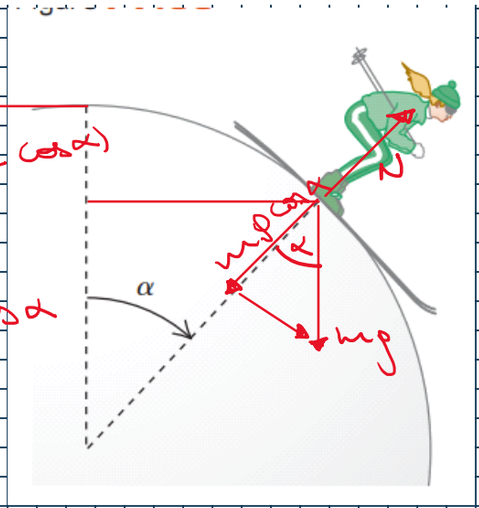


* Conservando la energía:

$$\frac{1}{2}mv^2 - mgh = 0 \Rightarrow v^2 = 2gh = 2gR(1-\cos\alpha)$$

Newton: $\sum F) -m\cdot a_c = N - mg\cos\alpha$

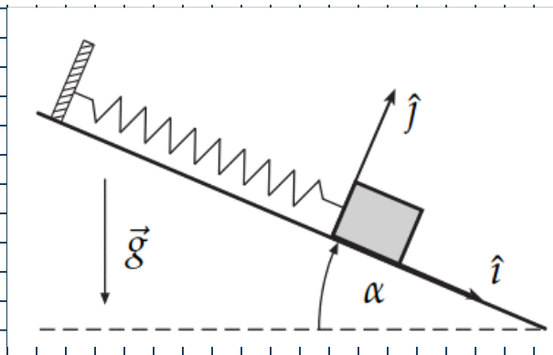
$$\Rightarrow -m\frac{v^2}{R} = N - mg\cos\alpha$$

Que se despeque $\Rightarrow N \overset{!}{=} 0$

$$\Rightarrow -m\,2g(1-\cos\alpha) = -mg\cos\alpha$$

$$\Rightarrow 2\cos\alpha - 2 = -\cos\alpha$$

$$\Rightarrow \boxed{\cos\alpha = \frac{2}{3}}$$



a) $\alpha^* = \arctan(\mu_e)$

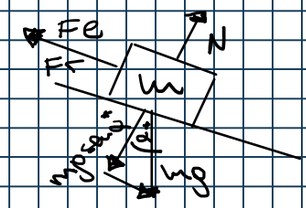
b) Usando el teorema del trabajo:

$\rightarrow W_{fi} = \Delta E = E_f - E_i$

+ calculando:

$\rightarrow W_{fi} = \vec{F} \cdot \vec{d} = \vec{F}_{roce} \cdot x\,\hat{i}$

DCL:



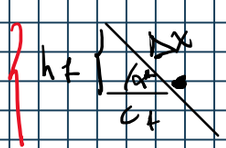
$\rightarrow \vec{F}_r = -\mu_d |N|\,\hat{i}$

$\rightarrow N = mg\cos\alpha^*$

$\Rightarrow W_{fi} = -\mu_d mg\cos\alpha^* \Delta x$

* Ahora, si definimos $y = 0$ en el inicio

$\Rightarrow E_i = 0$; $E_f = \frac{1}{2} k \Delta x^2 - mgh_f$



$\Rightarrow \operatorname{sen}\alpha^* = \frac{h_f}{\Delta x} \Rightarrow h_f = \Delta x \operatorname{sen}\alpha^*$

* Juntando todo:

$E_f - E_i = W_{fi} \Rightarrow \frac{1}{2} k \Delta x^2 - mg\Delta x \operatorname{sen}\alpha^* = -\mu_d mg\cos\alpha^* \Delta x$

+ De a) $\rightarrow \operatorname{sen}\alpha^* = \mu_e \cos\alpha^*$

$\Rightarrow \frac{1}{2} k \Delta x^2 - mg\cos\alpha^* (\mu_e - \mu_d)\Delta x = 0$

$\Rightarrow \left(\frac{1}{2} k \Delta x - mg\cos\alpha^* (\mu_e - \mu_d)\right)\Delta x = 0$

$\Rightarrow \boxed{\Delta x = \frac{2mg\cos\alpha^* (\mu_e - \mu_d)}{k}}$

(c) Muestre que el estiramiento donde la velocidad es máxima está dado por:

$$x(v_{max}) = \frac{mg}{k} cos(\alpha^*)(\mu_e - \mu_d)$$

* La ecuación de Newton para $\hat{N}$ es:

$$\sum \vec{F} = m \cdot a = \vec{F}_r + \vec{F}_e + mg \operatorname{sen}\alpha^*$$

$$\Rightarrow \quad m \cdot a = -\mu_d mg \cos\alpha^* - k\Delta x + mg\mu_e \cos\alpha^*$$

* Para $v_{máx}$ $a = 0$

$$\Rightarrow \quad \boxed{\Delta x^* = \frac{mg\cos\alpha^*(\mu_e - \mu_d)}{k}}$$

d) Para encontrar la velocidad máxima voy que volver a usar energía.

* Sea un tiempo arbitrario

$$\Rightarrow \quad E_f - E_i = W_{fi} = -\mu_d mg \cos\alpha^* \Delta x$$

$E_i = 0$ y $E_f = \frac{1}{2} k \Delta x^2 - mg \operatorname{sen}\alpha^* \Delta x + \frac{1}{2} m v^2$

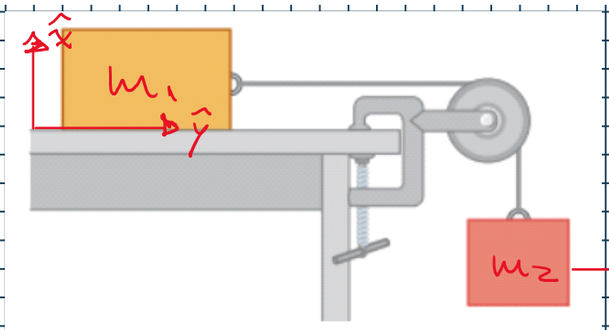
* Ahora, $v_{máx}$ se da en Δx^*

$$\Rightarrow \quad v_{máx}^2 = \frac{2}{m}\left[mg\cos\alpha^* \Delta x^* (\mu_e - \mu_d) - \frac{1}{2} k \Delta x^{*2} \right]$$

$$= \frac{2}{m}\left[\frac{(mg\cos\alpha^*(\mu_e - \mu_d))^2}{k} - \frac{k}{2}\left(\frac{mg\cos\alpha^*(\mu_e - \mu_d)}{k}\right)^2 \right]$$

$$= \frac{2}{m} \cdot \frac{1}{2} \left(\frac{mg\cos\alpha^*(\mu_e - \mu_d)}{k}\right)^2$$

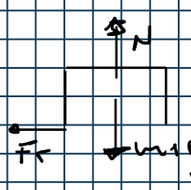
$$\Rightarrow \quad \boxed{v_{máx} = \sqrt{\frac{m}{k}}\, g\cos\alpha^*(\mu_e - \mu_d)}$$



* Utilizando el teorema del trabajo:

$$\rightarrow W_{ti} = E_f - E_i$$

* Del mose t:



$$\Rightarrow N = m_1 g \hat{y}$$

$$\Rightarrow F_r = -\mu_d m_1 g \hat{x}$$

$$\Rightarrow W_{ti} = -\mu_d m_1 g \hat{x} \cdot x\hat{x} = -\mu_d m_1 g x.$$

* Ahora: $E_i = m_1 g H$; $E_f = m_1 g H - m_2 g \Delta y + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$

$$\Rightarrow \frac{1}{2}(m_1 + m_2) v^2 - m_2 g \Delta y = -\mu_d m_1 g x.$$

* Pero como la cuerda es inextensible

$$\rightarrow x = \Delta y.$$

$$\Rightarrow v = \sqrt{\frac{2g\Delta y(m_2 - m_1)}{m_1 + m_2}}$$

b) Ahora, se tiene que:

$$\rightarrow E_i = m_1 g H + \frac{1}{2}(m_1 + m_2) V_0^2$$

$$\rightarrow E_f = m_1 g H - m_2 g \Delta y^*$$

$$\Rightarrow -m_2 g \Delta y^* - \frac{1}{2}(m_1 + m_2) V_0^2 = -\mu_d m_1 g \Delta y^*$$

$$\Rightarrow \mu_d = \frac{m_2 g \Delta y^* - \frac{1}{2}(m_1 + m_2) V_0^2}{m_1 g \Delta y^*}$$

$$\Rightarrow \boxed{\mu_d = \frac{m_2}{m_1} - \frac{1}{2} \frac{(m_1 + m_2) V_0^2}{m_1 g \Delta y^*}}$$