

Considerate $f: \mathbb{R} \rightarrow \mathbb{R}$ def

$$f(x) = \frac{(1 + \sin(x))}{(1 - \cos(x))} \quad \text{1) Dom, cos, signos, periodicidad, } \frac{\text{impr. dividida}}{\text{impr. dividida}}$$

$(-\frac{\pi}{2}, \frac{\pi}{2})$ Dom $1 - \cos(x) \neq 0$
 2π $1 \neq \cos(x)$

$$x = 2k\pi \pm 0 = 2k\pi, \quad k \in \mathbb{N}$$

1) $\Rightarrow \text{Dom } f(x) = \mathbb{R} - \{x / x = 2k\pi, k \in \mathbb{N}\}$

2) $\cos Z(f) = 1 + \sin(x) = 0$
 $\sin(x) = -1$
 $\Rightarrow x = k\pi + (-1)^k \frac{3\pi}{2} : \pi + \frac{3\pi}{2} < \frac{\pi}{2}$

$$\Rightarrow Z(f) = \{x / x = k\pi + (-1)^k \frac{3\pi}{2}, k \in \mathbb{N}\}$$

3) signos $\rightarrow f(x) = \frac{1 + \sin(x)}{1 - \cos(x)} \quad \left| \begin{array}{l} -1 \leq \sin(x) \leq 1 \\ 0 \leq \sin(x) + 1 \leq 2 \end{array} \right| \quad \left| \begin{array}{l} \cos(x) \leq 1 \wedge -\cos(x) \\ -1 \leq -\cos(x) \wedge -\cos(x) \leq 1 \end{array} \right|$
 $\Rightarrow -1 \leq -\cos(x) \leq 1$
 $0 \leq 1 - \cos(x) \leq 2$

$1 + \sin(x) \geq 0 \quad \forall x \in \text{Dom}$
 $1 - \cos(x) \geq 0 \quad \forall x \in \text{Dom}$

$$\Rightarrow f(x) \geq 0 \quad \forall x \in \text{Dom}(f(x))$$

4) Paridad Dado $x = \frac{3\pi}{2} \Rightarrow f(x) = 0$
 $x = -\frac{3\pi}{2} \Rightarrow f(x) \neq 0 / \nexists x \in \text{Dom } f(x) < 0$

$$\frac{1 + \sin(x)}{1 - \cos(x)} \neq \frac{1 + \sin(-x)}{1 - \cos(-x)} \neq \frac{-1 + \sin(x)}{1 - \cos(x)}$$

Periodo

Debe ser al menos 2π periódica, cociente 2π período

Periodo $f(x+p) = f(x)$, $\forall$ mono + $p \in \mathbb{R}$

Por contradicción

s) $p \in (0, 2\pi)$

$$f(x) = f(x+p), \quad x = \frac{3\pi}{2}$$

$$0 = \frac{1 + \sin\left(\frac{3\pi}{2} + p\right)}{1 - \cos\left(\frac{3\pi}{2} + p\right)} \Rightarrow -1 = \sin\left(\frac{3\pi}{2} + p\right)$$

$$\frac{3\pi}{2} + p = k\pi + (-1)^k \frac{3\pi}{2}$$

$$\forall k, s, k=0$$

$$p = k\pi + (-1)^k \frac{3\pi}{2} - \frac{3\pi}{2}$$

$$p = 0$$

Imyectividad

$$\forall x, y \quad \frac{1 + \sin x}{1 - \cos x} = \frac{1 + \sin y}{1 - \cos y} \Rightarrow x = y$$

$$\text{pero } x = \frac{3\pi}{2} \wedge x = \frac{7\pi}{2} \Rightarrow$$

$$\Rightarrow f(x) = 0$$

	$f(x) = \frac{1 + \sin(x)}{1 - \cos(x)}$	
	ec1 : $x = 2 \pi$	
	ec2 : $x = 4 \pi$	
	ec3 : $x = \frac{3 \pi}{2}$	
	Entrada...	

