P1 $\frac{\partial r}{\partial u} = (a \operatorname{Senh}(u)\operatorname{Cos}(v), a\cosh(u)\operatorname{Sen}(v), 0)$

$$h_u = \left\|\frac{\partial r}{\partial u}\right\| = \sqrt{a^2\left(\operatorname{Senh}(u)^2\operatorname{Cos}(v)^2 + \operatorname{Cosh}(u)^2\operatorname{Sen}(v)^2\right)}$$

$$= a\sqrt{(-1+\operatorname{Cosh}(u)^2)\operatorname{Cos}(v)^2 + \operatorname{Cosh}(u)^2\operatorname{Sen}(v)^2}$$

$$= a\sqrt{\operatorname{Cosh}(u)^2 - \operatorname{Cos}(v)^2} \qquad \text{¿ Positivo?}$$

$$\frac{\partial r}{\partial v} = (a\operatorname{Cosh}(u)(-\operatorname{Sin}(v)), a\operatorname{Sinh}(u)\operatorname{Cos}(v), 0)$$

$$h_v = \left\|\frac{\partial r}{\partial v}\right\| = \sqrt{a^2\left(\operatorname{Cosh}(u)^2\operatorname{Sin}^2(v) + \operatorname{Sinh}(u)^2\operatorname{Cos}(v)^2\right)} = h_u = a\sqrt{\operatorname{Cosh}(u)^2 - \operatorname{Cos}(v)^2}$$

$$\frac{\partial r}{\partial z} = (0,0,1) \Rightarrow h_z = 1$$

$$\hat{u} = \frac{1}{\sqrt{\operatorname{Cosh}(u)^2 - \operatorname{Cos}(v)^2}}(\operatorname{Senh}(u)\operatorname{Cos}(v), \operatorname{Cosh}(u)\operatorname{Sin}(v), 0)$$

$$\hat{v} = \frac{1}{\sqrt{\operatorname{Cosh}(u)^2 - \operatorname{Cos}(v)^2}}(-\operatorname{Cosh}(u)\operatorname{Sin}(v), \operatorname{Sinh}(u)\operatorname{Cos}(v), 0)$$

$$\hat{z} = (0,0,1)$$

directo que $\hat{u}\cdot\hat{z} = \hat{v}\cdot\hat{z} = 0$

$$\hat{u}\cdot\hat{v} = \frac{1}{\operatorname{Cosh}(u)^2 - \operatorname{Cos}(v)^2}\left[-\operatorname{Cosh}(u)\operatorname{Senh}(u)\operatorname{Cos}(v)\operatorname{Sin}(v) + \operatorname{Cosh}(u)\operatorname{Senh}(u)\operatorname{Cos}(v)\operatorname{Sin}(v)\right]$$

$$= 0$$

$\Rightarrow \hat{u}, \hat{v}, \hat{z}$ son ortogonales

P2 a) $\frac{\partial \phi}{\partial x} = e^x z \cos(y)$

$\Rightarrow \phi(x,y,z) = e^x z \cos(y) + C(y,z)$

Ahora $\frac{\partial \phi}{\partial y} = -e^x z \sin(y) + \frac{\partial C}{\partial y}(y,z) \overset{\downarrow}{=} -e^x z \sin(y)$

$\Rightarrow \frac{\partial C(y,z)}{\partial y} = 0 \Rightarrow C(y,z) = C(z)$

Ahora $\frac{\partial \phi}{\partial z} = e^x \cos(y) + \frac{\partial C}{\partial z} \overset{\downarrow}{=} e^x \cos(y) + 1$

$\Rightarrow \frac{\partial C}{\partial z} = 1 \Rightarrow C(z) = z + C$

$\Rightarrow \phi(x,y,z) = e^x z \cos(y) + z + C$ (Tomemos $C=0$, da lo mismo)

$\boxed{\phi = e^x z \cos(y) + z}$

b) $\int_C G\, d\vec{r} \overset{\nabla\phi = G}{=} \phi(\text{Punto Final}) - \phi(\text{Punto Inicial})$

$= \phi(0,2,0) - \phi(1,0,0)$

$= 0 - 0 = \boxed{0}$

Comentario: Como $\text{rot}(G) = 0$ $(\text{rot}(\nabla\phi) = 0)$ $\Rightarrow$ es conservativo $\Rightarrow$ Da lo mismo el camino

$r(t) = (1-t, 2t, 0) \Rightarrow \frac{dr}{dt} = (-1, 2, 0)$ $\quad G(r(t)) \cdot \frac{dr}{dt} = 0$

$\Rightarrow$ Trabajo $\int_C G\, d\vec{r} = 0$

otra manera de hacerlo

P3 $\quad \mathrm{rot}(F) = (0,0,1)$

Una esfera de radio 3 se parametriza como

$x = 3\cos\theta\sin\phi \quad , \quad y = 3\sin\theta\sin\phi \quad , \quad z = 3\cos\phi$

$$\frac{\partial\varphi}{\partial\phi} = (3\cos\theta\cos\phi \,,\, 3\sin\theta\cos\phi \,,\, -3\sin\phi)$$

$$\frac{\partial\varphi}{\partial\theta} = (-3\sin\theta\sin\phi \,,\, 3\cos\theta\sin\phi \,,\, 0)$$

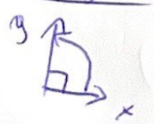
$$\frac{\partial\varphi}{\partial\phi} \times \frac{\partial\varphi}{\partial\theta} = 9\,(\cos\theta\sin\phi^2 \,,\, \sin\theta\sin\phi^2 \,,\, \cos\phi\sin\phi)$$

$$= 9\sin\phi\,(\cos\theta\sin\phi \,,\, \sin\theta\sin\phi \,,\, \cos\phi)$$

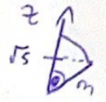
$$= 9\sin\phi \; F$$

Trozo de esfera restringir angulos

$\theta \in [0, \pi/2]$ hace donde



$\phi \in [0, \mathrm{Arcos}(\frac{\sqrt5}{3})]$



$\cos(\mathrm{Max}\,\phi) = \frac{\sqrt5}{3}$

$\mathrm{Max}\,\phi = \mathrm{Arcos}\left(\frac{\sqrt5}{3}\right)$

$$\Rightarrow \int_{C_1 \cup C_2 \cup C_3} F\,d\vec{r} = \int_0^{\frac{\pi}{2}} \int_0^{\mathrm{Arcos}(\frac{\sqrt5}{3})} \begin{pmatrix}0\\0\\1\end{pmatrix} \cdot \begin{pmatrix}\cos\theta\sin\phi\\ \sin\theta\sin\phi\\ \cos\phi\end{pmatrix} 9\sin\phi\, d\phi\, d\theta$$

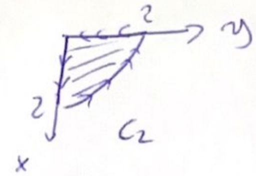
$$= \frac{\pi}{2}\int_0^{\mathrm{Arcos}(\frac{\sqrt5}{3})} 9\cos(\phi)\sin(\phi)\,d\phi = \cancel{\frac{9\pi}{4}\int_0^{\mathrm{Arcos}(\frac{\sqrt5}{3})} \sin(2\phi)\,d\phi = \boxed{\frac{9\pi}{4}\cos\left(2\,\mathrm{Arcos}\left(\frac{\sqrt5}{3}\right)\right)}}$$

$u = \cos(\phi)$
$du = -\sin\phi\, d\phi$

$$= \frac{\pi}{2}\int_1^{\frac{\sqrt5}{3}} 9u\,(-du) = \frac{9\pi}{4}\left(1 - \frac{5}{9}\right) = \boxed{\pi}$$

Opción 2 Solo usar

Bonus



(en verdad se usan los 3 pero los otros normales son $\hat{\imath}$, $\hat{\jmath}$

Por lo que con $rot(F) \cdot \hat{n}$ dan 0)

$$\int_{C_1 \cup C_2 \cup C_3} F \cdot d\vec{r} = \int_0^{\frac{\pi}{2}} \int_0^2 r \, dr \, d\theta = \boxed{\pi}$$

es más fácil, pero difícil que se ocurra

Opción 3 por def Creo que enreda mucho aparte C_1 y C_3 se deberían cancelar

(No recomiendo esta)

$\boxed{P4}$ a) Principalmente

$\ln(1 + \underbrace{x^2 + z^2}_{\geq 0})$ con $1 + x^2 + z^2 > 0$ $\quad \ln(x)$ es C^1 para $x > 0$

$\sqrt{\underbrace{1 + x^2 + y^2}_{> 0}}$ $\quad \sqrt{x}$ es C^1 para $x > 0$

El resto es álgebra de C^1

b)
$$Div(F) = 5 + (-5) + \sqrt{1+x^2+y^2} = \sqrt{1+x^2+y^2}$$

Por Gauss

$$\underbrace{\iint_S \vec{F}\cdot\hat{n}\,dA}_{①} + \underbrace{\iint_{\substack{\text{Tapa}\\ \text{Infer}}} \vec{F}\cdot\hat{n}\,dA}_{②} + \underbrace{\iint_{\substack{\text{Tapa}\\ \text{Superior}}} \vec{F}\cdot\hat{n}\,dA}_{③} = \underbrace{\iiint div(F)\,dV}_{④}$$

$$④ = \int_0^1\int_0^1\int_0^{2\pi} \sqrt{1+r^2}\, r\, d\theta\, dz\, dr = 2\pi \int_0^1 \sqrt{1+r^2}\, r\, dr$$

$u = 1+r^2$
$du = 2r\,dr$

$$= \pi \int_1^2 \sqrt{u}\, du$$

$$= \frac{2\pi}{3} u^{\frac{3}{2}}\Big|_1^2 = \frac{2}{3}\pi\left(\sqrt{8}-1\right)$$

$$② = \int_0^1\int_0^{2\pi} \begin{pmatrix} Algo \\ Algo \\ 0\sqrt{1+r^2}-5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} r\, d\theta\, dr = 5\pi \int_0^1 2r\, dr = \boxed{5\pi}$$

$$③ = \int_0^1\int_0^{2\pi} \begin{pmatrix} Algo \\ Algo \\ \sqrt{1+r^2}-5 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} r\, d\theta\, dr = 2\pi \int_0^1 \sqrt{1+r^2}\, r\, dr - 5\pi \int_0^1 2r\, dr$$

$$= \boxed{\frac{2}{3}\pi\left(\sqrt{8}-1\right) - 5\pi}$$

$$\boxed{① = 0}$$