theta} f(\theta) = \frac{\cos\theta\, R\sqrt{5-4\cos\theta} - \sin\theta\, R \frac{+4\sin\theta}{2\sqrt{5-4\cos\theta}}}{R^2(5-4\cos\theta)} \overset{!}{=} 0 \qquad /\cdot\, R(5-4\cos\theta)$$

$$\cos\theta\sqrt{5-4\cos\theta} - 2\sin^2\theta\,\frac{1}{\sqrt{5-4\cos\theta}} = 0 \qquad \cdot/\sqrt{5-4\cos\theta}$$

$$\cos\theta(5-4\cos\theta) - 2(1-\cos^2\theta) = 0$$

$$-2\cos^2\theta + 5\cos\theta - 2 = 0$$

$$\cos\theta = \frac{-5 \pm \sqrt{25-4\cdot 2\cdot 2}}{4}$$

$$\cos\theta = \frac{-5}{4} \pm \frac{\sqrt{9}}{4} = \frac{-5\pm 3}{4}$$

me quedo con el + porque el − me da $\cos\theta < -1$

$$\Rightarrow \cos\theta = \frac{-2}{4} = -\frac{1}{2} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

$$\max f(\theta) = \frac{\cancel{\sqrt{3}}/2}{R\underbrace{\sqrt{5-4\cdot 1/2}}_{=\cancel{\sqrt{3}}}} = \frac{1}{2R}$$

$$\hookrightarrow |\dot{x}|_{max} = 2R^2\omega_0 \underbrace{\max\frac{\sin\theta}{y}}_{\max f(\theta) = \frac{1}{2R}} = R\omega_0 \quad //$$

b) ahora queremos $\dot{x} = v_0$ cte y encontrar $\dot{\theta}(\theta = \pi/3)$

$\hookrightarrow \dot{y} = -v_0$

tenÍamos la ecuación :

$$2y\dot{y} = +4R^2\sin\theta\,\dot{\theta}$$

$$-y v_0 = 2R^2\sin\theta\,\dot{\theta}$$

usando

$y^2(t) = (2R)^2 + R^2 - 2(2R)R\cos\theta$

$y = R\sqrt{5-4\cos\theta}$

$$\dot{\theta} = -\frac{v_0\cancel{R}\sqrt{5-4\cos\theta}}{2R^{\cancel{2}}\sin\theta}$$

$\cos\pi/3 = \frac{1}{2}$

$\sin\pi/3 = \sqrt{3}/2$

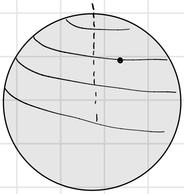
$$\dot{\theta}(\theta=\pi/3) = -\frac{v_0}{\cancel{2}R}\,\frac{\cancel{\sqrt{5-4/2}}}{\cancel{\sqrt{3}}/\cancel{2}}$$

$$= -\frac{v_0}{R}$$

y la aceleración centrípeta es $a_c = \frac{v^2}{R} = R\dot{\theta}^2$

$$= \frac{v_0^2}{R}$$

P3



$$r = R_0 \qquad \underbrace{\phi = N\theta \qquad \dot\theta = \omega_0}_{\dot\phi = N\dot\theta = N\omega_0} \qquad y \quad \ddot\phi = \ddot\theta = 0$$

1. Ocupamos las fórmulas en coords. esféricas

$$\vec v = \cancel{\dot r}^{\,0}\,\hat r + r\dot\theta\,\hat\theta + r\dot\phi\,\sin\theta\,\hat\phi \underset{\dot\theta = \omega_0}{\overset{r = R_0,\ \dot\phi = N\omega_0}{=}} R_0\omega_0\,\hat\theta + R_0 N\omega_0 \sin\theta\,\hat\phi$$

r cte

$$\begin{aligned}\vec a = {} & \left(\cancel{\ddot r}^{\,0} - r\dot\theta^2 - r\dot\phi^2\sin^2\theta\right)\hat r \\ & + \left(r\cancel{\ddot\theta}^{\,0} + 2\cancel{\dot r}^{\,0}\dot\theta - r\dot\phi^2\sin\theta\cos\theta\right)\hat\theta \\ & + \left(r\cancel{\ddot\phi\sin\theta}^{\,0} + 2\cancel{\dot r}^{\,0}\dot\phi\sin\theta + 2r\dot\phi\dot\theta\cos\theta\right)\hat\phi\end{aligned}$$

$\ddot\theta = 0$, $\ddot\phi = 0$

$$\begin{aligned} = {} & -\left(R_0\omega_0^2 + R_0(N\omega_0)^2\sin^2\theta\right)\hat r \\ & - R_0(N\omega_0)^2\sin\theta\cos\theta\ \hat\theta \\ & + 2R_0N\omega_0^2\cos\theta\ \hat\phi\end{aligned}$$

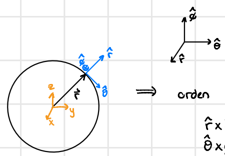
2. $\theta = \pi/2 \implies$ ($\cos\theta = 0$, $\sin\theta = 1$)

$$\vec v = R_0\omega_0\,\hat\theta + R_0N\omega_0\,\hat\phi$$

$$\vec a = -\underbrace{\left(R_0\omega_0^2 + R_0(N\omega_0)^2\right)}_{R_0\omega_0^2(N^2+1)}\hat r$$

$$\implies \underset{|\vec v|^2}{\vec v\cdot\vec v} = (R_0\omega_0)^2\underbrace{\hat\theta\cdot\hat\theta}_{1} + (R_0N\omega_0)^2\underbrace{\hat\phi\cdot\hat\phi}_{1} = R_0^2\omega_0^2(N^2+1) \qquad \hat\theta\cdot\hat\phi = 0$$

$$\implies |\vec v|^3 = \left[R_0^2\omega_0^2(N^2+1)\right]^{3/2}$$



$\implies$ orden $\hat r, \hat\theta, \hat\phi$

$$\hat r\times\hat\theta = \hat\phi \qquad \hat\theta\times\hat r = -\hat\phi$$
$$\hat\theta\times\hat\phi = \hat r \quad y \quad \hat\phi\times\hat\theta = -\hat r$$
$$\hat\phi\times\hat r = \hat\theta \qquad \hat r\times\hat\phi = -\hat\theta$$

$$\vec v\times\vec a = \left[R_0\omega_0\,\hat\theta + R_0N\omega_0\,\hat\phi\right]\times\left[-\left(R_0\omega_0^2(N^2+1)\right)\hat r\right]$$

$$= -R_0^2\omega_0^3(N^2+1)\underbrace{\hat\theta\times\hat r}_{-\hat\phi} - R_0^2N\omega_0^3(N^2+1)\underbrace{\hat\phi\times\hat r}_{\hat\theta}$$

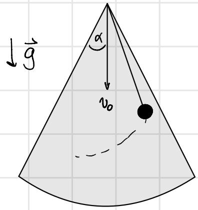
$$= +R_0^2\omega_0^3(N^2+1)\,\hat\phi - R_0^2N\omega_0^3(N^2+1)\,\hat\theta$$

Luego

$$\begin{aligned}|\vec v\times\vec a| &= \sqrt{R_0^4\omega_0^6(N^2+1)^2 + R_0^4N^2\omega_0^6(N^2+1)^2} \\ &= \sqrt{R_0^4\omega_0^6(N^2+1)^2(1+N^2)} \\ &= \left(R_0^4\omega_0^6(N^2+1)^3\right)^{1/2} = R_0^2\omega_0^3(N^2+1)^{3/2}\end{aligned}$$

$$r_v = \frac{\left[R_0^2\omega_0^2(N^2+1)\right]^{3/2}}{R_0^2\omega_0^3(N^2+1)^{3/2}} = \frac{R_0^{\cancel{3}}\,\cancel{\omega_0^3}\,\cancel{(N^2+1)^{3/2}}}{\cancel{R_0^2}\,\cancel{\omega_0^3}\,\cancel{(N^2+1)^{3/2}}} = R_0$$

P4|



Datos : $\theta = \alpha$ (usamos coord. esféricas) ojo: son coord. esféricas pero dadas vuelta, para que $\theta = \alpha$ cte y no $\theta = \pi - \alpha$

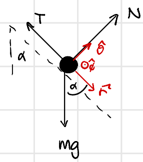
$\dot{\phi}(t=0) = \omega_0$

$r(t=0) = r_0$

Además se recoge la cuerda con velocidad v_0 $\Longrightarrow$ $\dot{r} = -v_0$

$\ddot{r} = 0$

DCL



$\Longrightarrow$ $\vec{F} = N\hat{\theta} + mg(\cos\alpha\,\hat{r} - \sin\alpha\,\hat{\theta})$

$$a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 \operatorname{sen}^2\theta = -r\dot{\phi}^2 \operatorname{sen}^2\alpha$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 \operatorname{sen}\theta\cos\theta = -r\dot{\phi}^2 \operatorname{sen}\alpha\cos\alpha$$

$$a_\phi = \frac{1}{r\operatorname{sen}\theta}\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right) = \frac{\sin^{\cancel{2}}\alpha}{r\cancel{\sin\alpha}}\frac{d}{dt}\left(r^2\dot{\phi}\right)$$

otra forma general de la aceleración en $\hat{\phi}$

ec. de mov

$\hat{r}$: $-mr\dot{\phi}^2 \operatorname{sen}^2\alpha = mg\cos\alpha - T$

$\hat{\theta}$: $-mr\dot{\phi}^2 \operatorname{sen}\alpha\cos\alpha = N - mg\sin\alpha$ $\longrightarrow$ $-mr\dfrac{\ell^2}{r^4}\operatorname{sen}\alpha\cos\alpha = N - mg\sin\alpha$

condición de despegue $N = 0$

$\hat{\phi}$: $\dfrac{m\sin\alpha}{r}\dfrac{d}{dt}\left(r^2\dot{\phi}\right) = 0$ $\longrightarrow$ $r^2\dot{\phi} = cte = \ell$

lo calculo con los valores en $t=0$

$\ell = r(t=0)^2\,\dot{\phi}(t=0)$

$= r_0^2\,\omega_0$

$$+\cancel{m}\frac{\ell^2}{r^3}\cancel{\operatorname{sen}\alpha}\cos\alpha = +\cancel{m}g\cancel{\sin\alpha}$$

$$r^3 = \frac{\ell^2\cos\alpha}{g}$$

$$\boxed{r^3_{despegue} = \frac{r_0^4\omega_0^2}{g}\cos\alpha}$$

Para calcular T reemplazamos todo en la ec. de $\hat{r}$

$$-m\,r_{despegue}\frac{\ell^2}{r^4_{despegue}}\sin^2\alpha = mg\cos\alpha - T$$

$$-m\frac{1}{r_{despegue}^3}r_0^4\omega_0^2\sin^2\alpha = mg\cos\alpha - T$$

$$-m\frac{g}{\cancel{r_0^4\omega_0^2}\cos\alpha}\cancel{r_0^4\omega_0^2}\sin^2\alpha = mg\cos\alpha - T$$

$$T = mg\Big(\underbrace{\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}}_{\frac{\cos^2\alpha + \sin^2\alpha}{\cos\alpha} = \frac{1}{\cos\alpha}}\Big)$$

$$\boxed{T = \frac{mg}{\cos\alpha}}$$