P1 $\frac{\partial \varphi}{\partial \alpha} = (-\mathrm{Sin}(\alpha)\,\mathrm{Sin}(\beta), \mathrm{Cos}(\alpha)\,\mathrm{Sin}(\beta), 0)$

$\left\|\frac{\partial \varphi}{\partial \alpha}\right\| = \sqrt{\mathrm{Sin}(\beta)^2(\mathrm{Cos}(\alpha)^2 + \mathrm{Sin}(\alpha)^2)} = |\mathrm{Sin}(\beta)| = \mathrm{Sin}(\beta)$, pues $\beta \in [0, \pi]$

$\Rightarrow \boxed{t_\alpha = (-\mathrm{Sin}(\alpha), \mathrm{Cos}(\alpha), 0)}$

$\frac{\partial \varphi}{\partial \beta} = (\mathrm{Cos}(\alpha)\,\mathrm{Cos}(\beta), \mathrm{Sin}(\alpha)\,\mathrm{Cos}(\beta), -\mathrm{Sin}(\beta))$

$\left\|\frac{\partial \varphi}{\partial \beta}\right\| = \sqrt{\mathrm{Cos}^2(\beta)\underbrace{(\mathrm{Cos}^2(\alpha) + \mathrm{Sin}^2(\alpha))}_{1} + \mathrm{Sin}^2(\beta)} = \sqrt{1} = 1$

$\boxed{t_\beta = (\mathrm{Cos}(\alpha)\,\mathrm{Cos}(\beta), \mathrm{Sin}(\alpha)\,\mathrm{Cos}(\beta), -\mathrm{Sin}(\beta))}$

Notando que $\hat{t}_\alpha \cdot \hat{t}_\beta = 0 \Rightarrow$ Son perpendiculares $\Rightarrow$ LI

Tambien se puede notar que

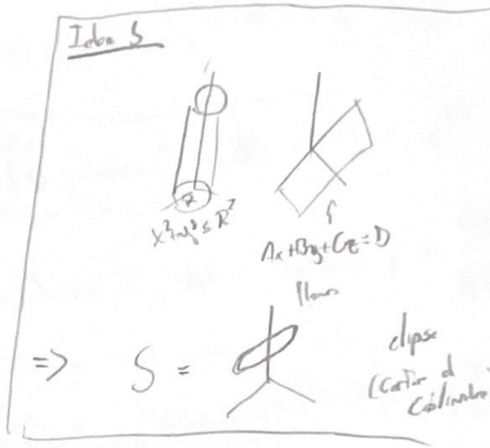
Para $\alpha = 0, \beta = 0$ $\hat{t}_\alpha(0,0) = (0, 1, 0)$, $\hat{t}_\beta(0,0) = (1, 0, 0)$ Son LI

Cualquiera $\Rightarrow$ regular

$$\hat{N} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -\mathrm{Sin}(\alpha) & \mathrm{Cos}(\alpha) & 0 \\ \mathrm{Cos}(\alpha)\mathrm{Cos}(\beta) & \mathrm{Sin}(\alpha)\mathrm{Cos}(\beta) & -\mathrm{Sin}(\beta) \end{vmatrix} = \boxed{(-\mathrm{Cos}(\alpha)\,\mathrm{Sin}(\beta)\,\hat{\imath} - \mathrm{Sin}(\alpha)\,\mathrm{Sin}(\beta)\,\hat{\jmath} - \mathrm{Cos}(\beta)\,\hat{k})}$$

P2| $\varphi(\rho,\theta) = (\rho\cos\theta, \rho\sin(\theta), 4+\rho\cos\theta+\rho\sin\theta)$

$$S = \left\{ \begin{array}{ll} \rho^2 \le 4, & z = 4+\rho\cos(\theta)+\rho\sin(\theta) \\ \rho \in [0,2] & \theta \in [0,2\pi] \end{array} \right\}$$



$$\frac{\partial\varphi}{\partial\rho} = (\cos\theta, \sin\theta, \cos\theta+\sin\theta)$$

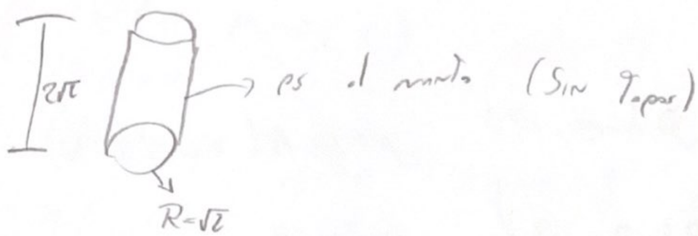
$$\frac{\partial\varphi}{\partial\theta} = (-\rho\sin\theta, \rho\cos\theta, -\rho\sin\theta+\rho\cos\theta)$$

$$\frac{\partial\varphi}{\partial\rho}\times\frac{\partial\varphi}{\partial\theta} = \Big[\rho(-\sin^2\theta + \cancel{\cos\theta\sin\theta} - \cos^2\theta - \cancel{\cos\theta\sin\theta})\,\hat{\imath} \quad = \rho(-1,-1,1)$$
$$\rho(-\cancel{\cos\theta\sin\theta} - \sin^2(\theta) + \cancel{\cos\theta\sin\theta} - \cos^2\theta)\,\hat{\jmath}$$
$$\rho(\cos^2\theta + \sin^2\theta)\,\hat{k}\Big]$$

$$\left\|\frac{\partial\varphi}{\partial\rho}\times\frac{\partial\varphi}{\partial\theta}\right\| = \rho\sqrt{3}$$

$$\int_0^2\int_0^{2\pi} \rho^2(4+\rho\cos\theta+\rho\sin\theta)\rho\sqrt{3}\,d\theta\,d\rho = \int_0^2\int_0^{2\pi}\rho^3 4\cdot\sqrt{3}\,d\theta\,d\rho$$
$$= 2\pi\sqrt{3}\int_0^2 4\rho^3\,d\rho$$
$$= 2\pi\sqrt{3}\,\rho^4\Big|_0^2$$
$$= \boxed{32\pi\sqrt{3}}$$

P3 $x^2+y^2=2 \Rightarrow z^2=2 \Rightarrow z=\pm\sqrt{2}$

$S_1 =$ 

Por def

Opción Cartesianas

$$\varphi(\theta,z)=(\sqrt{2}\cos\theta,\sqrt{2}\sin\theta,z)$$

$$\frac{\partial\varphi}{\partial\theta}=(-\sqrt{2}\sin\theta,\sqrt{2}\cos\theta,0)$$

$$\frac{\partial\varphi}{\partial z}=(0,0,1)$$

$$\frac{\partial\varphi}{\partial\theta}\times\frac{\partial\varphi}{\partial z}=\sqrt{2}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-\sin\theta&\cos\theta&0\\0&0&1\end{vmatrix}=\sqrt{2}(\cos\theta,\sin\theta,0)$$

Opción Cilíndricas

$$\varphi(\theta,z)=\sqrt{2}\,\hat{\rho}+z\hat{k}$$

$$\frac{\partial\varphi}{\partial\theta}=\sqrt{2}\,\hat{\theta},\quad \frac{\partial\varphi}{\partial z}=\hat{k}$$

$$\frac{\partial\varphi}{\partial\theta}\times\frac{\partial\varphi}{\partial z}=\sqrt{2}\,\hat{\rho}$$

Recordar $\frac{\partial\hat{\rho}}{\partial\theta}=\hat{\theta},\quad \frac{\partial\hat{\theta}}{\partial\theta}=-\hat{\rho}$

Yo recomiendo hacer lo siguiente en cartesianas aunque sepan en cilíndricas (Mucho trabajo pasar $\vec{F}$ a $\hat{\rho},\hat{\theta}$)

$$\text{Flujo}=\int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi}\begin{pmatrix}\sqrt{2}\cos\theta-\sqrt{2}\sin\theta\cdot z\\\sqrt{2}\sin\theta+\sqrt{2}\cos\theta\cdot z\\z+4\cos\theta\sin\theta\end{pmatrix}\cdot\begin{pmatrix}\sqrt{2}\cos\theta\\\sqrt{2}\sin\theta\\0\end{pmatrix}d\theta\,dz$$

$$=\int_{-\sqrt{2}}^{\sqrt{2}}\int_0^{2\pi}2\cos^2(\theta)+2\sin^2(\theta)\,d\theta\,dz$$

$$=\boxed{8\pi\sqrt{2}}$$

Por Gauss Hay que cerrar la superficie.

Para esto considerar las tapas superior e inferior

$t_{superior}$: $\rho\hat{\rho}+\sqrt{2}\hat{k} = (\rho\cos\theta, \rho\sin\theta, \sqrt{2})$ $\rho\in[0,\sqrt{2}]$, $\theta\in[0,2\pi]$

$t_{inferior}$: $\rho\hat{\rho}-\sqrt{2}\hat{k} = (\rho\cos\theta, \rho\sin\theta, \sqrt{2})$ $\rho\in[0,\sqrt{2}]$, $\theta\in[0,2\pi]$

Manto: $\sqrt{2}\hat{\rho}+z\hat{k} = (\sqrt{2}\cos\theta, \sqrt{2}\sin\theta, z)$ $\theta\in[0,2\pi]$, $z\in[-\sqrt{2},\sqrt{2}]$

$\mathrm{div}(V) = 1+1+1 = 3$

$$\Rightarrow \iiint \mathrm{div}(V)\,dV = 3\,\text{Volumen (Cilindro)} = 3\pi(2)(2\sqrt{2})$$
$$= 12\pi\sqrt{2}$$

$$\iint_{S_1} V\,d\vec{S} = \iiint \mathrm{div}(V)\,dV - \underbrace{\iint V\cdot d\vec{S}}_{t_{super}} - \underbrace{\int V\cdot d\vec{S}}_{t_{infer}}$$

tapa Super

$$\varphi(\rho,\theta) = \rho\hat{\rho}+\sqrt{2}\hat{k} \Rightarrow \frac{\partial\varphi}{\partial\rho} = \hat{\rho}, \quad \frac{\partial\varphi}{\partial\theta}\rho\hat{\theta} \Rightarrow \frac{\partial\varphi}{\partial\rho}\times\frac{\partial\varphi}{\partial\theta} = \rho\hat{k}$$

$$\Rightarrow \text{Flujo}_{sup} = \int_0^{\sqrt{2}}\int_0^{2\pi}(\sqrt{2}+\underbrace{2\rho^2\cos\theta\sin\theta}_{0\text{ (Argumentar)}})\,\rho\,d\theta\,d\rho = \pi\sqrt{2}\int_0^{\sqrt{2}}2\rho\,d\rho = 2\pi\sqrt{2}$$

$$\Rightarrow \text{Flujo}_{Inf} = \int_0^{\sqrt{2}}\int_0^{2\pi}(-\sqrt{2}+2\rho^2\cos\theta\sin\theta)\underbrace{(-\rho)}_{\text{es exterior}}\,d\theta\,d\rho \quad (\varphi(\rho,\theta)=\rho\hat{\rho}-\sqrt{2}\hat{k}) \Rightarrow \frac{\partial\varphi}{\partial\rho}\times\frac{\partial\varphi}{\partial\theta} = -\rho\hat{k}$$

$$= 2\pi\sqrt{2}$$

$$\Rightarrow \iint_{S_1} V\cdot d\vec{S} = 12\pi\sqrt{2} - 2\pi\sqrt{2} - 2\pi\sqrt{2} = \boxed{8\pi\sqrt{2}}$$

P4 Gauss

$div(F) = y^2 z + x^2 z = (x^2+y^2) z$

$tapa_S = (\rho \cos\theta, \rho \sin\theta, \pi) \quad \rho \in [0, \pi] \quad \theta \in [0, \frac{\pi}{2}], \quad \hat{n} = \hat{k},$
$= \rho\hat{\rho} + \pi\hat{k}$

$tapa_I = (\rho \cos\theta, \rho \sin\theta, 0) \quad \rho \in [0, \pi] \quad \theta \in [0, \pi/2], \quad \hat{n} = -\hat{k},$
$= \rho\hat{\rho}$

Entonces

$Manto = (\pi \cos\theta, \pi \sin\theta, z) \quad z \in [0, \pi] \quad \theta \in [0, \pi/2], \quad \hat{n} = \hat{\rho} = \cos\theta\,\hat{\imath}$
$= \pi\hat{\rho} + z\hat{k}$ $\quad \sin\theta\,\hat{\jmath}$

$div(\vec{F}) = y^2 z + x^2 z = (x^2+y^2) z$

$\psi(r, \theta, z) = r\hat{\rho} + z\hat{k} \quad r \in [0, \pi]$
$\theta \in [0, \pi/2]$
$dV = r\,dr\,d\theta\,dz \quad z \in [0, \pi]$
$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$

$$\iiint div(F)\,dV = \int_0^\pi \int_0^\pi \int_0^{\pi/2} r^2 \cdot z \cdot r\,d\theta\,dz\,dr$$

$$= \frac{\pi}{4} \int_0^\pi r^3 \int_0^\pi 2z\,dz\,dr$$

$$= \frac{\pi^3}{4} \int_0^\pi r^3\,dr$$

$$= \boxed{\frac{\pi^7}{16}}$$

$F = (\ldots, \ , x^2 e^y)$

tapa S

$$\int_0^\pi \int_0^{\frac{\pi}{2}} \rho^2 \cos^2\theta\, e^{\rho \sin\theta} \cdot \rho\,d\theta\,d\rho = \int_0^\pi \rho^3 \cos^2(\theta)\, e^{\rho\sin\theta}\,d\theta\,d\rho$$

tapa I

$$\int_0^\pi \int_0^{\frac{\pi}{2}} \rho^2 \cos^2(\theta)\, e^{\rho\sin\theta} \cdot (-\rho)\,d\theta\,d\rho = -\int_0^\pi \rho^3 \cos^2(\theta)\, e^{\rho\sin\theta}\,d\theta\,d\rho = -\text{Flujo}_S$$

$$\Rightarrow \boxed{\iint F \cdot d\vec{S} = \frac{\pi^7}{16}}$$

Flujo Manto

Propuestos

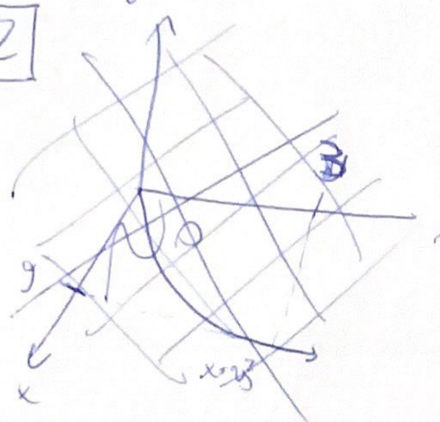
$\boxed{P1}$ $\frac{\partial \varphi}{\partial \gamma} = (\cos\gamma, 0, -\sin\gamma)$ (esto normalizado solo)

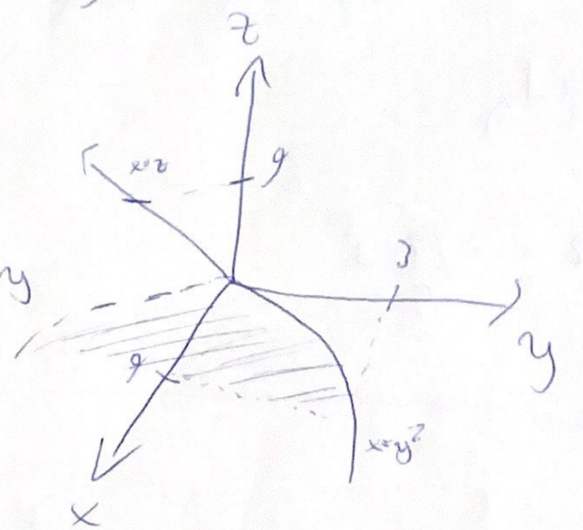
$\frac{\partial \varphi}{\partial \mu} = (0, 1, 0)$ (lo mismo)

$\frac{\partial \varphi}{\partial \gamma} \times \frac{\partial \varphi}{\partial \mu} = (\sin\gamma, 0, \cos\gamma)$ (Dio normalizado)

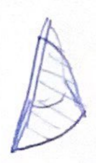
$\Rightarrow \vec{n} = (\sin\gamma, 0, \cos\gamma)$

$\boxed{P2}$



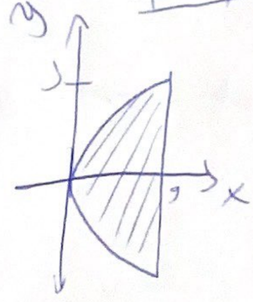


Ideas 3D

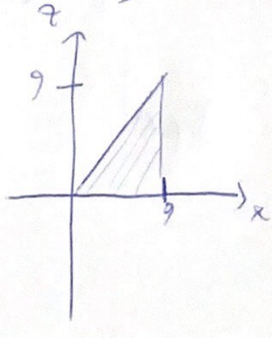


Vistas en los planos

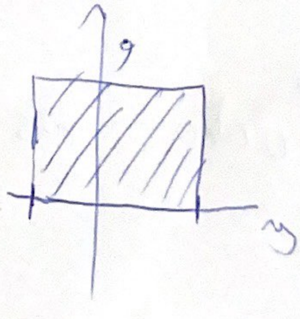
Tapa Inferior



Borde diagonal



Tapa Frontal



$$\mathrm{Div}(F) = 3 + (-2) + 8y$$
$$= 1 + 8y$$

$$W : \{ (x,y,z) \in \mathbb{R}^3 \mid x \in [0,9],\ 0 \leq z \leq x \ ;\ -\sqrt{x} \leq y \leq \sqrt{x} \}$$

$$\text{Flujo} = \int_0^9 \int_0^x \int_{-\sqrt{x}}^{\sqrt{x}} (1+8y)\, dy\, dz\, dx$$

$$= \int_0^9 \int_0^x 2\sqrt{x} + \cancel{4x} - \cancel{4x}\, dz\, dx$$

$$= \int_0^9 2x^{\frac{3}{2}}\, dx$$

$$= \frac{4}{5} x^{\frac{5}{2}} \Big|_0^9$$

$$= \boxed{\frac{4 \cdot 3^5}{5}}$$

Inferior Sor 4

$$\varphi(x,y) = (x,y) \quad x \in [0,9] \quad z=0$$
$$y \in [-\sqrt{x}, \sqrt{x}]$$

$$\text{Flujo tapa inferior} = \int_0^9 \int_{-\sqrt{x}}^{\sqrt{x}} (-8y\underset{0}{z})\, dy\, dx = 0$$

Frontal

$$\varphi(y,z) = (y,z) \quad y \in [-3,3] \quad x=9$$
$$z \in [0,9]$$

$$\text{Flujo Tapa frontal} = \int_0^9 \int_{-3}^{3} (27 - 5y)\, dy\, dz$$

$$= 2 \cdot 3^6 - \cancel{\tfrac{5}{2}\cdot}^{\,0} = 2 \cdot 3^6$$

Diagonal

$$\varphi(x,y) = (x, y, x) \qquad x \in [0,9]$$
$$y \in [-\sqrt{x}, \sqrt{x}]$$

$$\left.\begin{aligned} \frac{\partial \varphi}{\partial x} &= (1,0,1) \\ \frac{\partial \varphi}{\partial y} &= (0,1,0) \end{aligned}\right\} \times \quad \vec{n} = (-1, 0, 1)$$

positivo ✓

$$\text{Flujo diagonal} = \int_0^9 \int_{-\sqrt{x}}^{\sqrt{x}} -3x + 5y + 8yx\, dy\, dx = \int_0^9 -6x^{\frac{3}{2}}\, dx = -\frac{12}{5} x^{\frac{5}{2}} \Big|_0^9$$

$$= -\frac{12}{5} \cdot 3^5$$

Tapa Parábola

$$\varphi(y,z) = (y^2, y, z)$$

$$\frac{\partial\varphi}{\partial y} = (2y, 1, 0)$$

$$\frac{\partial\varphi}{\partial z} (0, 0, 1)$$

$$\vec{n} = (1, -2y, 0)$$

$$\vec{n}_{ext} = (-1, 2y, 0)$$

Apunta en $\hat{x}$ negativo

$y \in [-3, 3]$

$z \in [0, y^2]$

$$\text{Flujo} = \int_{-3}^{3}\int_{0}^{y^2} (3y^2 - 5y)(-1) + (4z - 2y)(2y) \, dz\, dy$$

$$= \int_{-3}^{3}\int_{0}^{y^2} \cancel{5y} - 3y^2 + 8zy - 4y^2 \, dz\, dy$$

$$= \int_{-3}^{3} -7y^4 \, dy$$

$$= -\frac{7}{5} 3^5 \cdot 2 = -\frac{14}{5} \cdot 3^5$$

$$2 \cdot 3^6 - \frac{14}{5} 3^5 - \frac{12}{5} \cdot 3^5 = \left(\frac{30}{5} - \frac{14}{5} - \frac{12}{5}\right) 3^5$$

$$= \boxed{\frac{4}{5} 3^5}$$ lo mismo

Mucho más difícil, Apreciar Gran Gauss