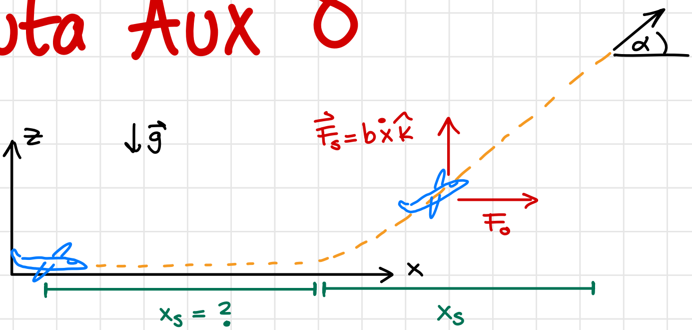
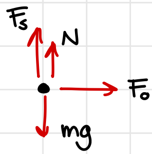
Pauta Aux 8

P1|



a) Mientras está en el suelo : DCL|



la ecuación de mov :

$$\hat{i}| \quad m\ddot{x} = F_0$$
$$\hat{k}| \quad \cancel{m\ddot{z}} = b\dot{x} + N - mg$$

cond. iniciales

$x(t=0) = \dot{x}(t=0) = 0$

y $z(t)=0$ mientras no despega $\rightarrow \dot{z} = \ddot{z} = 0$

$$x(t) = \frac{F_0}{2m}t^2 + At + B$$

C.I.

$$x(t) = \frac{F_0 t^2}{2m}$$

si lo ponemos en la ec. $\hat{k}|$

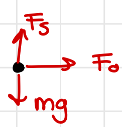
$$0 = \frac{bF_0}{m}t + N - mg$$

Cuando despega $N=0 \implies t = \dfrac{m^2 g}{bF_0}$, así que

$t_{despegue}$

$$x_s = \frac{F_0}{2m}\frac{m^4 g^2}{b^2 F_0^2}$$
$$= \boxed{\frac{m^3 g^2}{2b^2 F_0}}$$

b) Ahora vuela , DCL|



la ec. en $\hat{i}$ es la misma , $\implies x(t) = \dfrac{F_0 t^2}{2m}$ sigue siendo la solución , ¿En qué t^* $x(t^*) = 2x_s$?

y en $\hat{k}|$

$$m\ddot{z} = b\dot{x} - mg \quad \Big/ \int_{t_{despegue}}^{t} dt$$

$$\frac{F_0 t^{*2}}{2m} = \cancel{2}\frac{m^3 g^2}{\cancel{2}b^2 F_0}$$

$$t^* = \left(\frac{2m^4 g^2}{b^2 F_0^2}\right)^{1/2}$$

$$m(\dot{z}(t) - \underbrace{\dot{z}(t_d)}_{=0}) = b(\underbrace{x(t)}_{\frac{F_0 t^2}{2m}} - \underbrace{x(t_d)}_{x_s}) - mg(t - \underbrace{t_d}_{\frac{m^2 g}{bF_0}})$$

$$m\dot{z}(t) = \frac{bF_0 t^2}{2m} - \frac{m^3 g^2}{2b F_0} - mgt + \frac{m^3 g^2}{bF_0}$$

$$\dot{z}(t^*) = \frac{\cancel{bF_0}}{\cancel{2m^2}} \, \frac{\cancel{2}m^{\cancel{4}\,2} g^2}{\cancel{b^2}F_0^{\cancel{2}}} + \frac{m^2g^2}{2bF_0} - g\left(\frac{2m^4g^2}{b^2F_0^2}\right)^{1/2}$$

$$= \frac{m^2g^2}{bF_0} + \frac{m^2g^2}{2bF_0} - \sqrt{2}\,\frac{m^2g^2}{bF_0}$$

$$= \frac{m^2g^2}{bF_0}\left(1 + \frac{1}{2} - \sqrt{2}\right)$$

$$= \frac{m^2g^2}{bF_0}\left(\frac{3-2\sqrt{2}}{2}\right)$$

Tb $\quad \dot{x}(t^*) = \frac{F_0}{m}\left(\frac{2m^4g^2}{b^2F_0^2}\right)^{1/2} = \frac{mg}{b}\sqrt{2}$

luego

$$\alpha = \operatorname{arctg}\left(\frac{\dot{z}(t^*)}{\dot{x}(t^*)}\right) = \operatorname{arctg}\left(\frac{mg}{F_0}\,\frac{(3-2\sqrt{2})}{2\sqrt{2}}\right)$$

El profe

$$\dot{x}^* = \left(\frac{2x_sF_0}{m} + \frac{m^2g^2}{2b^2}\right)^{1/2} = \left(\frac{\cancel{2F_0}}{m}\,\frac{m^3g^2}{\cancel{2}b^2\cancel{F_0}} + \frac{m^2g^2}{2b^2}\right)^{1/2} = \left(\frac{m^2g^2}{b^2}\left(1+\tfrac{1}{2}\right)\right)^{\frac{1}{2}} = \frac{mg}{b}\sqrt{\frac{3}{2}}$$

$$m\dot{y}^* = bx_s - mg\int_{x_s}^{2x_s} \frac{dx}{(A(x-x_s)+B)^{1/2}}$$

$$= bx_s - mg\,\frac{2\sqrt{A(x-x_s)+B}}{A}\Bigg|_{x_s}^{2x_s}$$

$$= bx_s - mg\,\frac{2}{A}\sqrt{Ax_s+B} + \cdots \qquad B = \frac{m^2g^2}{2b^2}$$

$$= bx_s - \frac{2mg}{\frac{2F_0}{m}}\sqrt{\frac{\cancel{2F_0}}{m}\,\frac{m^3g^2}{\cancel{2}b^2\cancel{F_0}} + \frac{m^2g^2}{2b^2}}$$

$$= bx_s - \frac{m^2g}{F_0}\sqrt{m^2g^2/b^2 + m^2g^2/2b^2}$$

$$= \frac{bm^3g^2}{2b^2F_0} - \frac{m^2g}{F_0}\sqrt{\frac{3}{2}m^2g^2/b^2}$$

$$= \frac{m^3g^2}{2bF_0} - \frac{m^3g^2}{bF_0}\,\frac{\sqrt{3}}{2}$$

$$\dot{y}^* = \frac{m^2g^2}{2bF_0}\left(1-\sqrt{3}\right)$$

$$\rightsquigarrow \operatorname{tg}\alpha = \frac{\cancel{\frac{mg}{b}}\,\frac{mg}{F_0}\,\frac{(1-\sqrt{3})}{2}}{\cancel{\frac{mg}{b}}\sqrt{\frac{3}{2}}}$$

$$= \frac{mg}{F_0}\left(\frac{1}{2}\frac{\sqrt{2}}{\sqrt{3}} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{\sqrt{3}}\right)$$

$$= \frac{mg}{F_0}\frac{1}{2\sqrt{2}}\left(\frac{2}{\sqrt{3}} - 2\right)$$

$$\frac{1}{2\sqrt{2}}\left(\frac{2-2\sqrt{3}}{\sqrt{3}}\right)$$

$$m\ddot{x} = F_0$$

$$m\dot{x}\frac{d\dot{x}}{dx} = F_0 \quad \Big/ \int dx$$

$$m\frac{\dot{x}^2}{2}\Bigg|_{x_s}^{x} = F_0\,(x-x_s)$$

$$\dot{x}_s = \frac{F_0}{m}t_s$$

$$m\frac{\dot{x}^2}{2} = F_0(x-x_s) + \frac{m}{2}\left(\frac{\cancel{F_0}}{\cancel{m}}\frac{m^{\cancel{2}}g}{b\cancel{F_0}}\right)^2$$

$$= F_0(x-x_s) + \frac{m}{2}\frac{m^2g^2}{b^2}$$

$$\dot{x}^2 = \frac{2}{m}F_0\,(x-x_s) + \frac{m^2g^2}{b^2}$$

así $B = \frac{m^2g^2}{b^2}$

$$\dot{x}^2 = \frac{m^2g^2}{b^2} + \frac{m^2g^2}{b^2}$$

$$\dot{x} = \sqrt{2}\,\frac{mg}{b}$$

$$m\dot{y}^* = bx_s - mg\int_{x_s}^{2x_s}\frac{dx}{(A(x-x_s)+B)^{1/2}}$$

$$= bx_s - mg\,2\frac{\sqrt{A(x-x_s)+B}}{A}\Bigg|_{x_s}^{2x_s}$$

$$= bx_s - mg\frac{2}{A}\left(\sqrt{Ax_s+B} - \sqrt{B}\right)$$

$B = \frac{m^2g^2}{b^2}$

$$= bx_s - \frac{2mg}{\frac{2F_0}{m}}\left(\sqrt{\frac{\cancel{2F_0}}{m}\frac{m^3g^2}{\cancel{2}b^2\cancel{F_0}} + \frac{m^2g^2}{b^2}} - \frac{mg}{b}\right)$$

$$= bx_s - \frac{m^2g}{F_0}\left(\sqrt{m^2g^2/b^2 + m^2g^2/b^2} - \frac{mg}{b}\right)$$

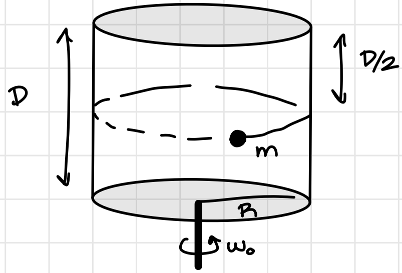
$$= \frac{bm^3g^2}{2b^2F_0} - \frac{m^2g}{F_0}\left(\sqrt{2\,m^2g^2/b^2} - \frac{mg}{b}\right)$$

$$= \frac{m^3g^2}{2bF_0} - \frac{m^3g^2}{bF_0}\left(\sqrt{2}-1\right)$$

$$\dot{y}^* = \frac{m^2g^2}{2bF_0}\left(\underbrace{\frac{1}{2} - \sqrt{2} + 1}_{\frac{3}{2}-\sqrt{2}}\right)$$

y ahí si da lo mismo

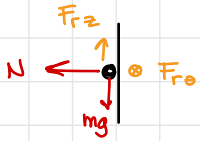
P2|



a) coord. cilíndricas

$\rho = R \;,\; \dot\theta = \omega_o \;,\; z = D/2$ $\quad \ddot\theta = 0$

$\hookrightarrow \dot\rho = \ddot\rho = 0 \qquad \hookrightarrow \dot z = \ddot z = 0$



$\hat\rho \,|\quad m(\ddot\rho - \rho\dot\theta^2) = -mR\omega_o^2 = -N$

$\hat z \,|\quad m\ddot z = 0 = -mg + F_{rz}$

$\hat\theta \,|\quad m(\cancel{\rho\ddot\theta}^{\,0} + 2\cancel{\dot\rho\dot\theta}^{\,0}) = 0 = F_{r\theta}$

luego $\quad \vec F_r = F_{rz}\,\hat z$

$\qquad\quad = mg\,\hat z$

y sabemos que $\quad |\vec F_{rz}| \leq \mu_e N$

$\qquad mg \leq \mu_e\, mR\omega_o^2$

$$\Longrightarrow \boxed{\mu_e \geq \frac{g}{R\omega_o^2}}$$

b) $\quad \rho = R$

$\quad \dot\theta = \omega_1 < \omega_o$

$\quad \dot z(t=0) = v_1$

$\hat\rho \,|\quad mR\omega_1^2 = N$

$\hat z \,|\quad m\ddot z = -mg - \mu_c N \longrightarrow \cancel{m}\ddot z = -\cancel{m}g - \mu_c \cancel{m}R\omega_1^2$

$$\dot z \frac{d\dot z}{dz} = -g - \mu_c R\omega_1^2 \quad \Big/ \int_{D/2}^{D} dz$$

para que llegue justo

$$\int_{v_1}^{0} \dot z\, d\dot z = -(g + \mu_c R\omega_1^2)\, D/2$$

$$-v_1^2/\cancel{2} = -(g + \mu_c R\omega_1^2)\, D/\cancel{2}$$

$\ddot z = -(g + \mu_c R\omega_1^2) \quad \Big/ \int dt$

$\dot z(t) - v_1 = -(g + \mu_c R\omega_1^2)\, t$

si en t^* llega arriba, $\dot z(t^*) = 0 \quad \Longrightarrow \quad t^* = \dfrac{v_1}{g + \mu_c R\omega_1^2}$

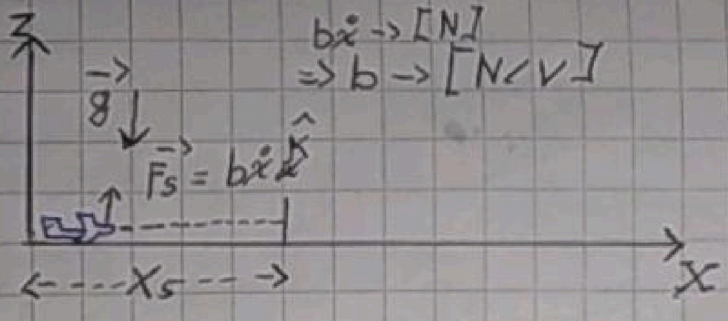
Problema 1

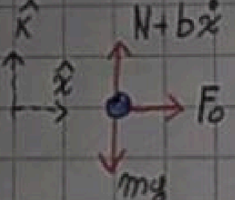
→ Datos importantes del enunciado: $\vec{F_x} = F_0\hat{x}$, $\vec{F_s} = b\dot{x}\hat{y}$, sin roce

a) reposo → $\dot{x}_0 = \dot{y}_0 = 0$

↳ Dimensión de b
$b\dot{x} \rightarrow [N]$
$\Rightarrow b \rightarrow [N/v]$

→ Podemos pensar en el avión como una partícula y analizar las fuerzas que se ejercen sobre el.





$\hat{y})$ $N + b\dot{x} - mg = m\ddot{y}$ (1)

Como el avión se mantiene pegado al suelo durante su despegue, entonces $y = cte \Rightarrow \dot{y} = \ddot{y} = 0$

(1): $N + b\dot{x} - mg = 0 \rightarrow N = mg - b\dot{x}$

$0 = mg - b\dot{x}_s$

$\dot{x}_s = \dfrac{mg}{b}$

Imponemos que despegue del suelo en x_s con $\dot{x}_s$ de velocidad horizontal

Ahora resolvemos para $\hat{x}$)

$$\hat{x})\ F_0 = m\ddot{x} \rightarrow \ddot{x} = F_0/m \rightarrow \dot{x}\frac{d\dot{x}}{dx} = F_0/m \rightarrow \int_0^{\dot{x}_s}\dot{x}\,d\dot{x} = \int_0^{x_s}\frac{F_0}{m}dx$$

esto nos entrega que

$$\dot{x}_s^2 = \frac{2F_0}{m}x_s$$

, despejando x_s y reemplazando $\dot{x}_s$ obtenemos,

$$x_s = \left(\frac{mg}{b}\right)^2\left(\frac{m}{2F_0}\right) = \frac{m^3g^2}{2F_0b^2}\quad \left[\frac{M^3 \cdot N^2}{N\cdot N^2/v^2}\right] \rightarrow \left[\frac{ML^2}{ML/T^2\,T^2}\right] \rightarrow [L]$$

Análisis Dimensional → ✓

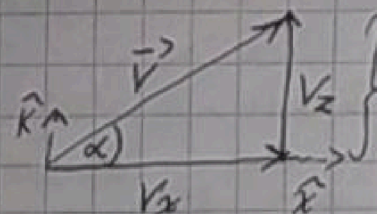
Por lo que obtenemos exactamente el punto de despegue, observamos que la fuerza ejercida tiene un aporte bajo en comparación a la importancia de la masa, la cual es dominante al cubo en esta cantidad x_s.

$$x_s = \left(\frac{g^2}{2b^2}\right)\cdot\frac{m^3}{F_0}$$

↳ constantes ambientales.

Si queremos variar la distancia de despegue cambiamos mucho más variar la masa, que la fuerza ejercida.

b) Para determinar el ángulo α en $2x_s$ debemos recordar que podemos hacer:



$$\tan(\alpha) = \frac{V_z}{V_x} = \frac{\dot{z}_f}{\dot{x}_f}$$

→ Escribimos las ecuaciones de movimiento

(2) $\hat{z}$) $b\dot{x} - mg = m\ddot{z}$; $(N=0)$

(3) $\hat{x}$) $\ddot{x} = F_0/m$, Hacemos las mismas integrales anteriores solo que ahora con $x(x_s \to 2x_s)$; $\dot{x}(\dot{x}_s \to \dot{x}_f)$

$$\int_{\dot{x}_s}^{\dot{x}_f} \dot{x}\,d\dot{x} = \frac{F_0}{m}\int_{x_s}^{2x_s} dx \to \dot{x}_f^2 = \dot{x}_s^2 + \frac{2F_0}{m}x_s \quad \Big| \quad \dot{x}_s = mg/b, \quad x_s = \frac{m^3 g^2}{2F_0 b^2}$$

$$\dot{x}_f^2 = \frac{m^2 g^2}{b^2} + \frac{2F_0}{m}\left(\frac{m^3 g^2}{2F_0 b^2}\right)$$

$$\dot{x}_f^2 = 2\frac{m^2 g^2}{b^2}$$

Luego, para obtener $\dot{z}_f$ usamos (2):

$$m\ddot{z} = b\dot{x} - mg \quad \Big| \quad \ddot{z} = \frac{d\dot{z}}{d\dot{x}}\frac{d\dot{x}}{dt} = \frac{d\dot{z}}{d\dot{x}}\ddot{x} \overset{(3)}{=} \frac{d\dot{z}}{d\dot{x}}\frac{F_0}{m}$$

$$\ddot{z} = b/m\,\dot{x} - g$$

$$\frac{d\dot{z}}{d\dot{x}}\frac{F_0}{m} = \frac{b}{m}\dot{x} - g \quad \Big| \quad \dot{z}(0 \to \dot{z}_f),\ \dot{x}(\dot{x}_s \to \dot{x}_f) \quad , \cdot \frac{m}{F_0}$$

$$\int_0^{\dot{z}_f} d\dot{z} = \frac{b}{F_0}\int_{\dot{x}_s}^{\dot{x}_f} \dot{x}\,d\dot{x} - \frac{mg}{F_0}\int_{\dot{x}_s}^{\dot{x}_f} d\dot{x} = \frac{b}{2F_0}(\dot{x}_f^2 - \dot{x}_s^2) - \frac{mg}{F_0}(\dot{x}_f - \dot{x}_s)$$

$$\dot{z}_f = \frac{b}{2F_0}\left(\frac{m^2 g^2}{b^2}\right) - \frac{mg}{F_0}(\sqrt{2} - 1)\left(\frac{mg}{b}\right)$$

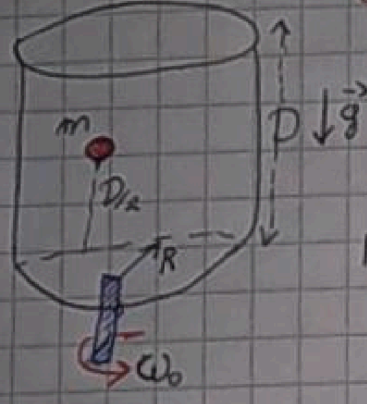
$$= \frac{m^2 g^2}{2F_0 b} - \frac{m^2 g^2}{F_0 b}(\sqrt{2} - 1) = \frac{m^2 g^2}{2F_0 b}\left(\frac{3 - 2\sqrt{2}}{2}\right)$$

Como poseemos $(\dot{x}_f, \dot{z}_f)$, entonces podemos obtener el ángulo por

$$\tan(\alpha) = \frac{\frac{m^2 g^2}{F_0 b}\left(\frac{3 - 2\sqrt{2}}{2}\right)}{\sqrt{2}\,\frac{mg}{b}} = \frac{mg}{bF_0}\left(\frac{3\sqrt{2} - 4}{4}\right)$$

$$\underline{\underline{\alpha}} = \mathrm{Arctan}\left(\frac{mg}{F_0}\left[\frac{3\sqrt{2} - 4}{4}\right]\right)$$

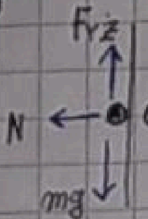
Problema 2



a) Utilizamos cilíndricos, $\ddot{\theta}=0$

$\rho=R$, $\dot{\theta}=\omega_0$, $z=D/2$

$\hookrightarrow \dot{\rho}=\ddot{\rho}=0$ $\quad\hookrightarrow \dot{z}=\ddot{z}=0$



$\hat{\rho}) \; m(\ddot{\rho}-\rho\dot{\theta}^2)=-N \rightarrow -mR\omega_0^2=-N$

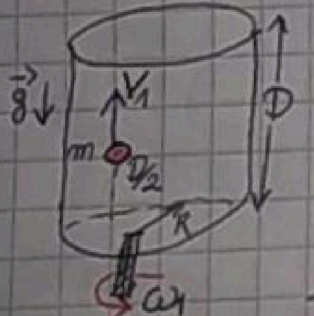
$\hat{\theta}) \; m(\rho\ddot{\theta}+2\dot{\rho}\dot{\theta})=0=F_{r\theta}$

$\hat{z}) \; m\ddot{z}=0=-mg+F_{rz}$

luego $\vec{F_r}=F_{rz}\hat{z}$
$=mg\hat{z}$

y sabemos que $|\vec{F_{rz}}| \leq \mu_e N$
$mg \leq \mu_e m R\omega_0^2$ $\Big\}$ $\mu_e \geq \dfrac{g}{R\omega_0^2}$

b) ahora con $\omega_1<\omega_0$, con coef cinético μ_c.



$\rho=R$

$\dot{\theta}=\omega_1<\omega_0$,

$\dot{z}(t=0)=V_1=\dot{z}_0$

$\hat{\rho}] \; mR\omega_1^2=N$

$\hat{z}] \; m\ddot{z}=-mg-\mu_c N \rightarrow \ddot{z}=-g-\mu_c R\omega_1^2$

$\dot{z}\dfrac{d\dot{z}}{dz}=-g-\mu_c R\omega_1^2 \; \Big/ \int_{D/2}^{D} dz$

$$\rightarrow \int_{V_1}^{0}\dot{z}\,d\dot{z}=-(g+\mu_c R\omega_1^2)\int_{D/2}^{D}dz$$

$$-\frac{V_1^2}{2}=-(g+\mu_c R\omega_1^2)D/2 \Rightarrow V_1^2=D(g+\mu_c R\omega_1^2)$$

c) Para determinar el tiempo usamos

$\ddot{z}=-(g+\mu_c R\omega_1^2) \; \Big/ \int dt$

$\dot{z}(t^*)-V_1=-(g+\mu_c R\omega_1^2)t^*$

si en t^* llega arriba, $\dot{z}(t^*)=0 \Rightarrow t^*=\dfrac{V_1}{g+\mu_c R\omega_1^2}$