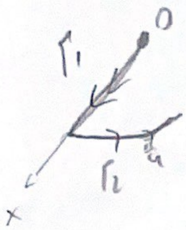
P1 $F = \rho^2 \hat{z} + z\rho\,\hat{\rho}$

Sección 1



$$\Gamma_1 = \Gamma_{r_1}(t) = (t, 0, 0) \qquad t \in [0, R]$$

$$\Gamma_2 = \Gamma_{r_2}(\theta) = (R\cos\theta, R\sin\theta, 0) = R\hat{\rho} \quad , \quad \theta \in [0, \tfrac{\pi}{4}]$$

$$\frac{d\Gamma_{r_1}}{dt} = (1, 0, 0)$$

$$\frac{d\Gamma_{r_2}}{d\theta} = (-R\sin\theta, R\cos\theta, 0) = R\hat{\theta}$$

$\boxed{\Gamma_1}$ $$\int_0^R \begin{pmatrix} 0\cdot\rho\cos\theta \\ 0\cdot\rho\sin\theta \\ \text{Algo} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} dt = 0$$

$\boxed{\Gamma_2}$ $$\int_0^{\frac{\pi}{4}} \underbrace{(R^2\hat{z} + 0\, R\hat{\rho}) \cdot R\hat{\theta}}_{0}\, d\theta = 0$$ (Se puede hacer en cartesianas)

Sección 2

$$\Gamma_3 = \Gamma_{r_3}(z) = (0, 0, H-z) \qquad z \in [0, H]$$

$$\frac{d\Gamma_{r_3}(z)}{dz} = (0, 0, -1)$$

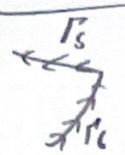
$$\Gamma_4 : \Gamma_{r_4}(z) = (R\cos(\pi/4), R\sin(\pi/4), z) \qquad z \in [0, H]$$

$$\frac{d\Gamma_{r_4}(z)}{dz} = (0, 0, 1)$$

$\boxed{\Gamma_3}$ $$\int_0^H (0\,\hat{z} + 0\,\hat{\rho}) \cdot (-\hat{z})\, dz = 0$$

$$\int_0^H (R^2\hat{z} + z\cdot R\,\hat{\rho}) \cdot (\hat{z})\, dz = \boxed{R^2 H}$$

Sección 3



$\Gamma_5 : r_{\Gamma_5}(t) = (0, R-t, H) \qquad t \in [0, R]$

$$\frac{dr_{\Gamma_5}(t)}{dt} = (0, -1, 0)$$

$\Gamma_6 : r_{\Gamma_6}(\theta) = (R\cos\theta, R\sin\theta, H) \qquad \theta \in [\frac{\pi}{4}, \frac{\pi}{2}]$

$$= R\hat{\rho} + H\hat{z}$$

$$\frac{dr_{\Gamma_6}}{d\theta} = (-R\sin\theta, R\cos\theta, 0)$$

$$= R\hat{\theta}$$

$\boxed{\Gamma_5}$

$$\int_0^R \left[H\cdot(R-t)\cos(\tfrac{\pi}{2})\hat{\imath} + H(R-t)\underbrace{\sin(\tfrac{\pi}{2})}_{1}\hat{\jmath} + (R-t)^2\hat{z}\right]\cdot\left[0\hat{\imath} - \hat{\jmath} + 0\hat{k}\right] dt$$

$$= \int_0^R H(t-R)\,dt = H\left[\frac{(t-R)^2}{2}\right]_0^R = -\frac{HR^2}{2}$$

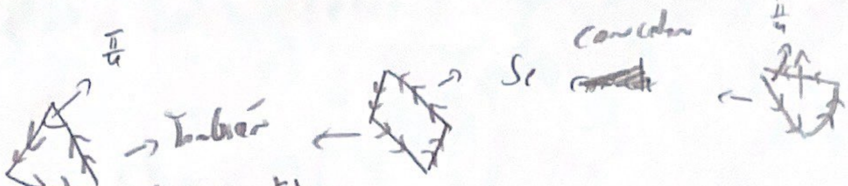
$\boxed{\Gamma_6}$

$$\int_0^R (R^2\hat{z} + HR\hat{\rho})\cdot(R\hat{\theta})\, d\theta = 0$$

$\Rightarrow$ Trabajo es $\boxed{\dfrac{HR^2}{2}}$

b) Con Stokes se usará sobre 3 superficies.

S_1 $\qquad$ S_2 $\qquad$ S_3



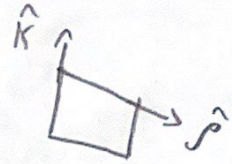
Se cancelan las orillas

Normal en S_1 y S_3 es $\hat{k}$

Calculando el
$$\text{rot}(F) = \frac{1}{\rho}\begin{vmatrix} \hat{\rho} & \rho\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ z\rho & 0 & \rho^2 \end{vmatrix} = -\rho\hat{\theta}$$

$$\Rightarrow \iint_{S_1} \text{rot}(F)\cdot\hat{n}\, dS = \iint_{S_3} \text{rot}(F)\cdot\hat{n}\, dS = 0$$

Notar que



por lo que el normal es $\hat{\theta}$

pero como apunta para atrás es $-\hat{\theta}$

$$\Rightarrow \iint_{S_2} \text{rot}(F)\cdot\hat{n}\, dS = \int_0^R\int_0^H \rho\, dz\, d\rho = H\int_0^R \rho\, d\rho = \frac{HR^2}{2}$$

P2 | a) $rot(F) = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{k}$

$$= (0-0)\hat{\imath} + (\underbrace{-3 + e^y\cos(z) - e^y\cos(z) + 3}_{0})\hat{\jmath} + (2x - 3x^2)\hat{k}$$

b) $\vec{r}$ es simple

$\vec{r}(t_1) = \vec{r}(t_2) \Rightarrow \sin(t_1) = \sin(t_2)$

$\Rightarrow \boxed{t_1 = t_2}$ o $t_1 = \pi - t_2$

Listo

Si $\pi - t_2 = t_1$ y $\cos(t_1) = \cos(t_2)$

$\Rightarrow \cos(t_1) = \cos(\pi - t_1)$

Pero esto es falso para todo t_1, por ejemplo $t_1 = 0$

$\Rightarrow \cos(0) = \cos(\pi)$ falso

$\Rightarrow \boxed{t_1 = t_2}$

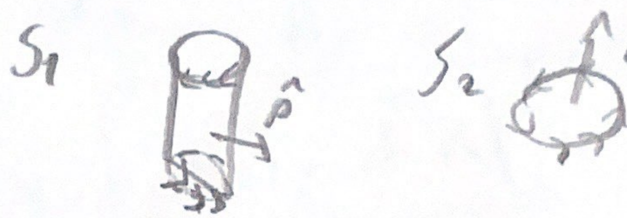
$\vec{r}$ es ~~regular~~ suave : Notar que $\cos(t)$ y $\sin(t)$ son

C^1

Γ regular $\quad \frac{d\vec{r}}{dt} = (-\sin(t), \cos(t), -\sin(t))$

$\left\|\frac{dr}{dt}\right\| = \sqrt{1+\sin^2(t)} \geq 1 > 0$ es regular

Por Stokes



Con esas orientaciones se recorre Γ y se cancela la unión

$$\oint_{\Gamma} \vec{F}\cdot dr = \underbrace{\iint_{S_1} rot(F)\cdot \hat{n}\, dS}_{0} + \iint_{S_2} rot(F)\cdot \hat{n}\, dS$$

pues rot(F)
solo $\hat{k}$

$$= \int_0^1 \int_0^{2\pi} \left[\overbrace{2\rho\cos(\theta)}^{0} - 3\rho^2\cos^2(\theta)\right]\rho\, d\theta\, d\rho$$

$$= \int_0^1 \left(-\frac{3}{2}\right)\rho^3\left(1+\overbrace{\cos(2\theta)}^{0}\right) d\theta\, d\rho = \left(-\frac{3}{2}\right)\int_0^1 (2\pi)\rho^3\, d\rho$$

$$= \boxed{\left(-\frac{3}{4}\right)\pi}$$

P3 a) $rot(F) = \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{k}$

el normal al plano XY es $\hat{k}$

Por Stokes

$$\int_{\partial S} F \cdot d\vec{r} = \boxed{\int F_1\,dx + F_2\,dy}$$
$$= \int rot(F) \cdot \hat{k}\,dS$$
$$= \boxed{\iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dS}$$

Con lo que se obtiene Green

b) Sea $F(x,y) = (-y, x) \Rightarrow \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} = 2$

$\Rightarrow 2\,Area(S) = \int_{\partial S} x\,dy - y\,dx$

c) Propuesta verificar hipótesis: Usando b)

$dx = 3a\cos^2(t)(-\sin(t))\,dt$
$= -3a\cos^2(t)\sin(t)\,dt$

$dy = 3a\sin^2(t)\cos(t)\,dt$
$= 3a\sin^2(t)\cos(t)\,dt$

$Area(S) = \frac{1}{2}\int_0^{2\pi} a\cos^3(t)\,dy - a\sin^3(t)\,dx$

b) Si uno quisiera usar stokes $rot(F) = 0$.

el problema es que F no es C^1, por lo que no se puede.

Por def

$$\frac{dr}{d\theta} = \left(-\left(\frac{\theta+h}{h}\right)\sin\theta + \frac{\cos(\theta)}{h}, \left(\frac{\theta+h}{h}\right)\cos\theta + \frac{\sin\theta}{h}, 1\right)$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left[-\frac{\sin\theta}{\cos\theta} + (\theta+h)^{-1}\right] + \left[\frac{\cos\theta}{\sin\theta} + (\theta+h)^{-1}\right] + \frac{1}{\theta+h}\, d\theta$$

$=$

$$= \ln(\cos(\theta)) + \ln(\theta+h) + \ln(\sin\theta) + \ln(\theta+h) + \ln(\theta+h)\Big]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \ln\left(\cos(\theta)\sin(\theta)\cdot(\theta+h)^3\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \ln\left(\frac{\cos(\pi/3)\,\cancel{\sin(\pi/3)}\left(\frac{\pi}{3}+h\right)^3}{\cancel{\sin(\pi/6)}\,\cancel{\cos(\pi/6)}\left(\frac{\pi}{6}+h\right)^3}\right)$$

$$= \boxed{\ln\left(\frac{2\cos(\pi/3)\,(\pi/3+h)^3}{(\pi/6+h)^3}\right)}$$

* Esto es mas feo que algo clásico no dar muchas vueltas

$$\text{Area}(S) = \frac{1}{2}\int_0^{2\pi} 3a^2 \cos^4(t)\sin^2(t) + 3a^2 \sin^4(t)\cos^2(t)\, dt$$

$$= \frac{3a^2}{2}\int_0^{2\pi} \cos^2(t)\sin^2(t)\, dt \qquad \sin(2t) = 2\sin(t)\cos(t)$$

$$= \frac{3a^2}{8}\int_0^{2\pi} \sin(2t)^2\, dt$$

$$= \frac{3a^2}{16}\int_0^{2\pi} (1 - \overset{0}{\cancel{\cos(4t)}})\, dt$$

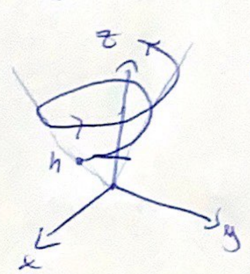
$$= \frac{3a^2}{16}\cdot 2\pi = \boxed{\frac{3a^2\pi}{8}}$$

<u>Propuesto</u> a) Como $\left.\begin{array}{l} z(0) = 1 \\ z(0) = h \end{array}\right\} \Rightarrow z(\theta) = \theta + h$

$\Rightarrow$ $x(\theta) = \rho(\theta)\cos(\theta)$, $y(\theta) = \rho(\theta)\sin(\theta)$ y $\rho(\theta)^2 = \frac{(\theta+h)^2}{h^2} \Rightarrow \rho(\theta) = \frac{\theta+h}{h}$

Finalmente $\Gamma : r(\theta) = \left(\frac{\theta+h}{h}\cos\theta, \frac{\theta+h}{h}\sin\theta, \theta+h\right)$

Bosquejo gira y sube



es como una espiral sobre un cono