- $x\frac{dx}{dy} = 4y$
- $csc(y)dx + sec^2(x)dy = 0$
- $\frac{dy}{dx} = x\sqrt{1-y^2}$
- $x^2\frac{dy}{dx} = y - yx, y(-1) = -1$
- $\sqrt{1-y^2}dx - \sqrt{1-x^2}dy = 0, y(0) = \frac{\sqrt{3}}{2}$
- $(1+x^4)dy + x(1+4y^2)dx = 0, y(1) = 0$

1. $x\frac{dx}{dy} = 4y$

Esto nos indica que x depende de y, relaø que debemos recordar cuando estemos resolviendo la EDO. Si cambiamos la notaø nos queda como:

$$xx' = 4y$$ ⇝ x: dependiente

$$\Leftrightarrow x(y)x'(y)$$ ⇝ y: independiente

$$\int x(y)x'(y)\,dy = \int 4y\,dy$$

Usando el teo. de cambio de variable, tenemos

$$x = x(y)$$
$$dx = x'(y)$$

Y así

$$\int x\,dx = \int 4y\,dy$$

$$\Leftrightarrow \frac{x^2}{\cancel{2}} = 4\frac{y^2}{\cancel{2}}$$

$$\Leftrightarrow x = 2y + C \;//$$

⇝ Muy importante.

2. $csc(y)dx + sec^2(x)dy = 0$

$$csc(y)\,dx = -sec^2(x)\,dy = 0$$

$$\frac{dx}{sec^2(x)} = -\frac{dy}{csc(y)} \;//$$

$$\underbrace{\int \frac{dx}{sec^2(x)}}_{(1)} = \underbrace{-\int \frac{dy}{csc(y)}}_{(2)}$$

(1) $\displaystyle\int \frac{dx}{\sec^2(x)} = \int \cos^2(x)\,dx = \frac{1}{2}\int 1+\cos(2x)\,dx$

$\displaystyle = \frac{1}{2}\left[x + \frac{\sin(2x)}{2} \right] + C_1$

$\displaystyle = \frac{x}{2} + \frac{\sin(2x)}{4} + C_1$

(2) $\displaystyle\int \frac{dy}{\csc(y)} = \int \sin(y)\,dy = -\cos(y) + C$

De aquí, tenemos que:

$$\frac{x}{2} + \frac{\sin(2x)}{4} = -\frac{y}{2} + \frac{\sin(2y)}{4} + C \quad / \cdot 2$$

$$x + \frac{\sin(2x)}{2} + C = -\cos(y)$$

$$\Leftrightarrow \quad \arccos\left(-\left[x + \frac{\sin(2x)}{2}\right]\right) + C = y$$

3. $\displaystyle\frac{dy}{dx} = x\sqrt{1-x}$ $\qquad$ y: dependiente

x: independiente

Reescribimos

$$y' = x\sqrt{1-x} \quad / \int dx$$

$$(\Rightarrow) \underbrace{\int dy}_{=y} = \underbrace{\int x\sqrt{1-x}\,dx}_{♡} \rightsquigarrow \quad y = y(x) = 1$$

$y' = y'(x)$

Calculemos ♡ :

Usando sustitución nos queda:

$u = 1 - x \qquad du = -dx$

$x = -(u-1)$

$$\Rightarrow \int (u-1)\sqrt{u}\, du$$

$$= \int (u^{3/2} - u^{1/2})\, du \qquad \frac{3}{2} + \frac{2}{2} = \frac{5}{2}$$

$$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$$

Volviendo a la variable original:

$$\int x\sqrt{1-x}\, dx = \frac{2}{5}(1-x)^{5/2} - \frac{2}{3}(1-x)^{3/2} + C$$

4. $x^2 \dfrac{dy}{dx} = y - yx \quad , \qquad y(-1) = -1$

y : dependiente

x : independiente

Cambiando la notación, tenemos

$$x^2 y' = y - yx$$

$$\Leftrightarrow x^2 y' = y(1-x)$$

$$\Leftrightarrow \frac{x^2}{(1-x)} y' = y$$

$$\Leftrightarrow \frac{y'}{y} = \frac{(1-x)}{x^2} \quad / \int dx$$

Regla cadena vara

$\downarrow$

$$\int \frac{dy}{y} = \int \frac{(1-x)}{x^2}\, dx = \int \left[\frac{1}{x^2} - \frac{1}{x}\right] dx \qquad \left(-\frac{1}{x} + \ln(x^{-1})\right)'$$

$$\Leftrightarrow \ln(y) = -\frac{1}{x} - \ln(x) + C \quad / e \qquad \frac{1}{x^2} + \frac{1}{\left(\frac{1}{x}\right)} \cdot \left(-\frac{1}{x^2}\right)$$

$$\Leftrightarrow y = e^{-1/x} \cdot e^{\ln(x^{-1})} \cdot C \qquad \frac{1}{x^2} - \frac{1}{x} \ \checkmark\checkmark$$

$$Y = \frac{e^{-1/x}}{x} \cdot C$$

Usemos la condió inicial:

$$Y(-1) = \frac{e^{-1/-1}}{-1} \cdot C = -eC = -1$$

$$\Leftrightarrow \quad C = 1/e$$

5. $\sqrt{1-Y^2}\,dx - \sqrt{1-x^2}\,dY = 0 \quad , \qquad Y(0) = \frac{\sqrt{3}}{2}$

$$\sqrt{1-Y^2}\,dx = \sqrt{1-x^2}\,dY$$

$$\Leftrightarrow \quad \frac{dx}{\sqrt{1-x^2}} = \frac{dY}{\sqrt{1-Y^2}} \quad / \int$$

$$\Leftrightarrow \quad \int \frac{dx}{\sqrt{1-x^2}} = \int \frac{dY}{\sqrt{1-Y^2}}$$

$$\Leftrightarrow \quad \arcsin(x) = \arcsin(Y) \quad / \sin(\cdot)$$

$$\Leftrightarrow \quad x + C = Y$$

Usando la condió inicial

$$Y(0) = C = \frac{\sqrt{3}}{2} \quad //$$

6. $(1+x^4)\,dY + x(1+4Y^2)\,dx = 0 \quad , \quad Y(1) = 0$

$$(1+x^4)\,dY = -x(1+4Y^2)\,dx$$

$$\frac{dY}{(1+4Y^2)} = \frac{-x}{(1+x^4)}\,dx \quad / \int$$

$$\underbrace{\int \frac{dY}{1+4Y^2}}_{(1)} = \underbrace{\int \frac{x}{1+x^4}\,dx}_{(2)}$$

Veamos las integrales

(1) $\int \frac{dY}{1+4Y^2}$, usemos sustituϕ:

$$u = 2Y \quad , \quad du = 2$$

Entonces:

$$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{\arctan(u)}{2} + C$$

Volviendo a la variable original:

$$(1) = \frac{\arctan(2Y)}{2} + C$$

(2) $\int \frac{x}{1+x^4}\,dx$, nueva% con sustituϕ:

$$u = x^2 \quad du = 2x$$

luego,

$$\frac{1}{2}\int \frac{du}{1+u^2} = \frac{\arctan(u)}{2}$$

Volviendo al cambio de variable:

$$(2) = \frac{\arctan(x^2)}{2} + C$$

Volviendo a la ec.

$$\frac{\arctan(2Y)}{\cancel{2}} = \frac{\arctan(x^2)}{\cancel{2}} + C \quad / \tan(\cdot)$$

$$Y = \frac{\tan(\arctan(x^2) + C)}{2}$$

Usemos la condiϕ inicial

$\rightsquigarrow \arctan(1) = \pi/4$

$$Y(1) = \tan(\arctan(1) + C) = 0$$

$$= \tan(\underbrace{\frac{\pi}{4} + C}_{=0}) = 0$$

$\Leftrightarrow \quad \frac{\pi}{4} + C = 0 \quad \Leftrightarrow \quad C = -\frac{\pi}{4}$ //

2. Determine una función cuyo cuadrado más el cuadrado de su derivada es igual a 1.

$$Y^2 + (Y')^2 = 1$$

1^er método

$$Y^2 + (Y')^2 = 1 \quad / \ (\ )'$$

1) $2YY' + 2Y'Y'' = 0 = 2Y'(Y + Y'')$

CASO 1 $\boxed{Y' = 0} \Rightarrow Y = C$

$$C^2 + 0 = 1 \Rightarrow C^2 = 1 \Rightarrow Y = 1 \ \vee \ Y = -1$$

CASO 2: $(Y + Y'') = 0$

$(1 + \lambda^2) = 0$ ⇝ Polinomio característico

$\lambda^2 = \pm i$

$\Leftrightarrow \quad Y = C_1 \cos(x) + C_2 \sin(x)$

$Y' = -C_1 \sin(x) + C_2 \cos(x)$

Reemplazando

$$C_1^2 \cos(x)^2 + 2C_1C_2 \cancel{\cos}(x) \sin(x) + C_2^2 \sin^2(x)$$
$$+ \ C_1^2 \sin^2(x) - 2C_1C_2 \cancel{\sin}(x)\cos(x) + C_2^2 \cos^2(x)$$
$$\overline{C_1^2(\cos^2(x) + \sin^2(x)) + C_2^2(\sin^2(x) + \cos^2(x)) = 1}$$

$$\begin{cases} C_1^2 + C_2^2 = 1 \\ Y = C_1 \cos(x) + C_2 \sin(x) \end{cases}$$

$C_1 = \sin\alpha \ , \quad C_2 = \cos\alpha$ ⇝ α parametriza un círculo de radio 1.

$Y = \sin\alpha \cos(x) + \cos\alpha \sin x$

$= \sin(x + \alpha)$