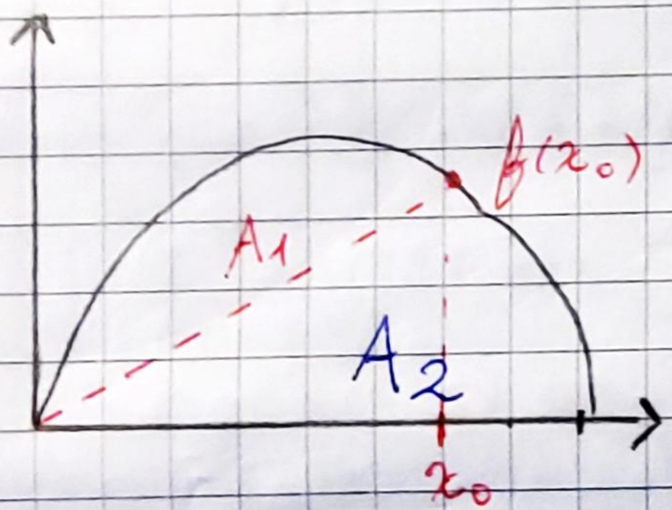
P 1| nuestra figura se vería más o menos así.



$$\Rightarrow \quad A_1 = \int_0^{x_0} 2x - x^2\,dx - \frac{f(x_0)\cdot x_0}{2}$$

$$= x^2 - \frac{x^3}{3}\Big|_0^{x_0} - \frac{x_0}{2}\left[2x_0 - x_0^2\right]$$

$$= x_0^2 - \frac{x_0^3}{3} - x_0^2 + \frac{x_0^3}{2}$$

$$= \frac{x_0^3}{6}$$

$$\wedge \quad A_2 = \int_0^{2} 2x - x^2\,dx - A_1$$

$$= x^2 - \frac{x^3}{3}\Big|_0^{2} - \frac{x_0^3}{6}$$

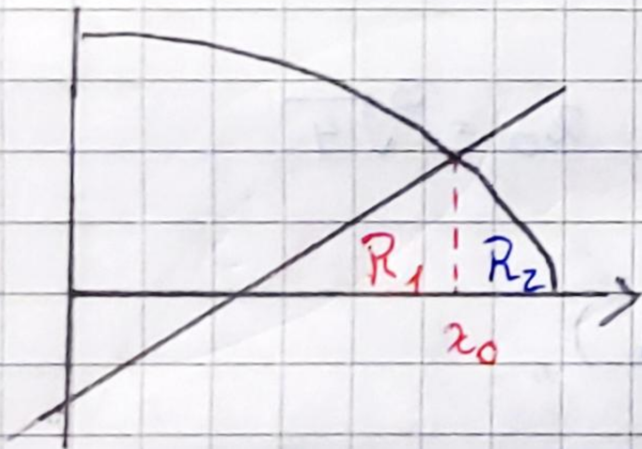
Pero $A_1 = A_2$

$\Rightarrow \quad 2A_1 = 4 - \frac{8}{3}$

$\Leftrightarrow \quad 2 \cdot \frac{x_0^3}{6} = \frac{4}{3} \quad \Rightarrow \quad x_0 = \sqrt[3]{4}$

$\Rightarrow \quad P\left(\sqrt[3]{4},\ 2\sqrt[3]{4} - \left[\sqrt[3]{4}\right]^2\right)$ //

P2| $f(x) = 2\sqrt{4-x} \wedge g(x) = x-1$



Dividimos R en 2 partes.

$\Rightarrow R = R_1 + R_2$

con $R_1 = \int_0^{x_0} x-1 \, dx$ y $\int_{x_0}^{4} 2\sqrt{4-x} \, dx$

con $x_0 \mid f(x_0) = g(x_0) \iff x_0 - 1 = 2\sqrt{4-x_0} \ /()^2$

$$x_0^2 - 2x_0 + 1 = 4(4 - x_0)$$

$$\iff x_0^2 - 2x_0 + 1 = 16 - 4x_0$$

$$\iff x_0^2 + 2x_0 - 15 = 0 \Rightarrow (x_0+5)(x_0-3) = 0$$

$$\therefore x_0 = 3$$

$$\Rightarrow R = \int_0^3 x-1 \, dx + \int_3^4 2\sqrt{4-x} \, dx$$

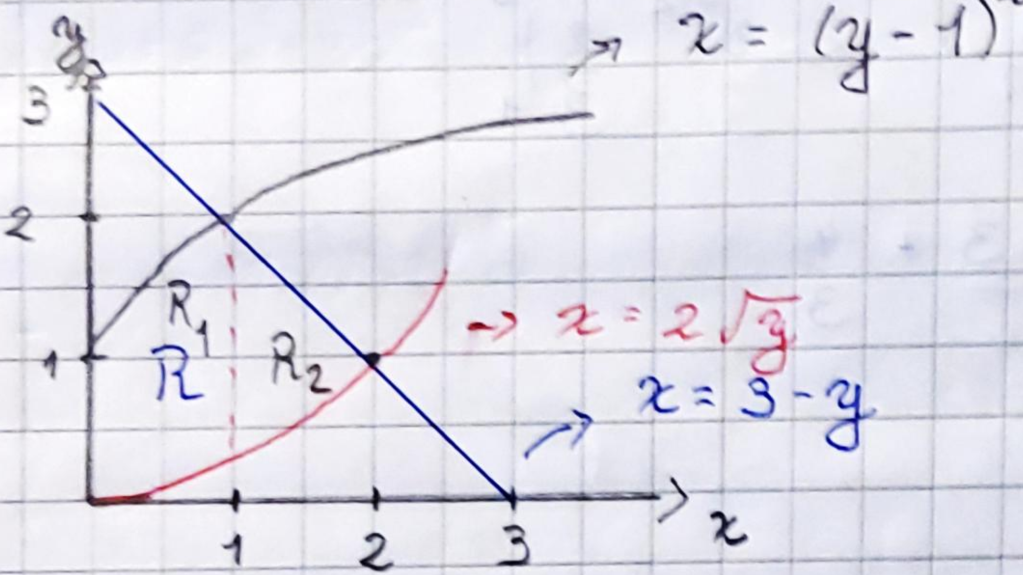
$u = 4-x$
$du = -dx$
$x=3 \Rightarrow u=1$
$x=4 \Rightarrow u=0$

$$= \frac{x^2}{2} - x \Big|_0^3 + \int_1^0 -2\sqrt{u} \, du$$

$$R = \frac{9}{2} - 3 - 2u^{3/2} \cdot \frac{2}{3}\Big|_1^0$$

$$= \frac{9}{2} - 3 + \frac{4}{3}$$

P3



Encontramos las intersecciones:

$$\overset{(1)}{(y-1)^2 = 3-y} \;\wedge\; \overset{(2)}{3-y = 2\sqrt{y}}$$

(1): $y^2 - 2y + 1 = 3 - y$

$\Leftrightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0$

$\Rightarrow y = 2 \Rightarrow x = 1$

(2): $3 - y = 2\sqrt{y} \quad /()^2$

$\Rightarrow 9 - 6y + y^2 = 4y$

$\Leftrightarrow y^2 - 10y + 9 = 0 \Rightarrow (y-9)(y-1) = 0$

$\Rightarrow y = 1 \Rightarrow x = 2$

Usamos que $R = R_1 + R_2$

$$<=> \quad R = \int_0^1 (\sqrt{x} + 1) - \frac{x^2}{4}\, dx + \int_1^2 (3 - x) - \frac{x^2}{4}\, dx$$

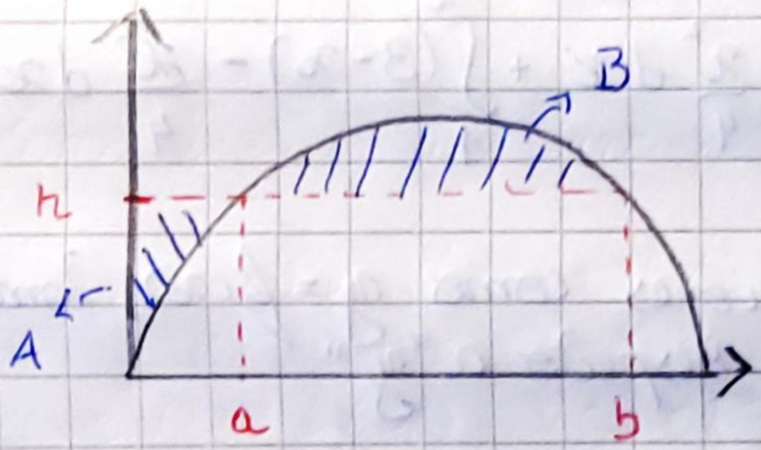
- Dejamos las funciones como $y = f(x)$, también se puede integrar respecto a "y"

$$R = \left[x^{3/2} \cdot \frac{2}{3} + x - \frac{x^3}{12}\right]\Bigg|_0^1 + \left[3x - \frac{x^2}{2} - \frac{x^3}{12}\Bigg|_1^2\right]$$

$$= \left[\cancel{\frac{2}{3}} + 1 - \cancel{\frac{1}{12}}\right] + \left[6 - 2 - \cancel{\frac{2}{3}}\right] - \left[3 - \frac{1}{2} - \cancel{\frac{1}{12}}\right]$$

$$= 2 + \frac{1}{2} = \frac{5}{2}\ _{,,}$$

P 4) $f(x) = 2x - 3x^3$



Sabemos que $h = f(a) = f(b)$

$$\Rightarrow \begin{cases} h = 2a - 3a^3 \\ h = 2b - 3b^3 \end{cases} \quad \text{y además } A = B$$

$$A = h \cdot a - \int_0^a 2x - 3x^3 \, dx$$

$$B = \int_a^b 2x - 3x^3 \, dx - h[b - a]$$

$$\Rightarrow A = h \cdot a - \left[x^2 - \frac{3}{4} x^4 \right] \Big|_0^a$$

$$= h \cdot a - a^2 + \frac{3}{4} a^4$$

$$B = \left[x^2 - \frac{3}{4} x^4 \right] \Big|_a^b - hb + ha$$

$$= b^2 - \frac{3}{4} b^4 - a^2 + \frac{3}{4} a^4 - h \cdot b + h \cdot a$$

Como $A = B \Rightarrow A - B = 0$

$$\cancel{h\cdot a} - a^2 + \frac{3}{4}a^4 - b^2 + \frac{3}{4}b^4 + \cancel{a^2} - \cancel{\frac{3}{4}a^4} + bh - \cancel{a\cdot h} = 0$$

$$\Rightarrow \quad -b^2 + \frac{3}{4}b^4 + b\cdot(2b - 3b^3) = 0$$

$$\Leftrightarrow \quad -b^2 + \frac{3}{4}b^4 + 2b^2 - 3b^4 = 0$$

$$\Leftrightarrow \quad b^2 - \frac{9}{4}b^4 = 0 \quad \Leftrightarrow \quad b^2\left(1 - \frac{9}{4}b^2\right) = 0 \quad \Rightarrow b = 2/3$$

$$\Rightarrow h = 2\cdot\frac{2}{3} - 3\cdot\frac{2^3}{3^3} = \frac{4}{9}$$

$$\cancel{\Rightarrow \frac{4}{9} = 2a - 3a^3 = \ldots}$$