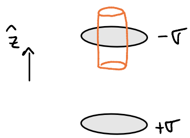
P1 a) Partimos por el campo eléctrico. Su forma es $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}$



$$\iint \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0} Q_{enc} \quad , \quad Q_{enc} = \sigma \cdot A$$

↳ área de la tapa del cilindro: $A = \pi\rho^2$

$$\vec{E} = E(z)(-\hat{z}) \Rightarrow \iint \vec{E}\cdot d\vec{S} = -E(z)(2A)$$

↳ el campo cruza ambas tapas del cilindro

⇒ el campo que genera la placa superior (con $-\sigma$) es

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$$

De forma análoga, el campo que genera la placa inferior (con $+\sigma$) es:

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z} \Rightarrow \vec{E}_{tot} = \frac{\sigma}{\varepsilon_0}\hat{z}$$

carga total en la placa

$\sigma = \sigma(t)$, pero conocemos que $I = cte \Rightarrow \sigma(t) = \frac{Q(t)}{\pi a^2}$ y $\dot{Q} = I \;\Big|\; \int_0^t dt'$

$$Q(t) - \underbrace{Q(0)}_{0,\ \text{por enunciado}} = It$$

$$\Rightarrow \vec{E}_{tot}(t) = \frac{It}{\pi\varepsilon_0 a^2}\hat{z}$$

integramos en un círculo, tangencial a las placas del cilindro

$$\nabla\times\vec{B} = \mu_0\varepsilon_0\partial_t\vec{E}_{tot} = \frac{\mu_0 I}{\pi a^2} \qquad \iint \cdot\, d\vec{S} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \frac{\mu_0 I}{\pi a^2}\iint dS$$

$$\Rightarrow \int_0^{2\pi} B\hat{\varphi}\cdot\rho\, d\varphi\,\hat{\varphi} = \frac{\mu_0 I}{\pi a^2}\pi\rho^2 \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi a^2}\rho\,\hat{\varphi}$$

b) $u_{EM} = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = \frac{1}{2}\left[\frac{1}{\varepsilon_0}\left(\frac{It}{\pi a^2}\right)^2 + \mu_0\left(\frac{I\rho}{2\pi a^2}\right)^2\right] = \frac{\mu_0 I^2}{2\pi^2 a^4}\left[(ct)^2 + (\rho/2)^2\right]$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\varepsilon_0}\frac{I^2\rho t}{2\pi^2 a^4}(-\hat{\rho}) = \frac{-I^2\rho t}{2\pi^2\varepsilon_0 a^4}\hat{\rho}$$

$$\frac{\partial u_{EM}}{\partial t} \stackrel{?}{=} -\nabla\cdot\vec{S}$$

$$\partial_t u_{EM} = \frac{\mu_0 I^2 c^2 t}{\pi^2 a^4} \qquad -\nabla\cdot\vec{S} = -\frac{1}{\rho}\frac{\partial(\rho S_\rho)}{\partial\rho} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{I^2 t\,\rho^2}{2\pi^2\varepsilon_0 a^4}\right) = \frac{I^2 t}{\pi^2\varepsilon_0 a^4}$$

$$c^2 = \frac{1}{\mu_0\varepsilon_0} \Rightarrow \frac{\partial u_{EM}}{\partial t} = \frac{I^2 t}{\pi^2\varepsilon_0 a^4} = -\nabla\cdot\vec{S} \quad \checkmark$$

c) La energía total es: $U_{EM} = \iiint u_{EM}\,dV = \frac{\mu_0 I^2}{2\pi^2 a^4}\int_0^w\int_0^{2\pi}\int_0^a\left[(ct)^2 + (\rho/2)^2\right]\rho\,d\rho\,d\varphi\,dz$

$$U_{EM} = \frac{\mu_0 I^2}{2\pi^2 a^4}\,2\pi w\left[(ct)^2\frac{a^2}{2} + \frac{1}{4}\frac{a^4}{4}\right] = \frac{\mu_0 I^2 w}{2\pi a^2}\left[(ct)^2 + \frac{a^2}{8}\right]$$

$$P_{in} = -\int\vec{S}\cdot d\vec{S} = -\int\vec{S}\cdot\rho\,d\varphi\,dz\,\hat{\rho} = \frac{I^2 t}{2\pi^2\varepsilon_0 a^4}\int_0^w\int_0^{2\pi}\rho^2\,d\varphi\,dz = \frac{I^2 t}{2\pi^2\varepsilon_0 a^4}\rho^2\,2\pi w$$

fijamos $\rho = a$, para ver la potencia total $\Rightarrow P_{in} = \frac{I^2 t w}{\pi\varepsilon_0 a^2}$

$$\frac{dU_{EM}}{dt} = \frac{\mu_0 I^2 w c^2 t}{\pi a^2} = \frac{I^2 w t}{\pi\varepsilon_0 a^2} = P_{in} \quad \checkmark\checkmark$$