Aux 9

a) $a_0 = 0$, $a_n = 0$

$$b_n = \int_{-1}^{1} x \sin(n\pi x)\,dx = 2\int_0^1 x \sin(n\pi x)\,dx$$

$$= 2\left[x\frac{\cos(n\pi x)}{(-n\pi)}\Big|_0^1 + \cancel{\int_0^1 \frac{\cos(n\pi x)}{n\pi}\,dx} \to 0 \right]$$

$$= \frac{-2}{n\pi}(-1)^n \Rightarrow \boxed{\text{Serie} = \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n\pi}\sin(n\pi x)}$$

b) $a_0 = \int_{-1}^{1} f(x)\,dx = \int_0^2 x\,dx = \frac{x^2}{2}\Big|_0^2 = 2 \Rightarrow \boxed{\frac{a_0}{2} = 1}$

$$a_n = \int_{-1}^{1} f(x)\cos(n\pi x)\,dx = \int_0^2 x\cos(n\pi x)\,dx$$

$$= \cancel{\frac{x\sin(n\pi x)}{n\pi}}^{0}\Big|_0^2 - \int_0^2 \frac{\sin(n\pi x)}{n\pi}\,dx$$

$$= \frac{\cos(n\pi x)}{(n\pi)^2}\Big|_0^2 = \frac{1-1}{(n\pi)^2} = \boxed{0}$$

$$b_n = \int_{-1}^{1} f(x)\sin(n\pi x)$$

$$= \int_0^2 x\sin(n\pi x)\,dx = x\frac{\cos(n\pi x)}{(-n\pi)}\Big|_0^2 - \cancel{\int_0^2 \frac{\cos(n\pi x)}{(-n\pi)}\,dx} \to 0$$

$$= \boxed{\frac{2}{-n\pi}}$$

$$S(x) = 1 + \sum_{N=1}^{\infty} \frac{(-2)}{N\pi} \sin(N\pi x)$$

c) es la a)

d) Completar con extensión par

$$f(x) = \begin{cases} x & x \in [0,1] \\ -x & x \in [-1,0] \end{cases} = |x|$$

$b_N = 0$ pues es par

$$a_0 = \int_{-1}^{1} f(x)\,dx = 2\int_0^1 x\,dx = x^2\Big|_0^1 = 1 \Rightarrow \boxed{\frac{a_0}{2} = \frac{1}{2}}$$

$$a_N = \int_{-1}^{1} f(x) \cos(N\pi x)\,dx = 2\int_0^1 x \cos(N\pi x)\,dx$$

$$= 2\left[\cancel{x \frac{\sin(N\pi x)}{N\pi}\Big|_0^1}^{\,0} - \int \frac{\sin(N\pi x)}{N\pi}\,dx \right]$$

$$= \frac{2\cos(N\pi x)}{(N\pi)^2}\Big|_0^1 = \frac{(-2)}{(N\pi)^2}\left[1 - (-1)^N\right]$$

Con esto lo

$$\text{Serie} = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{(-2)}{(n\pi)^2}\left[1-(-1)^n\right]\cos(n\pi x)$$

P2 $b_n = 0$

$$a_0 = \int_{-1}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{2}{3} \Rightarrow \boxed{\frac{a_0}{2} = \frac{1}{3}}$$

$$a_n = \int_{-1}^{1} x^2 \cos(n\pi x)\,dx = 2\int_0^1 x^2\cos(n\pi x)\,dx$$

$$= 2\left[\cancel{\frac{x^2 \sin(n\pi x)}{n\pi}\Big|_0^1}^{\,0} - \int_0^1 2x\,\frac{\sin(n\pi x)}{n\pi}\,dx\right]$$

$$= \frac{4}{n\pi}\left[\frac{x\cos(n\pi x)}{n\pi}\Big|_0^1 - \cancel{\int_0^1 \frac{\cos(n\pi x)}{n\pi}\,dx}^{\,0}\right]$$

$$= \frac{4}{(n\pi)^2}(-1)^n$$

$$\Rightarrow \text{Serie} = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2\pi^2}\cos(n\pi x)$$

Como x^2 coincide con la serie en $[-1,1]$

entonces evaluando en $\boxed{x=1}$

$$\Rightarrow 1 = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4}{\pi^2} \cdot \boxed{\frac{(-1)^n}{n^2} \cos(n\pi)} \rightarrow 1$$

$$1 = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\Rightarrow \frac{2}{3} \cdot \frac{\pi^2}{4} = \sum_{n=1}^{\infty} \frac{1}{n^2} \Rightarrow \boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}}$$

$\boxed{P3}$ $f(x) = \begin{cases} 1 & x \in [0,L] \\ -1 & x \in [-L,0] \end{cases}$

$$a_0 = a_n = 0, \quad b_n = \int_{-1}^{1} f(x) \sin(n\pi x)\,dx = 2\int_0^1 \sin(n\pi x)\,dx$$

$$= (-2)\frac{\cos(n\pi x)}{n\pi}\Big|_0^1 = \frac{2}{n\pi}(1-(-1)^n)$$

$$= \begin{cases} \frac{4}{(2k+1)\pi} & k \in \mathbb{Z}_{\geq 0} \text{ si } n \text{ impar} \\ 0 & n \text{ par} \end{cases}$$

$$\text{Serie}_f = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin((2k+1)\pi x)$$

b) $\text{Serie}_h = C$

$(g = f + c)$

c) $\text{Serie}_g = \text{Serie}_f + \text{Serie}_2$

$$\Rightarrow \boxed{\text{Serie}_g = 2 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin((2k+1)\pi x)}$$

Las fourier convergen a los puntos continuos salvo en las discontinuidades al promedio.