Conceptual Hydrological Models

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Abstract

Conceptual hydrological models, sometimes also called gray-box models, are precipitation-runoff models built based on observed or assumed empirical relationships among different hydrological variables. They are different from black-box models which consider precipitation-runoff relationship only statistically. They are also different from the physically based distributed hydrological models which are based on solving differential equations describing the physical laws of mass, energy, and momentum conservations. This chapter describes how conceptual hydrological models represent the different hydrological processes involved in converting precipitation to runoff over land, and then to streamflow discharge at the basin outlet, including precipitation, snow accumulation and ablation, infiltration, soil moisture storage, evapotranspiration, runoff generation, baseflow, and river routing. Some of the well-known models are also used for illustration.

Keywords

Conceptual hydrological codel • Precipitation • Infiltration • Soil moisture storage • Evaporation • Evapotranspiration • Runoff generation • River routing • Tank model • Xinanjiang model • Sacramento model

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Contents

1 Introduction

Hydrological cycle refers to the continuous circulation of water between the Earth and atmosphere (see Fig. 1). Water moves between land, sea, and atmosphere via processes such as evaporation, condensation, precipitation, deposition, runoff, infiltration, sublimation, transpiration, melting, and groundwater flow. Although these processes are fairly easy to grasp, they are far from easy to understand and quantify in detail. In order to do this, abstraction is necessary and various types of hydrological models have been created. In general, all hydrological models are designed to meet one of the two primary objectives: (1) to study the system operation, and (2) to predict its system behavior.

Hydrological models can be classified into three categories (1) black-box models, (2) conceptual model, and (3) physically based model. Black-box models, also referred as empirical models, consider the system input-output relationships from a statistical viewpoint. They do not aid in physical understanding of the system behaviors. Physical-based model (sometimes called white-box models or theoretical models), on the other hand, describe hydrological processes in details by solving differential equations describing the physical laws of mass, energy, and momentum conservations. Those equations are generally solved over some grid structure representing a spatial domain. Therefore, physically based models are often called distributed hydrological models. Conceptual models (sometimes called gray-box models) consider physical laws but in highly simplified forms. A conceptual model is a descriptive representation of hydrologic system that incorporates the modeler's understanding of the relevant physical, chemical, and hydrologic conditions. Conceptual rainfall-runoff models are designed to predict magnitude of streamflow by conceptualizing rainfall-to-runoff generating processes and simulating internal variables, such as soil moisture, by various types of response functions. Such models consider six major contributing physical processes and their relationships, including precipitation, infiltration, soil moisture storage, evapotranspiration, runoff generation (including both surface and subsurface runoff), and river routing.

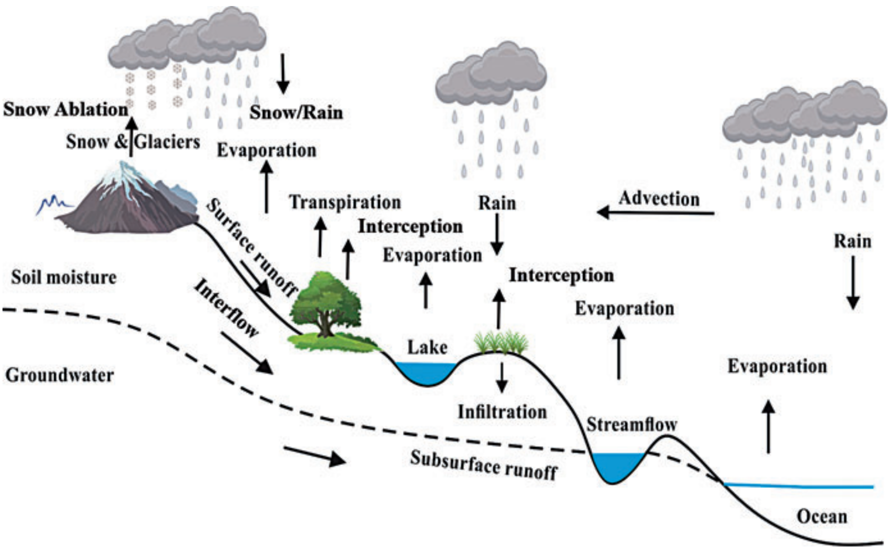


Fig. 1 A schematic description of hydrologic cycle

In the following sections, how those processes are represented in conceptual hydrological models are briefly described, followed by illustration with several well-known and widely used conceptual hydrological models.

2 Hydrological Processes Described by Conceptual Hydrological Models

2.1 Precipitation

Precipitation is the most important input to all hydrological models. Precipitation includes rainfall, snowfall, and other forms by which water falls from the air to the land surface (e.g., hail and sleet). The first two forms constitute the major part of precipitation and are of great importance to hydrological models. There are several methods to estimate areal rainfall from observed rain-gauge station data. In general, most models use areal average rainfall, i.e., arithmetic average or weighted average values, as inputs. The Thiessen polygon method (Thiessen 1911) is the most widely used method to estimate the station weights. Other popular methods include inverse distance and Krigging methods. In many models, rainfall input is implicitly assumed to be uniform in space. This can be problematic if the underlying basin is large. Some researchers use an exponential distribution function to account for spatial variability of rainfall for large river basins (Koren 1993). For example, the exponential distribution function is used for modeling areal rainfall in the simple water balance (SWB) model (Schaake et al. 1996). Since the development of remote sensing technology, remote-sensed precipitation products from radars and satellites have been widely used as precipitation data sources. The advantage of remote sensing precipitation product is that it can account for the areal distribution of precipitation. On the other hand, remote sensing precipitation data contain significant uncertainty because it is derived from radiative signals, which can be easily interfered by objects or noises.

Snowfall plays a significant part in the hydrological regime in many parts of the world, especially at middle and high latitude basins and high elevation basins. Whether precipitation falls as rain or snow can have a very significant influence on the estimation of runoff, especially for spring snowmelt-induced runoff. Model outputs are therefore sensitive to whether the form of precipitation is determined correctly. The determination of precipitation form is usually based on concurrent air temperature records. If air temperature is lower than a certain threshold value (usually set to 0°C), the precipitation form is snow, otherwise rainfall. It may be noted that some models use a fixed value for temperature threshold, whereas others treat it as a calibration parameter. Methods used in conceptual hydrological models for distinguishing the rainfall and snowfall could be summarized as following:

$$P_t = \begin{cases} P_r & if\ T \geq T_0 \\ P_s & \text{if } T < T_0 \end{cases} \tag{1}$$

where $\mathrm{P_r}$ is the amount of precipitation in the form of rain (mm), P_s is the amount of precipitation in the form of snow (mm), P_t is the total precipitation (mm), T is the daily air temperature (K), and $\mathrm{T_o}$ is the threshold temperature (K).

When snowfall is detected, the snow module in the hydrological model is activated. Snow accumulation and ablation is generally estimated by using an energy balance approach or a degree-day approach.

The energy balance of a snow cover can be expressed as (Anderson 1976; Price and Dunne 1976):

$$\Delta Q = Q_n + Q_e + Q_h + Q_m + Q_g \tag{2}$$

where ΔQ is the change in snow cover energy, Q_n is the net radiative heat flux, Q_e is the latent heat flux, Q_h is the sensible heat flux, Q_m is the heat gained from precipitation, and Q_g is the heat transfer across the snow-soil interface. The unit of each term is $\mathrm{cal \cdot cm^{-2}}$. A positive energy balance warms the snow cover, and results in melt. In general, frozen ground at the base of the snowpack persisted throughout the melt period, so that Q_g is usually assumed to be zero. Therefore, the four remaining components are as follows,

$$Q_n = Q_i - Q_r + \varepsilon Q_a - \Delta t \cdot \varepsilon \cdot \sigma \cdot T_s \tag{3}$$

where Q_i and Q_r are the incident (incoming) and reflected (outgoing) solar radiation ($\mathrm{cal \cdot cm^{-2}}$), respectively, Q_a is the incoming long-wave radiation ($\mathrm{cal \cdot cm^{-2}}$), ε is the emissivity in the long-wave portion of the energy spectrum, σ is the Stefan-Boltzmann constant ($1.355 \times 10^{-12}\ \mathrm{cal \cdot cm^{-2} \cdot K^{-4} \cdot s^{-1}}$), Δt is the time interval (s), and T_s is the snow surface temperature (K).

$$Q_e = 0.1\, L_S \cdot \rho_w \cdot \mathrm{f}(U_a) \cdot (e_a - e_s) \tag{4}$$

$$f(U_a) = \mathrm{a}\, U_a + \mathrm{b} \tag{5}$$

where L_S is the latent heat of sublimation (677 cal·g^{-1}), ρ_w is the density of water (g·cm^{-3}), U_a is the wind travel (mean wind speed multiplied by time interval) (km), $e_{a.}$ and e_s are the air and snow surface vapor pressure (mPa), a and b are the fitted coefficients.

$$Q_h = 0.16\, \rho_w \cdot c_a \cdot p \cdot \mathrm{f}(U_a) \cdot (T_a - T_s) \tag{6}$$

where c_a is the specific heat of dry air (cal·g^{-1}·K^{-1}), p is the atmospheric pressure (mPa), and T_a is the air temperature (K).

$$Q_m = 0.1\, c_w \cdot \rho_w \cdot P \cdot (T_w - 273.15) \tag{7}$$

where c_w is the specific heat of water (cal·g^{-1}·K^{-1}), P is the precipitation (mm), T_w is the wet-bulb temperature (K).

The degree-day method is used in most conceptual hydrological models as it needs the least amount of data (Hock 2003). In this approach, a simple degree-day expression to estimate snowmelt based on air temperature can be described as follows:

$$SM = DD \cdot (T - T_b) \tag{8}$$

where SM = snowmelt (mm·day^{-1}), DD = degree-day factor (mm·K^{-1}·day) indicating the snowmelt depth resulting from 1 degree-day, and T_b = a base temperature (K).

In most cases, T_b is assumed to be a constant (i.e., $T_b = 273.15$); it could also be estimated by model calibration. This method is used in the HBV model. Besides that, the SRM also considers snow-covered area in simulating snowmelt, which could be represented by a simplified equation,

$$SM = DD \cdot (T - T_b) \cdot A \tag{9}$$

where A is the snow-covered area in a region.

2.2 Infiltration

Infiltration is a key process in the hydrological cycle. Infiltration models usually employ simplified concepts of the infiltration rate or cumulative infiltration volume. It assumes that surface runoff begins when the precipitation rate exceeds the soil surface infiltration rate. Because of its fundamental role in land-surface and subsurface hydrology, infiltration has received a great deal of attention from soil and water scientists, and a large number of infiltration models have been developed to compute it. There are three types for infiltration models: empirical-based, semiempirical-based, and physically-based models. Physically-based models specify appropriate boundary conditions and normally require detailed data input. It requires solution of the Richards' equation (Richards 1931), which describes water flow in soils in terms of the hydraulic conductivity and the soil water pressure as functions of soil water content, for specified boundary conditions. However, it is extremely difficult to obtain all of data input required in the physically-based models. Therefore, for many applications, equations that simplify the concepts involved in the infiltration process are desirable for practical use (Rawls et al. 1993). The empirical and semiempirical approaches, which use simplified concepts for infiltration processes, are used in conceptual hydrological models. The empirical approaches generally relate infiltration rate or volume to elapsed time modified by certain soil properties. The most common empirical approaches are equations of Kostiakov, Horton, and Holtan. The semiempirical approaches such as of the Green and Ampt equation and the Philip equation apply the physical principles governing infiltration for simplified boundary and initial conditions.

The Kostiakov Equation

Kostiakov (1932) and independently Lewis (1937) proposed a simple empirical infiltration equation based on curve fitting from field data. It relates infiltration to time as a power function:

$$f_p = K_k \cdot t^{-\alpha} \tag{10}$$

where f_p is the infiltration capacity ($\text{mm} \cdot \text{s}^{-1}$), t is the time after infiltration starts (s), and K_k and α are constants depending on the soil type and initial conditions.

The parameters K_k and α must be evaluated from measured infiltration data, since they have no physical interpretation. The equation describes the measured infiltration curve and, given the same soil and same initial water condition, allows prediction of an infiltration curve using the same constants developed for those conditions.

The Kostiakov equation is widely used because of its simplicity, ease of determining the two constants from measured infiltration data, and reasonable fit to infiltration data for many soils over short-time periods (Clemmens 1983). Mezencev (1948) proposed a modification to Kostiakov's equation by adding a constant to the equation that represents the final infiltration rate reached when the soil becomes saturated after prolonged infiltration. The Kostiakov and modified Kostiakov equations tend to be the preferred models used for irrigation infiltration, probably because it is less restrictive as to the mode of water application than some other models.

The Horton Equation

Possibly, the best-known infiltration expression is that known as Horton's equation (11), which was proposed in 1940. Horton recognized that infiltration capacity (f_p) decreased with time until it approached a minimum constant rate (f_c). He attributed this decrease in infiltration primarily to factors operating at the soil surface rather than to flow processes within the soil. The equation is

$$f_p = f_c + (f_o - f_c)\, e^{-kt} \tag{11}$$

where f_c is the final or equilibrium infiltration rate (mm · s^{-1}), f_o is the initial infiltration capacity at $t = 0$ (mm · s^{-1}), and k is a constant dependent on soil type and the initial moisture content.

The parameters f_c, k, and f_o can be evaluated from measured infiltration data.

Horton's equation has advantages over the Kostiakov equation. First, at time that t equals 0, the infiltration capacity is not infinite but takes on the finite value f_o. Also, as t approaches infinity, the infiltration capacity approaches a nonzero constant minimum value of f_c (Horton 1940; Hillel 1998). Horton's equation has been widely used because it generally provides a good fit to data. Although the Horton equation is empirical in that β, f_c, and f_o must be estimated from experimental data, rather than measured in the laboratory, it does reflect the laws and basic equations of soil physics (Chow et al. 1988).

However, the Horton equation is cumbersome in practice since it contains three constants that must be evaluated experimentally (Hillel 1998). A further limitation is that it is applicable only when rainfall intensity exceeds f_c (Rawls et al. 1993). Horton's approach has also been criticized because he neglects the role of capillary potential gradients in the declination of infiltration capacity over time and attributes control almost entirely to surface conditions (Bevin 2004). Another criticism of the Horton model is that it assumes that hydraulic conductivity is independent of the soil water content (Novotny and Olem 1994).

The Holtan Equation

Holtan (1961) described an empirical equation based on a storage exhaustion concept. The infiltration rate is expressed in terms of cumulative infiltration, initial soil water content, and other soil variables:

$$f_p = f_c + a \cdot F_p{}^n \tag{12}$$

where F_p is the unfilled capacity of the soil to store water (equal to the initial available moisture storage minus the volume of water already infiltrated), a is a constant between 0.25 and 0.80, and n is a constant dependent on soil type.

The exponent n has been found to be about 1.4 for many soils. The value of Fp ranges from the maximum of the available water capacity to zero. This expression is well suited for inclusion in a watershed model, because it links infiltration capacity to the soil moisture level and is not time dependent. The Holtan model for infiltration has the advantage over the Horton model, in that it has a more physical basis and can describe infiltration and the recovery of infiltration capacity during periods low or no rainfall.

More details about infiltration approaches could be found in "Handbook of Hydrology" (Maidment 1993).

Infiltration is the process by which water enters the soil. In some conceptual hydrological models, surface runoff or overland runoff occurs when the infiltration capacity is smaller than the precipitation intensity. If the infiltration capacity is

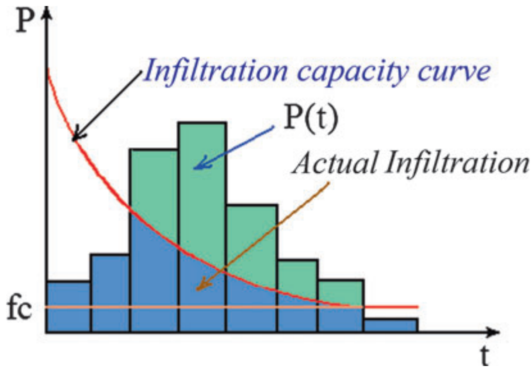


Fig. 2 Infiltration process modeling in conceptual hydrological models

greater than the precipitation intensity, all the water enters the soil profile and no overland flow occurs. In general, runoff generation based on infiltration in conceptual hydrological models can be illustrated in Fig. 2.

The red curve is the infiltration capacity curve. It could be one of the infiltration equations, such as the Horton and Holtan equations. With time, the value of the infiltration capacity decreases to the final or equilibrium infiltration rate fc. At first, because the precipitation intensity is smaller than the infiltration capacity, actual infiltration (marked blue) equals the precipitation intensity. In other words, all precipitation enters the soil without any surface runoff generated. When the precipitation intensity exceeds the infiltration capacity, surface runoff generates and equals to the difference value between the precipitation intensity and the infiltration capacity.

2.3 Soil Moisture Storage

When water infiltrates into the soil, it can storage and process considerable amounts of water. The soil would storage water until it is saturated. In this process, water infiltrates the soil surface and then moves laterally through the soil toward the stream channels, either as subsurface runoff or as underground (groundwater) runoff. Water storage in the soil can be quantified on the basis of its volumetric or gravimetric water content. The volumetric water content is the volume of water per unit volume of soil, expressed as a percentage of the volume. The gravimetric water content is the mass of water per unit mass of dry (or wet) soil. The volumetric water content is equal to the gravimetric water content times the soil's bulk density (on a dry soil basis).

In the conceptual hydrological models, it usually uses the soil moisture storage capacity to represent this process. The maximum soil moisture storage capacity is used in most of the conceptual hydrological models, including Tank, SCS, TOPMODEL, etc. In this concept, all the precipitation falling over the soil infiltrates unless the soil water content reaches the maximum soil moisture storage capacity (saturation).

Besides that, the soil moisture storage capacity curve is also applied in some conceptual hydrological models, including Xinanjiang, ARNO, and HBV models. It provides a nonuniform distribution of soil moisture storage capacity over the basin. In other words, the proportion of saturated areas, which generate surface runoff, is represented by the soil moisture storage capacity curve.

The Xinanjiang model was developed in 1973 and internationally published in 1980 (Zhao et al. 1980). The soil moisture storage capacity curve is the key of the model. It could be described as following,

$$\frac{f}{F} = 1 - \left(1 - \frac{W_m}{W_{mm}}\right)^B \tag{13}$$

where f is the saturated areas, F is the basin area, W_m is the soil moisture storage capacity, W_{mm} is the maximum soil moisture storage capacity, and B is the homogeneity of soil moisture storage capacity in the basin.

The soil moisture storage capacity curve used in the ARNO model is the same as that in the Xinanjiang model.

In the HBV model, the soil moisture accounting routine computes an index of the wetness of the entire basin and integrates interception and soil moisture storage. It is controlled by three free parameters, FC, BETA, and LP, as shown in Fig. 3. FC is the maximum soil moisture storage in the basin and BETA determines the relative contribution to runoff from rain or snowmelt at a given soil moisture deficit. LP controls the shape of the reduction curve for potential evaporation. At soil moisture values below LP, the actual evapotranspiration will be reduced.

The ideal soil is considered to be one which is homogeneous throughout the profile, and in which all of the pores are interconnected by capillaries. In addition, it is assumed that the applied rainfall falls uniformly over the soil surface. Because the movement of water into the soil is areally uniform, the infiltration process can be considered to be one dimensional. For this ideal case, perhaps the most important factors which affect the infiltration capacity are soil type and moisture content. The soil type determines the size and number of the capillaries through which the water must flow. However, actual soil conditions are seldom uniformly distributed in a basin. Therefore, a concept of variable infiltration capacity (VIC) was proposed by Liang et al. (1994). It is also similar to soil moisture storage capacity curve in the Xinanjiang model, and could be expressed by the following equation,

$$i = i_m(1 - A)^{\frac{1}{b_i}} \tag{14}$$

where i is the infiltration capacity, i_m is maximum infiltration capacity, A is the fraction of an area for which the infiltration capacity is less than i, and b_i is the infiltration shape parameter, which is a measure of the spatial variability of the infiltration capacity, defined as the maximum amount of water that can be stored in the soil column.

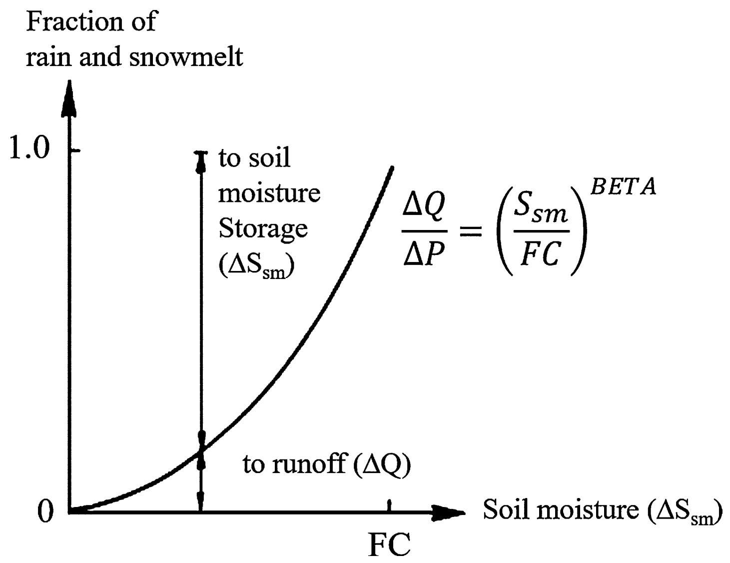


Fig. 3 The soil moisture storage capacity curve in the HBV model

Antecedent soil moisture is the degree of soil water content prior to a precipitation event. It is one of the most important factors controlling hydrological processes. It is usually estimated as a parameter in many conceptual hydrological models, including SCS, Xinanjiang, et al.

2.4 Evapotranspiration

Evapotranspiration, which include evaporation from soil and water surface and transpiration from vegetation, is usually a key variable in hydrological models. Generally, actual evapotranspiration is considered as a function of soil and vegetation properties, or a function upon potential evapotranspiration (PET). PET is generally considered to be the maximum rate of evaporation from vegetation-covered land surfaces when water is freely available. Following infiltration, evaporation from bare soil follows an atmosphere-controlled stage where evaporation is largely independent of soil moisture content and evaporation occurs near the free-water rate. Then there is a soil-controlled stage in which evaporation rate is determined by the rate at which water can be conducted to the surface rather than by atmospheric conditions.

In conceptual hydrological models, a linear relationship between actual and potential evaporations is usually used to estimate actual evaporation. Potential evaporation could be represented by pan evaporation rate, which is observed at most regions. Therefore, actual evaporation is estimated from pan evaporation rate by a linear formula in some conceptual hydrological models, when pan evaporation data is available. For example, in the Xinanjiang and the PRMS models, measured pan evaporation is used for estimating actual evaporation.

However, pan evaporation data is lacking at many regions. Potential evaporation is usually estimated by evaporation models. A great number of evaporation models have been developed and validated through field measurements, from the single climatic variable-driven equations to the energy balance and aerodynamic principle combination methods. Among them, the Thornthwaite equation (Thornthwaite 1948), Penman equation (Penman 1948), and Penman–Monteith equation (Allen et al. 1998) are widely used.

The Streamflow Synthesis and Reservoir Regulation model directly applied the Thornthwaite equation to estimate potential evapotranspiration as a function of air temperature. Other climatic variables are also used to estimate potential evapotranspiration in different conceptual hydrological models. For example, in the PRMS model, it include two equations,

$$PET = C_m \cdot SD \cdot \rho \tag{15a}$$

$$PET = C_m \cdot (T_{mean} - T_b) \cdot R_d \tag{15b}$$

where PET is the potential evapotranspiration, C_m is a monthly coefficient, SD is the Sunshine duration (hours), ρ is the absolute humidity (g/m^3), T_{mean} is the daily mean air temperature, T_b is a coefficient, and $R_{d.}$ is the daily solar radiation.

In the HBV model, the following equation is used to estimate potential evapotranspiration,

$$PET = (1 + C \cdot (T_{mean} - T_m)) \cdot PE_m \tag{16}$$

where C is an empirical model parameter, T_m is the monthly long-term average temperature, and PE_m is the monthly average potential evapotranspiration.

In the UBC model, maximum air temperature is used to estimate potential evapotranspiration,

$$PET = 0.133\, C_m \cdot T_{max} \tag{17}$$

where C_m is a monthly coefficient and T_{max} is the maximum air temperature.

If water is not readily available from soil surface, actual evapotranspiration is usually considered as a function upon soil moisture deficiency and potential evapotranspiration.

2.5 Runoff Generation

In a basin, runoff generation is equal to net precipitation, which is precipitation amount subtracted by precipitation losses. Precipitation losses include infiltration, soil moisture storage, and evapotranspiration as mentioned above. Net precipitation need to be transformed into runoff (flow) at the basin outlet, because point net precipitation flow to the basin outlet is different in physical meanings. However, duration of runoff generation from net precipitation is considered as uniformly over the entire area of the basin in conceptual hydrological models.

1. Unit Hydrograph Method

The unit hydrograph method is widely used for runoff generation from net precipitation. It is defined as the direct runoff hydrograph resulting from a unit volume of net precipitation of constant intensity and uniformly distributed over the drainage area. The duration of the unit volume of net precipitation, sometimes referred to as the effective duration, defines and labels the particular unit hydrograph. The unit volume is usually considered to be associated with 1 cm (1 inch) of net precipitation distributed uniformly over the basin area. The fundamental assumptions implicit in the use of unit hydrographs for modeling hydrological systems are: (a) Watersheds respond as linear systems; (b) The net precipitation is uniformly distributed over the entire basin; (c) Net precipitation is of constant intensity throughout the precipitation duration; (d) The duration of runoff hydrograph depends on the net precipitation duration.

The discrete convolution equation allows the calculation of flow for a given net precipitation

$$Q_n = \sum_{m=1}^{n \le M} P_m \times U_{n-m+1} \tag{18}$$

where Q is the flow, P is the net precipitation, U is the unit hydrograph, M is the net precipitation steps, n is the flow steps, and The unit hydrograph has N-M + 1 pulses.

The determination of unit hydrographs for particular basins can be carried out either using the theoretical developments of linear system theory or using empirical techniques. For either case, simultaneous observations of both precipitation and flow must be available. In other words, UH is applicable only for gauged basins and for the point on the stream where data are observed. Thus, the resultant UH is specific to a basin defined by the point on the stream where flow observations were made. When no direct observations are available, or when UH's for other locations on the stream in the same basin or for nearby basins of similar characteristics, the synthetic unit hydrograph method needs to be used.

$$Q_p = \frac{640 \times C_p \times A}{C_t \times (L \times L_c)^{0.3}} \tag{19}$$

where Q_p is the peak flow rate in English unit, cubic feet per second (cfs), 640 is the parameter (it is 2.75 for metric system), C_p is a storage coefficient ranging from 0.4 to 0.8 where larger values of C_p are associated with smaller values of C_t, A is the basin area in square miles (mi^2), C_t is a coefficient ranging from 1.8 to 2.2, L is the length of the basin outlet to the basin divide in miles (mi), and L_c is the length along the main stream to a point nearest the basin centroid (in mi).

More details about the unit hydrograph method could be found in Ramírez (2000).

2. Soil Conservation Service (SCS) Method

The SCS method is also widely used for estimating runoff at both gauged and ungauged basins. It has been adopted as the required procedure by many municipal and regional authorities. It was developed originally as a procedure to estimate runoff volume and peak discharge for design of soil conservation works and flood-control projects. It could be described as follows,

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \tag{20}$$

where Q is the actual runoff, P is the actual precipitation, I_a is the initial precipitation losses, and S is potential maximum retention after runoff begins. The unit of each item is inches.

The initial precipitation losses include infiltration, soil moisture storage, and evapotranspiration as mentioned above. It can be determined from observed rainfall-runoff events for small basins, where lag time is minimal, as the rainfall that occurs before runoff begins. Interception and surface depression storage may be estimated from cover and surface conditions, but infiltration during the early part of the storm is highly variable and dependent on such factors as rainfall intensity, soil crusting, and soil moisture. Establishing a relationship for estimating I_a is not easy. Thus, I_a is assumed to be a function of the maximum potential retention, S. In the SCS method, an empirical relationship between I_a and S was expressed as

$$I_a = 0.2S \tag{21}$$

Therefore, the equation could be simplified as

$$Q = \frac{(P - 0.2S)^2}{P + 0.8S} \tag{22}$$

For convenience and to standardize application of this equation, the maximum potential retention is expressed in the form of a dimensionless runoff curve number CN, where

$$CN = \frac{1000}{10 + S} \tag{23a}$$

If S is in millimeters:

$$CN = \frac{1000}{10 + \frac{S}{25.4}} \tag{23b}$$

The variability in the CN results from rainfall intensity and duration, total rainfall, soil moisture conditions, cover density, stage of growth, and temperature. Its practical range is from 40 to 98. CN is also used only as an integer value.

The CN is related to soil type, soil infiltration capability, land use, and the depth of the seasonal high water table. To account for different soils' ability to infiltrate, NRCS has divided soils into four hydrological soil groups. In practical use, the CN value could be found in National Engineering Handbook Hydrology Chapters 9: (http://www.nrcs.usda.gov/wps/portal/nrcs/detailfull/national/water/?cid=stelprdb1043063).

2.6 River Routing

Hydrologic routing is a way to predict how water moves from an upper-stream location to a downstream location. There exist two kinds of routing: routing surface runoff from hillslope to the nearest stream and routing water in a river from upper-stream to downstream and eventually to the basin outlet. The latter is also called river routing. Here we focus on river routing.

In simple terms, river routing is a way to describe the movement of water from one point to another along a river. River routing methods account for storage as water moves through stream channels and water control structures. Generally, river routing is described as the difference between the inflow at the upstream end and the outflow at the downstream end and is equal to storage changes.

$$I(t) - O(t) = \frac{dS}{dt} \tag{24}$$

where I is the inflow at the upstream, O is the outflow at the downstream, and S is the storage in the river.

Solution of this equation for $O(t)$ with various approximations for the storage constitutes lumped flow routing. Both graphical and mathematical techniques for solving this equation have been used. The advantage of lumped flow routing is its relative simplicity compared to physically flow routing. However, lumped flow routing methods for rivers neglect backwater effects and are not accurate for rapidly rising hydrographs routed through mild to flat sloping rivers. Those methods simulate stage and discharge in stream channels. There are several lumped river routing methods used in conceptual hydrological models. Some common techniques include the simple stage-storage method and the modified PULS method and the slightly more complicated Muskingum method.

1. The Stage-Storage Method

The simplest method of flood routing defines the storage in terms of the mean gage height in the reach. Thus a gage-height record for both ends of the reach must be available if the flood is to be routed. The necessary stage-storage relation is, generally defined on the basis of past flood-discharge records, although in certain reaches it may be defined from topographic data. It could be described as follows,

$$O = I - A\frac{\Delta h}{\Delta t} \tag{25}$$

where A is the average area of water surface in the reach during time Δt and Δh is the average change in water surface elevation in the reach in time Δt.

The water-surface area at a given stage is the slope of the stage-storage curve and may be easily computed from a stage-storage table, since it is the "first difference." Hence, the slope or first difference is a function of the mean stage in the reach. The outflow may be computed from the mean stage, the rate of change of stage, and the inflow.

2. The Modified PULS

The Modified PULS routing method utilizes the simple concept that storage is a function of outflow. Correct computation of the outflow hydrograph rests on the assumption that storage depends primarily, if not solely, on outflow rate. For this reason, the Modified PULS routing method is typically used for reservoir routing where a unique storage-outflow relation is likely. Strelkoff (1980) stated that determination of this relationship is a key factor in the application of the Modified PULS method.

To perform the routing, a relationship between storage and outflow is calculated and plotted as a curve. The following form of the continuity equation is then solved for each time step.

$$\frac{S_2}{\Delta t} + \frac{O_2}{2} = \left(\frac{S_1}{\Delta t} + \frac{O_1}{2}\right) - O_1 + \left(\frac{I_1 + I_2}{2}\right) \tag{26a}$$

or

$$\frac{2S_2}{\Delta t} + O_2 = I_1 + I_2 + \left(\frac{S_1}{\Delta t} - O_1\right) \tag{26b}$$

where S is the storage in the river, O is the outflow at t_1 and t_2 (m^3/s), and I is the inflow at t_1 and t_2 (m^3/s), $\Delta t = t_2 - t_1$.

The Modified PULS routing method proves valid for reservoirs when the effects of a flood wave (differences in storage due to rising and falling stages) are dampened, if not eliminated, by the reservoir. The Modified PULS method can be used for channel routing in a similar manner where each subsection of the reach is considered to behave like a cascading reservoir. It is assumed that a unique and single-valued stage-storage outflow relationship exists for each reach, and that changing downstream conditions will not alter this relationship.

In the application, this method requires either a known stage-storage-discharge relationship, or hydraulic geometry data adequate to calculate this relationship for each reach. An appropriate computation time step also must be selected, which requires an estimate of the travel time through the reach.

The Modified PULS method is available in the HEC-1 model. The HEC-1 program allows the user to enter the storage-outflow relationships directly, or they may be computed from eight-point cross-sectional data provided to the model. If cross section data are entered, the normal depth is calculated for each cross section using Manning's equation. The danger in using this method is that downstream effects cannot be taken into account for each cross section. Also, if this eight-point cross section is not truly representative of the reach, then the stage-storage relationships cannot be developed accurately.

3. The Muskingum Method

In a stream channel (river), a flood wave may be reduced in magnitude and lengthened in travel time, i.e., attenuated, by storage in the reach between two sections. The storage in the reach may be divided into two parts – prism storage and wedge storage, since the water surface is not uniform during the floods. The volume that would be stored in the reach if the flow were uniform throughout, i.e., below a line parallel to the stream bed, is called "prism storage" and the volume stored between this line and the actual water surface profile due to outflow being different from inflow into the reach is called "wedge storage." During rising stages, the wedge storage volume is considerable before the outflow actually increases, while during falling stages, inflow drops more rapidly than outflow, the wedge storage becoming negative.

In the case of stream-flow routing, the solution of the storage equation is more complicated, than in the case of reservoir routing, since the wedge storage is involved. While the storage in a reach depends on both the inflow and outflow, prism storage depends on the outflow alone and the wedge storage depends on the difference (I – O). A common method of stream flow routing is the Muskingum method (McCarthy 1938) where the storage is expressed as a function of both inflow and outflow in the reach as

$$S = K\,[xI + (1 - x)\,O] \tag{27}$$

where I is the inflow at the upstream, O is the outflow at the downstream, K is slope of storage – weighted discharge relation and has the dimension of time, and x is a dimensionless constant which weights the inflow and outflow.

It assumes that the water-surface profile is uniform and unbroken between the upstream and downstream points on the reach, that the stage and discharge are uniquely defined at these two places, and that K and x are sensibly constant throughout the range in stage experienced by the flood wave.

The factor x is chosen so that the indicated storage volume is the same whether the stage is rising or falling. The value of x ranges from 0 to 0.50 with a value of 0.25 as average for natural river reaches. If $x = 0.5$, the storage depends equally on inflow and outflow. If $x = 0$, the storage depends only on the outflow, as in the case of a large body of water such as a reservoir. No way is known for determining the value of x from the hydraulic characteristics of a channel system in the absence of discharge records.

The factor K has the dimension of time and is the slope of the storage-weighted discharge relation, which in most flood problems approaches a straight line. Analysis of many flood waves indicates that the time required for the center of mass of the flood wave to pass from the upstream end of the reach to the downstream end is equal to the factor K. The time between peaks only approximates the factor K. Ordinarily, the value of K can be determined with much greater ease and certainty than that of x.

After determining the values of K and x, the outflow O from the reach may be obtained by combining and simplifying the two equations.

$$\left(\frac{I_1 + I_2}{2}\right) \times \mathrm{t} - \left(\frac{Q_1 + Q_2}{2}\right) \times \mathrm{t} = S_2 - S_1 \tag{28a}$$

$$S_2 - S_1 = K\left[x\left(I_2 - I_1\right) + (1 - x)\left(O_2 - O_1\right)\right] \tag{28b}$$

For a discrete time interval, the following equation may be obtained

$$\mathrm{O_2 = C_0 I_2 + C_1 I_1 + C_2 O_1} \tag{29a}$$

$$C_0 = -\frac{Kx - 0.5t}{K - Kx + 0.5t} \tag{29b}$$

$$C_1 = -\frac{Kx + 0.5t}{K - Kx + 0.5t} \tag{29c}$$

$$C_0 = -\frac{K - Kx - 0.5t}{K - Kx + 0.5t} \tag{29d}$$

where t is the routing period. The routing period should be less than the time of travel for the flood wave through the reach; otherwise, it is possible that the wave crest may pass completely through the reach during the routing period.

3 Typical Conceptual Hydrological Models

Below we provide a brief introduction of three popular conceptual models: the Tank model, the Xinanjiang model, and the Sacramento model. These models are formulated from different conceptual views of the rainfall-runoff processes. The Tank model assumes that the watershed consists of a series of linear reservoirs (i.e., tanks), with each tank representing certain physical processes (i.e., evaporation and surface runoff, interflow, subsurface flow, and baseflow). The key concept contained in the Xinanjiang model is the empirical water storage capacity curve which plays a critical role in computing infiltration, runoff, evaporation, and soil water storage change. The Sacramento model also uses linear reservoir concepts. But the relationships between the reservoirs are much more complicated than that of the Tank model. There are two reservoirs representing the shallow soil water storages and three reservoirs representing the deep soil water storages, with each depicting certain physical processes such as evapotranspiration, surface runoff, interflow, and fast and slow baseflow.

3.1 The Tank Model

The Tank model (Sugawara 1972) is known as a lumped conceptual model, and is recommended by the World Meteorological Organization (WMO) as a hydrological forecasting model (WMO 1981). The model is simple and practical, and has been successfully applied to river basins in Asia, Africa, Europe, and USA (WMO 1981).

A Tank model is a simple concept that uses one or more tanks are illustrated as reservoirs in a watershed that considering rainfall as the input and generate the output as the surface runoff, subsurface flow, intermediate flow, sub-base flow, and base flow as output, as well as the phenomenon of infiltration, percolation, deep percolation, and water storages in the tank can be explained by the model. Generally, it is composed of a few (usually four) tanks laid vertically in series as shown in Fig. 4.

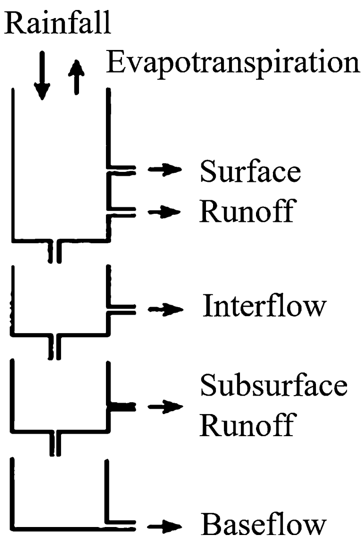
The top tank has two side outlets corresponding to the conceptual structure of the surface discharge, and one bottom outlet representing the infiltration. The second and third tanks have two outlets each, while the fourth tank has only one outlet.

Precipitation is put in the top tank, and evaporation or evapotranspiration is subtracted from the top tank. If there is no water in the top tank, evaporation or evapotranspiration is subtracted from the second tank; if there is no water in both the top and the second tank, evaporation or evapotranspiration is subtracted from the third tank; and so on.

The output from the side outlets are the calculated runoff. The output from the second tank is as intermediate runoff, the output from the third tank as sub-base runoff, and the output from the fourth tank as base flow.

Water in the second tank partly moves to the stream channel through the side outlet and this corresponds to the interflow. This model structure may be considered to correspond to the zonal structure of the surface and subsurface water. The process of water inflow to soil is considered as infiltration and if the infiltration is constant the percolation is appeared.

Fig. 4 Schematic diagram of the Tank model



3.2 The Xinanjiang Model

The Xinanjiang model was developed in 1973 and was initially used for streamflow forecasting (Zhao et al. 1995). The model is based on the concept of saturation excess overland flow, i.e., runoff occurs when soil content in the zone of aeration reaches field capacity. Runoff then is the rainfall excess infiltration without further loss. Saturation area where runoff generates is described by an empirical storage capacity curve (Fig. 5a):

$$\alpha = 1 - \left(1 - \frac{WM'}{WMM}\right)^b, \tag{30}$$

where α is the fraction of area where storage less or equal to WM′; b is a parameter related to the watershed characteristics; WM′ is the water storage in the zone of aeration at each point in the basin; WMM is the maximum of WM′; and the mean watershed storage (WM) is estimated as

$$WM = \frac{WMM}{1 + b}. \tag{31}$$

For an initial watershed storage of *W* and effective precipitation of *PE*, runoff generation (Fig. 5b) is calculated by

$$R = P + W - WM + WM\left(1 - \frac{P + a}{WMM}\right)^{b+1}, \tag{32}$$

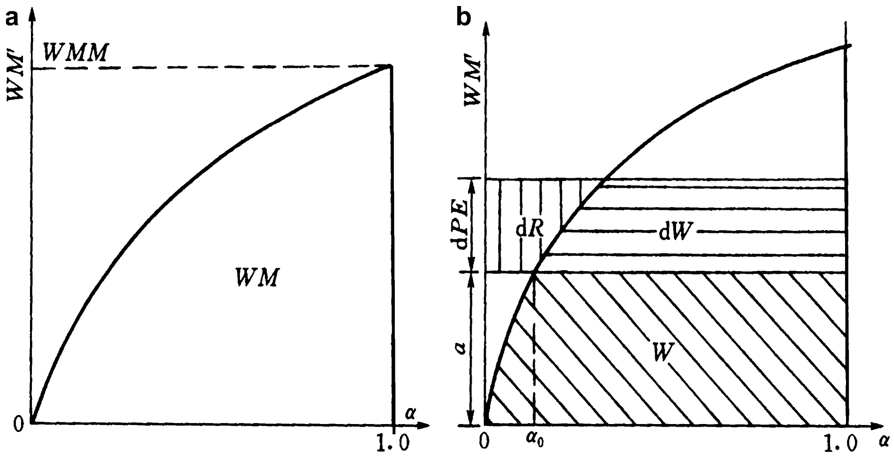


Fig. 5 Illustration of the Xinanjiang model storage capacity curve and runoff generation (**a**) Storage capacity curve. (**b**) Runoff generation

where *a* is the maximum value of *W*.

Evaporation in the Xinanjiang model is usually estimated based on pan-evaporation (or potential evaporation) by an adjustment coefficient.

In the three-source Xinanjiang model, total runoff consists of surface runoff (RS), subsurface runoff (RI), and deep ground runoff (RG). Surface runoff generates when soil water excess its storage capacity (SM), and subsurface runoff RI = KI × SM, and ground runoff RG = KG × SM; when soil water is not excess its storage capacity, no surface runoff generates, and RI = KI × PN, RG = KG × PN, where PN is net rainfall, KI and KG are outflow coefficients for subsurface and ground runoff, respectively.

Unit hydrograph method is used for surface runoff routing and linear reservoir model is used for subsurface or ground runoff routing. Muskingum method is used for streamflow routing after runoff reaches river channels. Vertical movement of subsurface runoff is characterized by the Darcy law.

The Xinanjiang model has been widely used in humid and semihumid regions in China (Zhao 1992) and has great impact on later hydrological model studies in China. Infiltration excess runoff was further introduced in the Xinanjiang model to enhance its application in arid regions (Bao 1995). The storage capacity curve was also used in the VIC model (Liang et al. 1994).

3.3 The Sacramento Model

Sacramento Soil Moisture Accounting (SAC-SMA) model was initially developed by the US National Weather Service (Burnash 1995). In the Sacramento model, precipitation on the impervious area will be the direct runoff. In each basin, the pervious area is represented vertically by two zones (Fig. 6): (i) an upper zone for

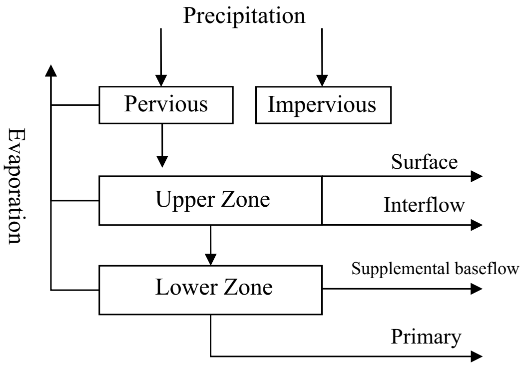


Fig. 6 Structure of Sacramento model

short-term storage and (ii) a lower zone for the longer ground water storage. In the upper zone, precipitation first supplies as tension water until it reaches the tension water capacity (UZTWM), the excess then is the effective precipitation (PAV), equal to $P + UZTWC + UZTWM$, where UZTWC is the current tension water storage in the upper zone.

Direct runoff from the impervious area is counted as a proportion of precipitation: $P \times PCTIM$, where PCTIM is the fraction of impervious area. Variable impervious area also consists of two zones but no free water in the zones. Runoff from the variable impervious area is estimated as $PAV \times \left(\frac{ADIMC - UZTWC}{LZTWM}\right)^2$, where ADIMC is the total tension water storage of the two zones, UZTWC is the tension water storage in the upper zones, LZTWM is the free water storage capacity of the lower zone.

Surface runoff generates on the pervious area when the free water in the upper zone reaches its capacity (UZFWM), and the effective precipitation (PAV) then is $P + UZFWC + UZFWM$, where UZFWC is the current free water storage. Surface runoff (ADSUR) is estimated as PAV × PAREA, where PAREA is the fraction of pervious area in a basin.

The volume of water laterally moves through the soil in the upper zone to provide the interflow, which is supposed to be linearly related to storage: $UZFWC \times UZK \times PAREA$, where UZK is the outflow coefficient.

Ground water in the lower zone consists of supplemental baseflow which supplements the baseflow after a period of relatively recent rainfall, and primary baseflow which is very slow draining and providing baseflow over a long time. Both the two baseflows drain independently by subjecting to the Darcy's Law, and are assumed to be linearly related to storage in the lower zone. The daily supplemental baseflow is $LZFSC \times LZSK \times PAREA$, where $LZFSC$ is supplemental baseflow storage, and $LZSK$ is the outflow coefficient.

Potential evaporation (PE) is estimated with adjusted pan-evaporation. Evaporation in the Sacramento model is estimated based on PE, and the evaporable water first from the tension water in the upper zone (E1), then from the free water in the upper zone (E2), and finally from the tension water in the lower zone (E3). Evaporation from surface water is estimated according to water area and PE.

Percolation in the Sacramento model is estimated before the interflow computation. Horizontal interflow occurs only when precipitation rate is greater than percolation rate from the upper zone free water. The vertical water movement from the upper zone to lower zone is driven by the lower zone percolation demand. The percolation into the lower zone is equal to the runoff draining out from the lower zone if the lower zone is saturated. Percolation rate is the greatest when the upper zone is saturated and the lower zone is dry. When the upper zone is not saturated, the percolation is controlled by the storage in upper zone free water. Percolation into the lower zone will first separated as tension water and free water, and the latter then divide into the supplemental baseflow and primary baseflow.

Direct runoff and surface runoff drain into river channel directly, while draining of interflow and baseflow into river is simulated by linear reservoir method. Flow routing in river channel is usually simulated by dimensionless unit hydrograph. The Muskingum method is recommended for complex river channels conditions.

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