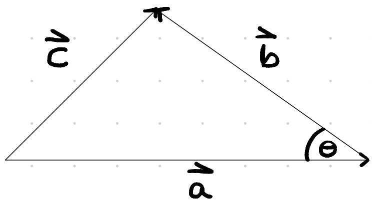
Pauta Aux #1

P1

a) Teorema del coseno

Podemos crear un triángulo arbitrario recordando la diferencia entre dos vectores:



$$\Rightarrow \vec{b} = \vec{c} - \vec{a}$$

$$\Rightarrow \vec{c} = \vec{a} + \vec{b}$$

Aplicando el módulo:

$$\Rightarrow \|\vec{c}\| = \|\vec{a} + \vec{b}\| \quad // (\ )^2$$

$$\Rightarrow \|\vec{c}\|^2 = (\|\vec{a} + \vec{b}\|)^2$$

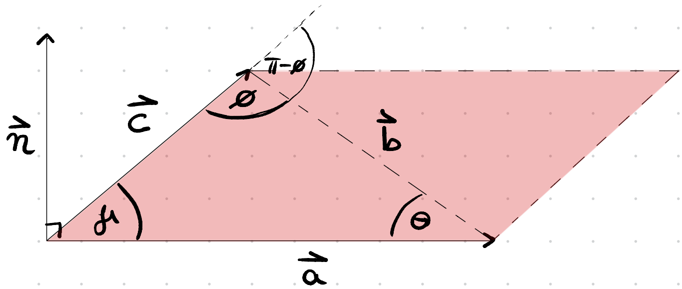
$$= \|\vec{a}\|^2 + 2\vec{a}\cdot\vec{b} + \|\vec{b}\|^2$$

$$= a^2 + b^2 + 2ab\cos(\theta)$$

$$\Rightarrow \boxed{c^2 = a^2 + b^2 + 2ab\cos(\theta)} \quad //$$

b) Teorema del seno

Recordando el sentido geométrico del producto vectorial o cruz:



$$\underset{\text{paralelepipedo}}{\text{Área}} = A_p = \|\vec{c} \times \vec{a}\| \Rightarrow A_\Delta = \frac{1}{2}\|\vec{c} \times \vec{a}\|$$

Pero también: $A_\Delta = \frac{1}{2}\|\vec{a} \times \vec{b}\| = \frac{1}{2}\|\vec{c} \times \vec{b}\|$

Entonces: $\|\vec{a} \times \vec{c}\| = ac\sin\mu$

$$\|\vec{a} \times \vec{b}\| = ab\sin\theta$$

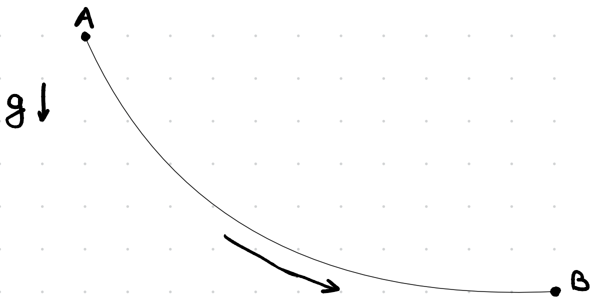
$$\|\vec{c} \times \vec{b}\| = cb\sin(\pi - \phi) = cb\sin(\phi)$$

Juntando todo, llegamos a que:

$$\frac{1}{2}cb\sin\phi = \frac{1}{2}ab\sin\theta = \frac{1}{2}ac\sin\mu \quad / \cdot \frac{2}{abc}$$

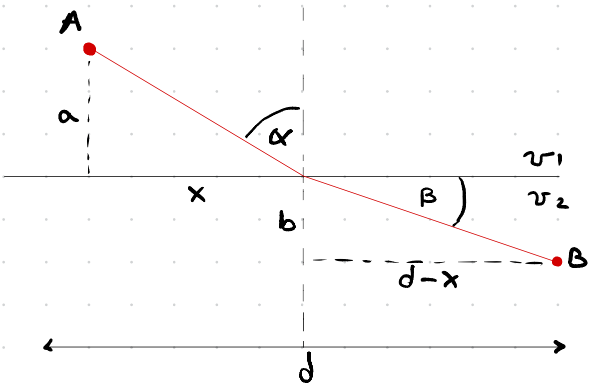
$$\Rightarrow \boxed{\frac{\sin\phi}{a} = \frac{\sin\theta}{c} = \frac{\sin\mu}{b}} \quad //$$

P21



Idea:

Ley de snell:



$$L_1 = \sqrt{a^2+x^2}$$

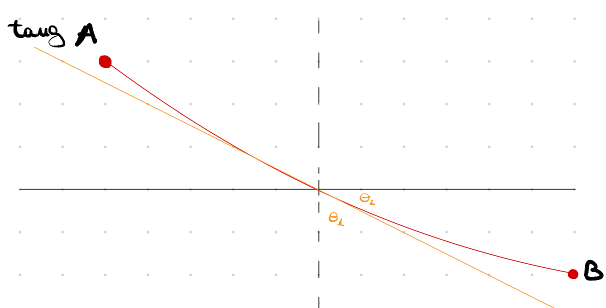
$$L_2 = \sqrt{b^2+(d-x)^2}$$

$$\min T = \min\left(\frac{L_1}{v_1} + \frac{L_2}{v_2}\right)$$

$\Rightarrow$ Ley de snell: $\dfrac{\sin\alpha}{v_1} = \dfrac{\sin\beta}{v_2} = C$

Por conservacion de la energía: $mgy = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gy}$

En un pedacito de la trayectoria se proyecta la tangente



$\theta_1 + \theta_2 = \pi/2$

$$\Rightarrow \sin\theta_1 = \cos\theta_2 = \frac{1}{\sec\theta_2} = \frac{1}{\sqrt{1+\tan^2\theta_2}}$$

$$\tan\theta_2 = \frac{dy}{dx}$$

$$\Rightarrow \sin\theta_1 = \frac{1}{\sqrt{1+y'^2}}$$

Usando la ley de snell: "c absorbe las ctes" → constante cualquiera

$$\frac{\sin^2\theta_1}{v^2} = c \Rightarrow \frac{1}{1+y'^2} = cv^2 = 2cgy \Rightarrow \frac{1}{y(1+y'^2)} = c$$

$$\Rightarrow \boxed{y(1+y'^2) = c}$$

$$\Rightarrow y\left(1+\left(\frac{dy}{dx}\right)\right) = c \qquad \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{c}{y} - 1$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{c-y}{y}}$$

$$\Rightarrow \sqrt{\frac{y}{c-y}}\,dy = dx \quad \text{(Separación de variables)}$$

integrando: $\displaystyle\int \sqrt{\frac{y}{c-y}}\,dy = \int dx$

c.v $\quad y = c\sin^2\phi \qquad dy = 2c\sin\phi\cos\phi\,d\phi$

$$\Rightarrow x = \int \sqrt{\frac{c\sin^2\phi}{c\cos^2\phi}} \cdot 2c\sin\phi\cos\phi\,d\phi$$

$$\Rightarrow x = 2c\int \frac{\sin\phi}{\cos\phi}\sin\phi\cos\phi\,d\phi$$

$$= 2c\int \sin^2\phi \, d\phi = 2c\int \frac{(1-\cos(2\phi))}{2}\, d\phi$$

$$\Rightarrow x = \phi c - c\int \cos 2\phi \, d\phi = \phi c - \frac{c}{2}\sin 2\phi + \cancel{A}$$

cond inicial $y(t=0) = x(t=0) = 0$

$$\Rightarrow \boxed{x = \frac{c}{2}(2\phi - \sin 2\phi)}$$

Como $y = c\sin^2\phi$

$$\Rightarrow \boxed{y = \frac{c}{2}(1-\cos 2\phi)}$$

Ecuación Parametrica!

$$\boxed{R = c/2 \qquad \theta = 2\phi}$$

$\Rightarrow$ Cicloide!!