P1 $u = x^2 - y^2 + 2x \Rightarrow \dfrac{\partial u}{\partial x} = 2x + 2 = \dfrac{\partial v}{\partial y} \Rightarrow v(x,y) = 2xy + 2y + C(x)$

$$\frac{\partial v}{\partial x} = 2y + \frac{\partial C(x)}{\partial x} \quad / \quad \frac{\partial u}{\partial y} = -2y \xRightarrow{\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}} \frac{\partial C(x)}{\partial x} = 0 \Rightarrow C(x) = C$$

Finalmente $f(x,y) = (x^2 - y^2 + 2x) + i(2xy + 2y + C)$

ws $f(i) = 2i \underset{\text{Typo}}{-1}$ $\quad f(\underbrace{0,1}_{0+1i}) = (-1) + i(2 + C) = -1 + 2i + Ci = -1 + 2i$

$$\Rightarrow C = 0 \Rightarrow \boxed{v = 2xy + 2y}$$

P2

$$f(x,y) \begin{cases} \dfrac{x + iy + 1}{x + iy} & \text{si } (x,y) \neq (0,0) \\ 0 & \end{cases}$$

Para $(x,y) \neq (0,0)$

$$f(x,y) = 1 + \frac{1}{x + iy} = 1 + \frac{x - iy}{x^2 + y^2}$$

$$= \underbrace{1 + \frac{x}{x^2 + y^2}}_{u} - i \underbrace{\frac{y}{x^2 + y^2}}_{v}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2 + y^2) - 2x^2}{(x^2 + y^2)^2} = \boxed{\frac{y^2 - x^2}{(x^2 + y^2)^2}}$$

$$\frac{\partial v}{\partial y} = \frac{-(x^2+y^2-2y^2)}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2} = \frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{-2xy}{(x^2+y^2)^2} \quad , \quad \frac{\partial v}{\partial x} = \frac{-(-2xy)}{(x^2+y^2)^2} = \frac{2xy}{(x^2+y^2)^2} \Rightarrow \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Ahora satisface C-R en $\mathbb{C}-\{(0,0)\}$

Pero no es holomorfa

Notemos que · holomorfa $\Rightarrow$ Continua

y $$u(x,y) = \begin{cases} 1+\frac{x}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & \text{si } (x,y)=(0,0) \end{cases}$$

$$\lim_{(x,y)\to(0,0)} u(x,y) = \lim_{(x,y)\to(0,0)} 1+\frac{x}{x^2+y^2} \underset{\substack{\text{Por} \\ \text{unicidad}}}{=} \lim_{\substack{x=0 \\ y=\frac{1}{n}}} 1+\frac{x}{x^2+y^2} = \boxed{1} \neq 0$$

No Continua

Por lo que no es holomorfa en $\mathbb{C}$

(Si lo es en $\mathbb{C}-\{0,0\}$

P3 a) $|a_N| = 1 \Rightarrow R = 1$

b) $a_N = N + 2^N \Rightarrow a_{1,N} = N$ and $a_{2,N} = 2^N$

$$\left.\begin{array}{l} \Rightarrow \sqrt[N]{|a_{1,N}|} = \sqrt[N]{N} \longrightarrow 1 \\ \sqrt[N]{|a_{2,N}|} = 2 \longrightarrow 2 \end{array}\right\} \Rightarrow R = 1/2$$ se toma el más pequeño

c) $a_N = N^2$ No sirve por que

$(z+2)^{2^N} \longrightarrow$ No es N

Idea: N^2

Cambio de variable $\Rightarrow$ $a_K = \begin{cases} \log_2(K)^2 & K = 2^N \\ 0 & \text{Si no} \end{cases}$

$\sqrt[K]{|a_K|} \simeq \sqrt[K]{\log_2(K)^2}$ Como $1 \leq \log_2(K)^2 \leq K^2$ para K grande

$\Rightarrow \limsup \sqrt[K]{|a_K|} = \lim \sqrt[K]{\log_2(K)^2} = 1$

d) $\sum_{N=0}^{\infty} (3+(-1)^N)^N z^N \Rightarrow a_N = \begin{cases} 4^N \\ 2^N \end{cases} \Rightarrow \limsup \sqrt[N]{|a_N|} = \lim \sqrt[N]{4^N} = 4$

$\Rightarrow \boxed{R = 1/4}$

P4 $f(z)=\dfrac{1}{1-z}=\displaystyle\sum_{k=0}^{\infty} z^k$ $\quad\boxed{R=1}$

$$f(z)=\frac{1}{1-z}=\frac{1}{1-(z+1)+1}=\frac{1}{2-(z+1)}=\frac{1}{2}\left(\frac{1}{1-\left(\frac{z+1}{2}\right)}\right)$$

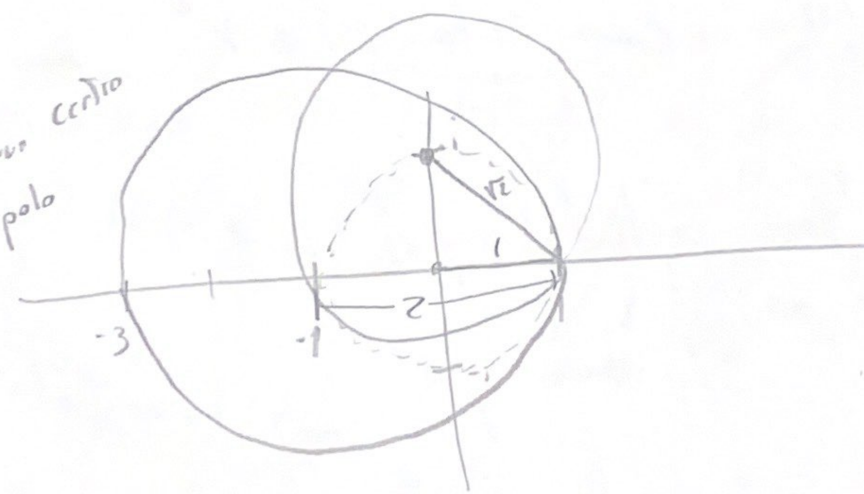
$$=\frac{1}{2}\sum_{k=0}^{\infty}\frac{(z+1)^k}{2^k}=\sum_{k=0}^{\infty}\frac{(z+1)^k}{2^{k+1}} \qquad \boxed{R=2}$$

$$f(z)=\frac{1}{1-z}=\frac{1}{1-(z-i)-i}=\frac{1}{1-i}\left(\frac{1}{1-\frac{(z-i)}{1-i}}\right)$$

$$=\frac{1}{1-i}\sum_{k=0}^{\infty}\frac{(z-i)^k}{(1-i)^k}=\sum_{k=0}^{\infty}\frac{(z-i)^k}{(1-i)^{k+1}} \qquad \boxed{R=\sqrt{2}}$$

Idea

R = distancia centro a polo



P5 $f(z)=\dfrac{z}{z^2+1}$ Como es en Torno $\underline{z_0=1}$ no nos sirve ~~$\dfrac{z}{1-(-z^2)} = z\sum (-1)^k z^{2k}$~~

HAciendo fracciones Parciales

$$\frac{z}{z^2+1} = \frac{1}{i}\left(\frac{1}{z-i} - \frac{1}{z+i}\right) = \frac{i}{i-z} + \frac{-i}{-i-z}$$

$$\frac{1}{i-z} = \frac{1}{i-1-(z-1)} = \left(\frac{1}{i-1}\right)\left(\frac{1}{1-\frac{(z-1)}{i-1}}\right) = \frac{1}{i-1}\left(\sum_{k=0}^{\infty}\frac{(z-1)^k}{(i-1)^k}\right)$$

$$= \sum_{k=0}^{\infty}\frac{(z-1)^k}{(i-1)^{k+1}} \Rightarrow \frac{i}{i-z} = \sum_{k=0}^{\infty}\frac{i}{(i-1)^k}(z-1)^k$$

$$\frac{1}{-i-z} = \frac{1}{-(1+i)-(z-1)} = \frac{1}{-(1+i)}\left(\frac{1}{1-\left(-\frac{(z-1)}{1+i}\right)}\right) = \frac{-1}{1+i}\sum_{k=0}^{\infty}\frac{(-1)^k}{(1+i)^k}(z-1)^k$$

$$= \sum_{k=0}^{\infty}(z-1)^k\left(\frac{(-1)}{(1+i)}\right)^{k+1} \Rightarrow \frac{-i}{-i-z} = \sum_{k=0}^{\infty}\frac{i(-1)^k}{(1+i)^{k+1}}(z-1)^k$$

$$\Rightarrow \boxed{\frac{z}{z^2+1} = \sum_{k=0}^{\infty} i\left[\frac{(-1)^k}{(1+i)^{k+1}} + \frac{1}{(i-1)^{k+1}}\right](z-1)^k}$$

$R=\sqrt{2}$

Se pueden separar y cada una Tiene $\sqrt{2}$