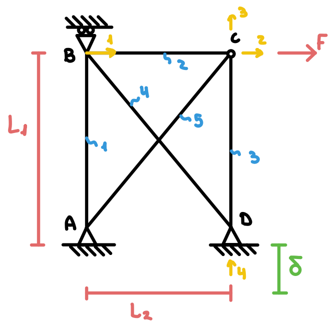

Elemento 1:

$$[K_1] = 0$$

$$L_3 = \sqrt{L_1^2 + L_2^2}$$

Elemento 2:

$$[K_2] = \frac{EA}{L_2}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & & S \\ -1 & 1 & \\ 0 & 0 & 0 \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Elemento 3

$$[K_2] = \frac{EA}{L_1}\begin{array}{c} \begin{array}{ccc} 4 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & & S \\ 0 & 0 & \\ -1 & 0 & 1 \end{bmatrix} \end{array}\begin{array}{c} 4 \\ 2 \\ 3 \end{array}$$

Elemento 4

$$c = \frac{L_2}{\sqrt{L_1^2+L_2^2}} \qquad s = \frac{-L_1}{\sqrt{L_1^2+L_2^2}}$$

$$[K_4] = \frac{EA}{\sqrt{L_1^2+L_2^2}}\begin{array}{c} \begin{array}{cc} 1 & 4 \end{array} \\ \begin{bmatrix} \frac{L_2^2}{L_1^2+L_2^2} & \\ \frac{-L_1 L_2}{L_1^2+L_2^2} & \frac{L_1^2}{L_1^2+L_2^2} \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 4 \end{array}$$

Elemento 5:

$$c = \frac{L_2}{\sqrt{L_1^2+L_2^2}} \qquad s = \frac{L_1}{\sqrt{L_1^2+L_2^2}}$$

$$[K_5] = \frac{EA}{\sqrt{L_1^2+L_2^2}}\begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \frac{L_2^2}{L_1^2+L_2^2} & \\ \frac{L_1 L_2}{L_1^2+L_2^2} & \frac{L_1^2}{L_1^2+L_2^2} \end{bmatrix} \end{array}\begin{array}{c} 2 \\ 3 \end{array}$$

- La estructura posee 2 grados de Hiperestaticidad, 3 grados de libertad libres, y 1 grado de libertad prescrito.

Definiendo la matriz de rigidez de los grados de libertad libres K_{FF}

$$[K_{FF}] = EA\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} \frac{1}{L_2} + \frac{L_2^2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} & & S \\ -\frac{1}{L_2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_2^2}{L_1^2+L_2^2} + \frac{1}{L_2} & \\ 0 & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1 L_2}{L_1^2+L_2^2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1^2}{L_1^2+L_2^2} + \frac{1}{L_1} \end{bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Definiendo el vector de fuerzas externas

$$\{F\} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

Definiendo la matriz de rigidez de los grados de libertad prescritos K_{FP}

$$[K_{FP}] = EA\begin{array}{c} 4 \\ \begin{Bmatrix} \frac{L_1 L_2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} \\ 0 \\ \frac{-1}{L_1} \end{Bmatrix} \end{array}\begin{array}{c} 1 \\ 2 \\ 3 \end{array}$$

↳ Sabemos además que nuestra ecuación de equilibrio dice que:

$$[K_{FF}][U] = \{F\} - \underbrace{[K_{FE}]_i \cdot \delta_i}_{\{F_\delta\}} \longrightarrow \text{Debemos encontrar } F_\delta$$

Encontrando F_δ

$$\{F_\delta\} = [K_{FE}]\cdot\delta = EA\begin{Bmatrix} \frac{L_1 L_2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} \\ 0 \\ \frac{-1}{L_1} \end{Bmatrix}\cdot\delta = \begin{Bmatrix} EA\cdot\delta\cdot\frac{L_1 L_2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} \\ 0 \\ \frac{-EA}{L_1}\cdot\delta \end{Bmatrix}$$

Planteando la ecuación final, y resolviendo

$$
EA\begin{bmatrix}
\frac{1}{L_2}+\frac{L_2^2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} & & S \\
-\frac{1}{L_2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_2^2}{L_1^2+L_2^2}+\frac{1}{L_2} & \\
0 & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1L_2}{L_1^2+L_2^2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1^2}{L_1^2+L_2^2}+\frac{1}{L_1}
\end{bmatrix}\cdot\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}=\begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}-\begin{Bmatrix} EA\cdot\delta\cdot\frac{L_1L_2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} \\ 0 \\ -\frac{EA}{L_1}\cdot\delta \end{Bmatrix}
$$

$$
\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}=\left(EA\cdot\begin{bmatrix}
\frac{1}{L_2}+\frac{L_2^2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} & & S \\
-\frac{1}{L_2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_2^2}{L_1^2+L_2^2}+\frac{1}{L_2} & \\
0 & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1L_2}{L_1^2+L_2^2} & \frac{1}{\sqrt{L_1^2+L_2^2}}\cdot\frac{L_1^2}{L_1^2+L_2^2}+\frac{1}{L_1}
\end{bmatrix}\right)^{-1}\cdot\begin{Bmatrix} -EA\cdot\delta\cdot\frac{L_1L_2}{L_1^2+L_2^2}\cdot\frac{1}{\sqrt{L_1^2+L_2^2}} \\ F \\ \frac{EA}{L_1}\cdot\delta \end{Bmatrix}
$$

↳ Recordar hacer la multiplicación Antes de invertir la matriz.