p1 $T: \mathbb{R}^3 \to \mathbb{R}^3$

$$T\begin{pmatrix}1\\2\\1\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix} \quad T\begin{pmatrix}-2\\1\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$T\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}$$

a) la expresión de T. Quiero encontrar $T\begin{pmatrix}x\\y\\z\end{pmatrix}$

lo luego usando $T\begin{pmatrix}x\\y\\z\end{pmatrix} = xT(e_1) + yT(e_2) + zT(e_3)$

1) $T\begin{pmatrix}1\\2\\1\end{pmatrix} = 1T\begin{pmatrix}1\\0\\0\end{pmatrix} + 2T\begin{pmatrix}0\\1\\0\end{pmatrix} + 1T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

2) $T\begin{pmatrix}-2\\1\\-1\end{pmatrix} = -2T\begin{pmatrix}1\\0\\0\end{pmatrix} + 1T\begin{pmatrix}0\\1\\0\end{pmatrix} - 1T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

3) $T\begin{pmatrix}1\\1\\1\end{pmatrix} = 1T\begin{pmatrix}1\\0\\0\end{pmatrix} + 1T\begin{pmatrix}0\\1\\0\end{pmatrix} + 1\cdot T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}$

si resto 1) $-$ 3) tenemos

$$T\begin{pmatrix}1\\2\\1\end{pmatrix} - T\begin{pmatrix}1\\1\\1\end{pmatrix} = T\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}2\\1\\3\end{pmatrix} =$$

$$\begin{pmatrix}-1\\1\\0\end{pmatrix} = \quad \therefore \boxed{T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix}}$$

notemos que en 2) $-2T\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}-1\\1\\0\end{pmatrix} - T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$$\Rightarrow T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix} - 2T\begin{pmatrix}1\\0\\0\end{pmatrix} \quad (4)$$

y en 3) $T\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}-1\\1\\0\end{pmatrix} + T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}$

$$\Rightarrow T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}3\\0\\3\end{pmatrix} - T\begin{pmatrix}1\\0\\0\end{pmatrix} \quad (5)$$

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$\therefore$ (4) = (5) queda

$$\begin{pmatrix}-1\\1\\0\end{pmatrix} - 2T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}3\\0\\3\end{pmatrix} - T\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}-1\\1\\0\end{pmatrix} - \begin{pmatrix}3\\0\\3\end{pmatrix} = T\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\Rightarrow \boxed{T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}}$$

Luego con esto en (4)

$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix} - 2\begin{pmatrix}-4\\1\\-3\end{pmatrix} = \begin{pmatrix}7\\-1\\6\end{pmatrix}$$

$\therefore$ $T\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}-4\\1\\-3\end{pmatrix}$ $T\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\\0\end{pmatrix}$

$$T\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}7\\-1\\6\end{pmatrix}$$

$\therefore$ $T\begin{pmatrix}x\\y\\z\end{pmatrix} = xT\begin{pmatrix}1\\0\\0\end{pmatrix} + yT\begin{pmatrix}0\\1\\0\end{pmatrix} + zT\begin{pmatrix}0\\0\\1\end{pmatrix}$

$\therefore\ T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4x - y + 7z \\ x + y - z \\ -3x + 6z \end{pmatrix}$ } esta es nuestra función.

b) $\ker T$; $x \in \mathbb{R}_3$ tq $T(x) = 0$

$\therefore$ vemos

$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\rightarrow$ usamos la def que encontramos.

$\Rightarrow \begin{pmatrix} -4x - y + 7z \\ x + y - z \\ -3x + 6z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

de 3) $6z = 3x \Rightarrow \boxed{x = 2z}$

luego en 2)

$x + y - z = 0$

$\Rightarrow 2z + y - z = 0$

$\therefore \boxed{y = -z}$

$\therefore\ T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ tienen la forma $x = 2z$ y $y = -z$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2z \\ -z \\ z \end{pmatrix} = z\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

$\therefore\ \ker T = \left\langle \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \right\rangle$ $\dim \ker T = 1$

c) determinar $\operatorname{Im} T$ y su dimensión

TNI:

$$T: U \to V$$

$$\dim(U) = \dim(\ker T) + \dim(\operatorname{Im} T)$$

Aquí $T: \mathbb{R}^3 \to \mathbb{R}^3$ y $\dim \ker T = 1$

$\therefore \quad \dim(\mathbb{R}^3) = 3 = 1 + \dim \operatorname{Im} T$

$\therefore \quad \dim(\operatorname{Im} T) = 2$

Sabemos que:

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} 7 \\ -1 \\ 6 \end{pmatrix}$$

$\therefore$ una generador de la $\operatorname{Im} T$ sería

pero es li? notemos que;

$$-2\begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ -1 \\ 6 \end{pmatrix}$$

$\therefore \left\langle \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle$ es li, es generador $\Big\}$ Base

Cumple con la dimensión

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d) Inyect. y epiyct.

ojo ; $\dim(\ker T) \neq 0$

pues es L $\therefore$ $\ker T \neq \{0\}$

$\therefore$ no es inyectiva

pero ojo $\dim(\operatorname{Im} T) = 2$

y $\dim(\mathbb{R}_3) = 3$

$\therefore$ $\operatorname{Im} T \neq \mathbb{R}_3$ $\therefore$ no es epiy

e) determinar la matriz represent. en los canonicos:

$$T(e_1) = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} = -4e_1 + 1e_2 + -3e_3$$

$$T(e_2) = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} = -1e_1 + 1e_2 + 0 \cdot e_3$$

$$T(e_3) = \begin{pmatrix} 7 \\ -1 \\ 6 \end{pmatrix} = 7e_1 + -1e_2 + 6e_3$$

$$\therefore \ [T]_{C_{\mathbb{R}_3} \leftarrow C_{\mathbb{R}_3}} = \begin{bmatrix} -4 & -1 & 7 \\ 1 & 1 & -1 \\ -3 & 0 & 6 \end{bmatrix}$$

f) Calculemos ahora

$$[T]_{\beta' \leftarrow \beta}$$

$$\begin{tikzcd} \beta \arrow[r, "T"] \arrow[d, "id"'] & \beta' \\ C_{\mathbb{R}^3} \arrow[r, "T"'] & C_{\mathbb{R}^3} \arrow[u, "id"'] \end{tikzcd}$$

Tenemos $[T]_{C_{\mathbb{R}^3} \leftarrow C_{\mathbb{R}^3}}$

$$\therefore \quad [T]_{\beta' \leftarrow \beta} = [id]_{\beta' \leftarrow C_{\mathbb{R}^3}} [T]_{C_{\mathbb{R}^3} \leftarrow C_{\mathbb{R}^3}} [id]_{C_{\mathbb{R}^3} \leftarrow \beta}$$

$\therefore \quad [id]_{\beta' \leftarrow C_{\mathbb{R}^3}}$ calculando

$$id(e_1) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \underline{1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \underline{-1} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \underline{0} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$id(e_2) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \underline{0} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \underline{1} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \underline{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$id(e_3) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \underline{0} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \underline{0} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \underline{1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore [id]_{\beta' \leftarrow C_{\mathbb{R}_3}} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Luego Calculamos;

$[id]_{C_{\mathbb{R}_3} \leftarrow \beta}$;

$$id\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{\quad}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{0}{\quad}\begin{pmatrix}0\\1\\0\end{pmatrix} + \frac{0}{\quad}\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$id\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} = 1 \qquad 1 \qquad 0$$

$$id\begin{pmatrix}0\\2\\-3\end{pmatrix} = \begin{pmatrix}0\\2\\-3\end{pmatrix} = 0 \qquad 2 \qquad -3$$

$$\therefore [id]_{C_{\mathbb{R}_3} \leftarrow \beta} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\therefore [T]_{\beta' \leftarrow \beta} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} -4 & -1 & +7 \\ 1 & 1 & -1 \\ -3 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

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p2) $T: \mathbb{M}_{22} \to \mathbb{R}^3$

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a-b \\ c-d \\ a-b+c-d \end{pmatrix}$$

a) Sea $A, B \in \mathbb{M}_{2\times2}$ y $\alpha, \beta \in \mathbb{R}$.

$$T(\alpha A + \beta B) = T\left(\alpha \begin{pmatrix} a & b \\ c & d \end{pmatrix} + \beta \begin{pmatrix} e & f \\ g & h \end{pmatrix}\right)$$

$$= T\begin{pmatrix} \alpha a + \beta e & \alpha b + \beta f \\ \alpha c + \beta g & \alpha d + \beta h \end{pmatrix}$$

$$= \begin{pmatrix} (\alpha a + \beta e) - (\alpha b + \beta f) \\ (\alpha c + \beta g) - (\alpha d + \beta h) \\ (\alpha a + \beta e) - (\alpha b + \beta f) + (\alpha c + \beta g) - (\alpha d + \beta h) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha(a-b) + \beta(e-f) \\ \alpha(c-d) + \beta(g-h) \\ \alpha(a-b+c-d) + \beta(e-f+g-h) \end{pmatrix}$$

$$= \begin{pmatrix} \alpha(a-b) \\ \alpha(c-d) \\ \alpha(a-b+c-d) \end{pmatrix} + \begin{pmatrix} \beta(e-f) \\ \beta(g-h) \\ \beta(e-f+g+h) \end{pmatrix}$$

$$= \alpha T\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \beta T\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

es lineal

$$= \alpha T(A) + \beta T(B)$$

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b) Base y dim Ker T. $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ tq $T(\ ) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$\therefore \quad T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a-b \\ c-d \\ (a-b)+(c-d) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

$$\Rightarrow \quad a-b=0 \qquad c-d=0$$
$$(a-b)+(c-d)=0$$

$$\Rightarrow \quad \boxed{a=b} \quad y \quad \boxed{c=d}$$

$\therefore$ son aquellas matrices

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & a \\ c & c \end{pmatrix} = a\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$$

$$\left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \right\}$$ genera y es li
$\therefore$ Base
dim Ker $T = 2$

c) por TNI: $\dim(\mathbb{M}_{2\cdot 2}) = \dim(\text{Ker}\,T) + \dim(\text{Im}\,T)$
$$4 = 2 + \dim(\text{Im}\,T)$$

$\therefore \dim \text{Im}\,T = 2$

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a-b \\ c-d \\ (a-b)+(c-d) \end{pmatrix} = \begin{pmatrix} a \\ 0 \\ a \end{pmatrix} + \begin{pmatrix} 0 \\ c \\ c \end{pmatrix} + \begin{pmatrix} -b \\ \cancel{0} \\ -b \end{pmatrix} + \begin{pmatrix} 0 \\ -d \\ -d \end{pmatrix}$$

$$= a\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + b\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} + d\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$$

$\therefore \quad \Big\{ \underbrace{\begin{pmatrix}1\\0\\1\end{pmatrix}}_{a_1}, \underbrace{\begin{pmatrix}0\\1\\1\end{pmatrix}}_{a_2}, \underbrace{\begin{pmatrix}-1\\0\\-1\end{pmatrix}}_{a_3}, \underbrace{\begin{pmatrix}0\\-1\\-1\end{pmatrix}}_{a_4} \Big\}$ es li?

pues no $-1a_3 = a_1$ y $-a_4 = a_2$

$\therefore \quad \Big\{ \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix} \Big\} \to$ es li
dim $\operatorname{Im} T = 2$

Base

d) $\ker T \neq \{0\} \therefore$ no es inyect.

y $\underbrace{\dim(\mathbb{R}^3)}_{\text{el conj. de llegada}} = 3$ pero $\dim(\operatorname{Im} T) = 2$

$\therefore \operatorname{Im} T \subseteq \mathbb{R}^3$
pero no es igual a $\mathbb{R}^3 \therefore$ no es epiyect

e) Calcular $[T]_{C_{\mathbb{R}^3} \leftarrow C_{M_{22}}}$

$\therefore \quad T\begin{pmatrix}1&0\\0&0\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} = 1\cdot e_1 + 0\cdot e_2 + 1\cdot e_3$

$T\begin{pmatrix}0&1\\0&0\end{pmatrix} = \begin{pmatrix}-1\\0\\-1\end{pmatrix} = -1\cdot e_1 + 0\cdot e_2 + -1\cdot e_3$

$$T\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 0 \cdot e_1 + 1 \cdot e_2 + 1\, e_3$$

$$T\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} = 0\, e_1 + -1 \cdot e_2 + -1 \cdot e_3$$

$$\therefore \quad [T]^{\leftarrow}_{C_{\mathbb{R}_3} C_{M_{22}}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

f) Calculemos:

$[T]^{\leftarrow}_{\beta\alpha}$ $\therefore$ hagamos el recuadro.

$$\begin{tikzcd} \alpha \arrow[r, "T"] \arrow[d, "id"'] & \beta \\ C_{M_{22}} \arrow[r, "T"] & C_{\mathbb{R}_3} \arrow[u, "id"'] \end{tikzcd}$$

$$\therefore \quad [T]^{\leftarrow}_{\beta\alpha} = [id]^{\leftarrow}_{\beta C_{\mathbb{R}_3}} [T]^{\leftarrow}_{C_{\mathbb{R}_3} C_{M_{22}}} [id]^{\leftarrow}_{C_{M_{22}}\alpha}$$

$\therefore$ Calcular $\underbrace{[id]^{\leftarrow}_{C_{M_{22}}\alpha}}_{①}$ y $\underbrace{[id]^{\leftarrow}_{\beta C_{\mathbb{R}_3}}}_{②}$

① $\longrightarrow$ ojo aquí $e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $e_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $e_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $e_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$id\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = 1\, e_1 + 0 \cdot e_2 + 0 \cdot e_3 + 0 \cdot e_4$$

$$id\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = 1 \quad 1 \quad 0 \quad 0$$

$$id\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = 1 \quad 1 \quad 1 \quad 0$$

$$id\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 1 \quad 1 \quad 1 \quad 1$$

$$\therefore \quad [id]^{\leftarrow}_{C_{M_{22}}\alpha} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$-2 + 2 + 4$$

$$1\cdot 1 + 0\cdot 2 + -\tfrac{1}{2}\cdot 2 = 0$$

$$1\cdot 2 + 0\cdot 0 + -\tfrac{1}{2}\cdot 2 = 1$$

$$[id]_{\beta \leftarrow C_{B_3}}$$

$$id\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} = -1\begin{pmatrix}1\\2\\0\end{pmatrix} + 0\begin{pmatrix}2\\0\\1\end{pmatrix} + 1\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$$id\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} = 1\begin{pmatrix}1\\2\\0\end{pmatrix} + 0\begin{pmatrix}2\\0\\1\end{pmatrix} + -\tfrac{1}{2}\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$$id\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} = +2\begin{pmatrix}1\\2\\0\end{pmatrix} + 1\begin{pmatrix}2\\0\\1\end{pmatrix} + -2\begin{pmatrix}2\\2\\0\end{pmatrix}$$

$\therefore$

$$\left.\begin{aligned} 0 &= a\cdot 1 + b\cdot 2 + c\cdot 2 \\ 1 &= b\cdot 1 \\ 0 &= 2a + 2c \Rightarrow a = c \end{aligned}\right\} \begin{aligned} &2 + 2 - (2\cdot 2) \\ &2\cdot 2 + 0 - (2\cdot 2) \end{aligned}$$

$$\therefore\ [id]_{\beta \leftarrow C_{B_3}} = \begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & -\tfrac{1}{2} & -2 \end{bmatrix}$$

$$\therefore\ [T]_{\beta \leftarrow a} = \underset{3\times 3}{\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 1 \\ 1 & -\tfrac{1}{2} & -2 \end{bmatrix}} \underset{3\times 4}{\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{bmatrix}} \underset{4\times 4}{\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Sea V finita y $T: V \to V$, $S: V \to V$
dos transf. lineales tq

$$T \circ T = S \circ S = 0$$

y $T \circ S + S \circ T = I$

a) pruebe $\underbrace{\ker T = \operatorname{Im} T}_{①}$ y $\underbrace{\ker(S) = \operatorname{Im}(S)}_{②}$

para ver ① usamos $\subseteq$ y $\supseteq$

1) $y \in \operatorname{Im} T \Rightarrow \exists x \in V$ tq $T(x) = y$

tenemos

$$T(x) = y \quad / T(\,)$$
$$(T \circ T)(x) = T(y)$$
$$0 = T(y)$$
$$\therefore y \in \ker T$$

2) Estar en $\operatorname{Im} T$ es que $\exists x \in V$ tq
$$T(x) = y$$

$\therefore$ Sea $x \in \ker T \Rightarrow T(x) = 0$

sabemos que

$$T \circ S(x) + S \circ T(x) = \cancel{x}$$

$$T(S(x)) + S(\overset{0}{\cancel{T(x)}}) = x$$
$$T(S(x)) + 0 = x$$

$S(0) = 0$ pues es lineal

$\Rightarrow$

$$T(\underbrace{S(x)}_{\in V}) = x$$

llamemosle $\bar{x} \in V$

$\therefore \exists \bar{x} \in V$ tq $T(\bar{x}) = x \Rightarrow x \in \operatorname{Im} T$

$\therefore \ker T = \operatorname{Im} T$

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② → ejercicio para ustedes

b) $\dim(\operatorname{Im} T) = \dim(\operatorname{Im} S)$

por TNI la dim de V es finita
$\therefore \dim(V) = n$ ← la dejamos $n \in \mathbb{N}$ cualquiera

$\therefore \dim V = \dim \ker T + \dim \operatorname{Im} T$
$n = \dim \ker T + \dim \operatorname{Im} T$

por a) $\ker T = \operatorname{Im} T$
$\Rightarrow n = 2 \dim \ker T$ ó $n = 2 \dim \operatorname{Im} T$

$\Rightarrow \dim \operatorname{Im} T = n/2$

Luego con S; $\dim V = \dim \ker S + \dim \operatorname{Im} S$
$n = 2 \dim \operatorname{Im} S$
↳ por a) del ejercicio

$\Rightarrow \dim \operatorname{Im} S = n/2$

$\Rightarrow \dim(\operatorname{Im} T) = \dim(\operatorname{Im} S)$