P 1)

$$f(x) = \begin{cases} x^2 \text{ si } x \in [0,1] \\ 4 \text{ si } x \in (1,2] \end{cases}$$

Como $f(x)$ es una función creciente, con $x_i = i/n$

tenemos que $e^-(x) = f(x_{i-1}) \ \forall x_i \in [x_{i-1}, x_i]$

y $e^+(x) = f(x_i) \ \forall x_i \in [x_{i-1}, x_i]$,,

$$\Rightarrow \int_0^2 e^+ - e^- = \sum_{i=1}^{2n} [f(x_i) - f(x_{i-1})] \cdot \frac{1}{n} \quad / \text{ telescópica}$$

$$= [f(x_{2n}) - f(x_0)] \cdot \frac{1}{n}$$

$$= [f(2) - f(0)] \cdot \frac{1}{n}$$

$$= [4 - 0] \cdot \frac{1}{n}$$

$\therefore$ Basta tomar $n \geq 1000$ para cumplir lo pedido.

P21

Entonces tenemos que $A(x) = \int_0^x \tan(t^2)\, dt$

y $B(x) = \frac{x \cdot f(x)}{2} = \frac{x \cdot \tan(x^2)}{2}$

$$\Rightarrow \quad L = \lim_{x \to 0^+} \frac{A(x)}{B(x)} = \lim_{x \to 0^+} 2 \cdot \frac{\int_0^x \tan(t^2)\, dt}{x \cdot \tan(x^2)}$$

$= 0/0$ usamos L'H

$$\Rightarrow \quad L = \lim_{x \to 0^+} 2 \cdot \frac{\tan(x^2)}{\tan(x^2) + x \cdot 2x \cdot \sec(x^2)} \qquad / : \tan(x^2)$$

$$= \lim_{x \to 0^+} 2 \cdot \frac{1}{1 + \underbrace{\frac{2}{\cos(x^2)}}_{\to 2} \cdot \underbrace{\frac{x^2}{\sin(x^2)}}_{\to 1}} = 2 \cdot \frac{1}{1+2} = \frac{2}{3}$$

P3) muestre que $g(x) = \int_0^x f(t)\,dt + \int_0^{f(x)} f^{-1}(t)\,dt$

satisface $g'(x) = f(x) + f'(x) \cdot x$

Si $g(x) = \int_0^x f(t)\,dt + \int_0^{f(x)} f^{-1}(t)\,dt \quad / \, ()'$

$$\Rightarrow g'(x) = f(x) \cdot x' - f(0) \cdot 0' + f^{-1}(f(x)) \cdot f'(x) - f^{-1}(0) \cdot 0'$$

$$= f(x) + x \cdot f'(x)$$

$$\therefore \quad g'(x) = f(x) + x \cdot f'(x)$$

$$= f(x) \cdot x' + x \cdot f'(x)$$

$$= [x \cdot f(x)]'$$

$$g'(x) = [x \cdot f'(x)] \quad / \int_0^x$$

$$g(x) - g(0) = x \cdot f'(x) - 0 \cdot f(0)$$

$$g(0) = \int_0^0 f(t)\,dt + \int_0^{f(0)=0} f^{-1}(t)\,dt = 0$$

$$\Rightarrow \quad g(x) = x \cdot f'(x)\,_{//}$$

P41 $\lim_{x\to 0} \dfrac{\int_0^{x^2} \sin(t^2)\,dt}{1-e^{x^6}} = \dfrac{\int_0^0 \sin(t^2)\,dt}{1-1} = \dfrac{0}{0}$

usamos L'H

$$\lim_{x\to 0} \frac{\sin(x^4)\cdot 2x}{e^{x^6}\cdot 6\cdot x^5} = \lim_{x\to 0} \underbrace{\frac{\sin(x^4)}{3x^4}}_{\to \frac{1}{3}} \cdot \underbrace{\frac{1}{e^{x^6}}}_{\to 1}$$

$$= \frac{1}{3}$$ //