P 1/

- $\int \frac{\cot^2(x)}{\sin(x)-1}\,dx$ Recordar que $\cot(x) = \frac{\cos(x)}{\sin(x)}$

$\Leftrightarrow \int \frac{\cos(x)}{\sin(x)[\sin(x)-1]}\,dx$ / usamos el C.V

$u = \sin(x)$

$\Rightarrow du = \cos(x)\,dx$

$\Rightarrow \int \frac{du}{u(u-1)}$ / acá podemos usar fracciones parciales.

$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} \Leftrightarrow \frac{A(u-1)+B\cdot u}{(u-1)\cdot u}$

$\Leftrightarrow \frac{Au - A + Bu}{u(u-1)} = \frac{1}{u(u-1)} \Rightarrow \begin{cases} A+B=0 \\ -A = 1 \end{cases}$

$\therefore\ A=-1 \wedge B=1$

$\Rightarrow \int \frac{du}{u(u-1)} = \int -\frac{1}{u} + \frac{1}{u-1}\,du$

$\Leftrightarrow -\ln|u| + \ln|u-1| + C$ / Devolvemos el C.V

$\Leftrightarrow \ln|(\sin(x)-1)/\sin(x)| + C$,,

- $\int \frac{1}{1+\sin(x)}\,dx$ / acá usamos $t = \tan(x/2)$

$\Rightarrow \sin(x) = 2t/(1+t^2)$

$\cos(x) = (1-t^2)/(1+t^2)$

$dx = 2\,dt/(1+t^2)$

$$\Rightarrow \int \frac{1}{1+\frac{2t}{(1+t^2)}} \cdot \frac{2\,dt}{(1+t^2)} \Leftrightarrow \int \frac{2\,dt}{1+t^2+2t}$$

$$\Leftrightarrow \int \frac{2\,dt}{(t+1)^2}$$ / usamos c.v $u = 1+t$

$\Rightarrow du = dt$

$$\Rightarrow \int \frac{2\,du}{u^2} = \frac{-2}{u} + C$$ / Devolvemos los c.v

$$\frac{-2}{1+\tan(x/2)} + C$$

P21

- $\displaystyle\int \frac{x}{(4x+1)(x+2)}\,dx$ / Usamos la indicación

$$\frac{x}{(4x+1)(x+2)} = \frac{A}{4x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(4x+1)}{(4x+1)(x+2)}$$

$$\Rightarrow \begin{cases} (A+4B)x = x & (1) \\ 2A+B = 0 \ (2) & \Rightarrow B = -2A \end{cases}$$

Reemplazamos en (1)

$$\Rightarrow A - 4\cdot 2A = 1 \quad \Rightarrow A = -\frac{1}{7} \quad \Rightarrow B = \frac{2}{7}$$

$\Rightarrow$ La integral nos quedaría

$$\int -\frac{1}{7}\underbrace{\frac{1}{(4x+1)}} + \frac{2}{7}\underbrace{\frac{1}{(x+2)}}\,dx$$

usamos $u = 4x+1$ usamos $V = x+2$

$\Rightarrow du = 4dx$ $\Rightarrow dV = dx$

$$\Rightarrow \int -\frac{1}{7}\,\frac{1}{u}\cdot\frac{du}{4} + \int \frac{2}{7}\cdot\frac{1}{V}\,dV$$

$$\Leftrightarrow \frac{-1}{28} \cdot \ln|u| + \frac{2}{7} \cdot \ln|v| + C$$ / Devolvemos los C.V

$$\Rightarrow \frac{-1}{28} \cdot \ln|4x+1| + \frac{2}{7} \cdot \ln|x+2| + C$$

- $\int \frac{x^2 - x + 6}{x^3 + 3x}\, dx$ Usamos fracciones parciales

$$\frac{x^2 - x + 6}{x(x^2+3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3}$$

$$= \frac{A(x^2+3) + (Bx + C)x}{x(x^2+3)}$$

$$= \frac{(A+B)x^2 + Cx + 3A}{x(x^2+3)}$$

$$\Rightarrow \begin{cases} A + B = 1 \\ C = -1 \\ 3A = 6 \end{cases} \Rightarrow A = 2,\ B = -1,\ C = -1$$

$$\Rightarrow \int \frac{2}{x} - \frac{(x+1)}{x^2+3}\, dx = \underbrace{\int \frac{2}{x}\, dx}_{2\ln|x|} - \int \frac{x}{x^2+3}\, dx - \underbrace{\int \frac{1}{x^2 + (\sqrt{3})^2}\, dx}_{\frac{-1}{\sqrt{3}} \tan^{-1}(x/\sqrt{3})}$$

$$-\int \frac{x}{x^2+3}\,dx \quad / \text{C.V} \quad u = x^2+3$$
$$\Rightarrow du = 2x\,dx$$

$$\Rightarrow -\int \frac{1}{2}\frac{du}{u} = -\frac{1}{2}\,\text{LN}\,|u| = -\frac{1}{2}\,\text{LN}\,|x^2+3|$$

$$\therefore \int \frac{x^2-x+6}{x^3+3x} = 2\,\text{LN}\,|x| - \frac{1}{2}\,\text{LN}\,|x^2+3| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

$$+ C_{\shortparallel}$$

P3) • $\int e^x \cdot \sin(x)\, dx$

Para este ejercicio usaré un truco llamado método DI (ver descripción de la portada)

	D	I
+	$\sin(x)$	e^x
−	$\cos(x)$	e^x
+	$[-\sin(x)]$	e^x

$$\Rightarrow \int e^x \cdot \sin(x) = e^x \cdot \sin(x) - e^x \cdot \cos(x) - \int e^x \cdot \sin(x)$$

$$\Leftrightarrow 2\int e^x \cdot \sin(x) = e^x \cdot \sin(x) - e^x \cdot \cos(x) + C$$

$$\Rightarrow \int e^x \cdot \sin(x) = \frac{e^x \cdot \sin(x) - e^x \cdot \cos(x)}{2} + C$$

//

• $\int \sin(ax)\cdot\cos(bx)\,dx$

Usaré el mismo truco de acá

	D	I
+	$\sin(ax)$	$\cos(bx)$
-	$a\cos(ax)$	$\sin(bx)/b$
+	$[-a^2\cdot\sin(ax)]$	$-\cos(bx)/b^2$

$$\Rightarrow \int \sin(ax)\cos(bx)\,dx = \frac{\sin(ax)\cdot\sin(bx)}{b} + \frac{a}{b^2}\cdot\cos(ax)\cdot\cos(bx) + \int \frac{a^2}{b^2}\cdot\sin(ax)\cdot\cos(bx)$$

$$\Leftrightarrow \left(1-\frac{a^2}{b^2}\right)\int \sin(ax)\cos(bx)\,dx = \underbrace{\frac{\sin(ax)\cdot\sin(bx)}{b} + \frac{a}{b^2}\cos(ax)\cdot\cos(bx) + C}_{*}$$

$$\Rightarrow \int \sin(ax)\cdot\cos(bx)\,dx = \frac{*}{\left(1-\frac{a^2}{b^2}\right)}$$ //

• $\int x \cdot \sin(x)\, dx$ / usamos "la vaca"

$x = u \quad , \quad dv = \sin(x)\, dx$

$\Rightarrow \quad dx = du \quad , \quad v = -\cos(x)$

$\Rightarrow \int x \cdot \sin(x)\, dx = -x\cos(x) - \int -\cos(x)\, dx$

$\left(u \cdot dv = u \cdot v - \int v\, du \right)$

$\Rightarrow \int x \sin(x)\, dx = -x\cos(x) + \sin(x) + C$

• $\int \ln(x^2 + a^2)\, dx$ / como no sabemos integrar directo, tomamos $u = \ln(x^2+a^2)$

$\Rightarrow \quad u = \ln(x^2+a^2) \quad , \quad dv = dx$

$\Rightarrow \quad du = \dfrac{2x}{x^2+a^2} \quad , \quad v = x$

$$\Rightarrow \int \ln(x^2+a^2)\, dx = x \cdot \ln(x^2+a^2) - \int \frac{2x^2}{x^2+a^2}\, dx$$

$$= x \cdot \ln(x^2+a^2) - \int \frac{2(x^2+a^2)}{x^2+a^2} - \frac{2a^2}{x^2+a^2}\, dx$$

$$= x \cdot \ln(x^2+a^2) - 2x + \int \frac{2\, dx}{\left(\frac{x}{a}\right)^2 + 1}$$

$$= x \cdot \ln(x^2+a^2) - 2x + 2a \cdot \tan^{-1}(x/a) + C$$

P4| P.D.Q si $I_n = \int x^n/\sqrt{1+x}\, dx$ se tiene

$$(1+2n)\, I_n = 2x^n\sqrt{1+x} - 2n \cdot I_{n-1}$$

- notemos que al lado derecho tenemos un I_{n-1}, entonces tendría sentido tener que derivar x^n, ¿pero cómo? Probemos IPP

$$\Rightarrow \quad u = x^n, \quad (1+x)^{-1/2}\, dx = dV$$

$$\Rightarrow \quad du = n \cdot x^{n-1}\, dx, \quad 2\sqrt{1+x} = V$$

$$\Rightarrow \quad I_n = x^n \cdot 2\sqrt{1+x} - \int 2\sqrt{1+x} \cdot n \cdot x^{n-1}\, dx$$

$$\Leftrightarrow \quad I_n = 2x^n\sqrt{1+x} - 2n\int \frac{(1+x)\cdot x^{n-1}}{\sqrt{1+x}}\, dx$$

$$\Leftrightarrow \quad I_n = 2x^n\cdot\sqrt{1+x} - 2n\int \frac{x^{n-1}}{\sqrt{1+x}}\, dx - 2n\int\frac{x^n}{\sqrt{1+x}}\, dx$$

$$\Leftrightarrow \quad I_n = 2x^n\sqrt{1+x} - 2n\cdot I_{n-1} - 2n\cdot I_n$$

$$\Rightarrow \quad (1+2n)\, I_n = 2x^n\cdot\sqrt{1+x} - 2n\cdot I_{n-1} \,//$$

P5] $\cdot \int \cos^n(x)\,dx$

Llamaremos $I_n = \int \cos^n(x)\,dx$

Para resolverlo usamos IPP

$$\int \cos^n(x)\,dx = \int \cos(x)\cdot \cos^{n-1}(x)\,dx$$

$$u = \cos^{n-1}(x) \qquad , \qquad \cos(x)\,dx = dv$$

$$\Rightarrow du = (n-1)\cos^{(n-2)}(x)\cdot -\sin(x)\,dx \quad , \quad \sin(x) = v$$

$$\Rightarrow \int \cos^n(x)\,dx = \sin(x)\cdot \cos^{(n-1)}(x) - \int -(n-1)\cos^{(n-2)}(x)\cdot \sin^2(x)$$

Usamos que $\sin^2(x) = 1 - \cos^2(x)$

$$\Leftrightarrow \int \cos^n(x)\,dx = \sin(x)\cdot \cos^{(n-1)}(x) + \int (n-1)\left[\cos^{(n-2)}(x)\,(1-\cos^2(x))\right]$$

$$= \sin(x)\cdot \cos^{(n-1)}(x) + \int (n-1)\cos^{(n-2)}(x) - (n-1)\int \cos^n(x)$$

$$= \sin(x)\cdot \cos^{(n-1)}(x) + (n-1)\,I_{n-2} - (n-1)\cdot I_n$$

$$n\int \cos^n(x)\,dx = \sin(x)\cdot \cos^{(n-1)}(x) + (n-1)\,I_{n-2}$$

$$\Rightarrow I_n = \frac{\sin(x)\cdot \cos^{(n-1)}(x) + (n-1)\,I_{n-2}}{n} \;//$$

- $\int x^n \cdot \sin(x)\, dx = I_n$

Usamos el mismo procedimiento.

	D	I
+	x^n	$\sin(x)$
-	$n x^{n-1}$	$-\cos(x)$
+	$n(n-1)x^{n-2}$	$-\sin(x)$

$--->$

$$\Rightarrow I_n = -x^n \cdot \cos(x) + n x^{n-1} \cdot \sin(x) - n(n-1)\int x^{n-2} \cdot \sin(x)\, dx$$

$$\Leftrightarrow I_n = -x^n \cos(x) + n x^{n-1} \sin(x) - n(n-1) \cdot I_{n-2}$$