Trigonometría.

- Propiedades. $\cos(2x) = 1 - 2\,\text{sen}^2(x)$
- Resuelva $\text{sen}(x) + 1 = 0$, $x = ?$
- Geometría

1) $\cos(x) = a$, $\quad \alpha = \text{arc}\cos(a)$

$x = 2k\pi \pm \alpha$

$\rightarrow x = 2k\pi \pm \underbrace{\text{arc}\cos(a)}$, $\quad k \in \mathbb{Z}$

2) $\text{sen}(x) = a$ $\qquad \alpha = \text{arc}\sin(a)$

$x = k\pi + (-1)^k \alpha$

$x = k\pi + (-1)^k\, \text{arc}\sin(a)$, $\quad k \in \mathbb{Z}$

3) $\tan(x) = a$, $\quad \alpha = \text{arctg}(a)$

$x = k\pi + \alpha$

$x = k\pi + \text{arc tg}(a)$, $\quad k \in \mathbb{Z}$

Para resolver una ecuación de este estilo vemos lo siguiente

$$(\text{algo}(\operatorname{sen}(x)) - \text{algo}(\cos(x))$$

1) Pasar todo a sen y cos, si tengo $tg = \frac{s}{c}$ | $sec = \frac{1}{c}$

$ctg = \frac{c}{s}$

2) Armaré un producto o cociente comparado con 0.

i) $a \cdot b = 0 \iff \underbrace{a = 0}_{\sin(x) = \text{algo}} \vee \underbrace{b = 0}_{\cos(x) = \text{algo}}$

$$\frac{a}{b} = 0 \Rightarrow a = 0$$

Luego de eso unir soluciones

duda 1

$$\frac{\sin^{\cancel{2}}(x)}{\cos(x)\cdot\cancel{\sin(x)}} = \frac{\sin(x)}{\cos(x)}$$

$$\frac{tg(x)}{tg(x)}$$

1) Impongo $\sin(x) \neq 0$

$S \neq 0$

$C \neq 0$

1) $\sin(2x) + \cos(x) = 0$

$\Leftrightarrow 2\sin(x)\cos(x) + \cos(x) = 0$

$\Leftrightarrow \cos(x)\left[2\sin(x) + 1\right] = 0$

1) $\cos(x) = 0 \quad \vee \quad$ 2) $2\sin(x) + 1 = 0$

$\sin(x) = -\frac{1}{2}$

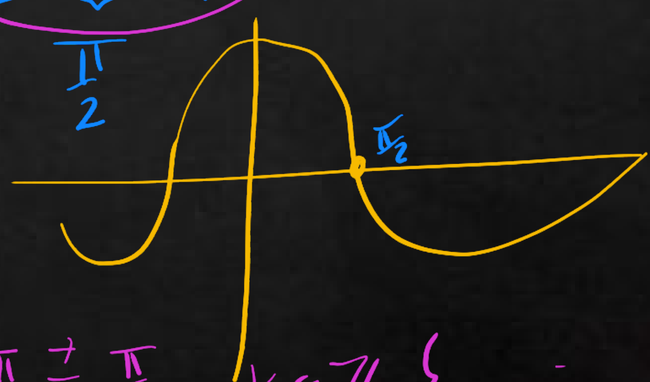
Solucionemos 1)

$\cos(x) = 0 \quad x = \pm\frac{\pi}{2}$

que ángulo en el $\cos(x)$ (fn original) me entrega el 0?

$$x = 2k\pi \pm \underbrace{\arccos(0)}_{\frac{\pi}{2}}$$

$$x = 2k\pi \pm \frac{\pi}{2}$$



$k \in \mathbb{Z}$

$S_1 = \{x \in \mathbb{R},\ x = 2k\pi \pm \frac{\pi}{2},\ k \in \mathbb{Z}\}$

forma / pro p.

2) $\operatorname{sen}(x) = -\frac{1}{2}$

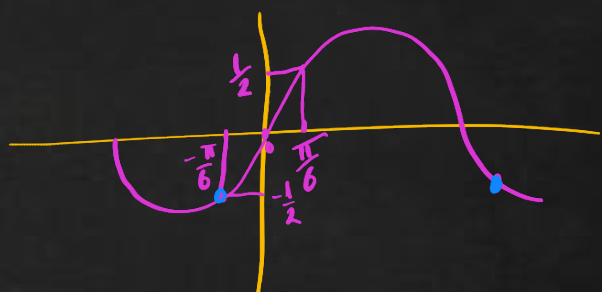
$\hookrightarrow x = k\pi + (-1)^k \underbrace{\arcsin\left(-\tfrac{1}{2}\right)}$

$$x = k\pi - (-1)^k \frac{\pi}{6}, \quad k \in \mathbb{Z}$$

	0	$\boxed{\frac{\pi}{6}}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\operatorname{sen}(x)$	0	1	2	3	4
$\cos(x)$	4	3	2	1	0

2

$\operatorname{sen}\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$



$S_2 = \{x \in \mathbb{R},\ x = k\pi - (-1)^k \frac{\pi}{6},\ k \in \mathbb{Z}\}$

$S_F = S_1 \cup S_2$

$S_F = \{x \in \mathbb{R},\ x = 2k\pi \pm \frac{\pi}{2} \ \vee\ x = \bar{k}\pi - (-1)^k \frac{\pi}{6},\ k, \bar{k} \in \mathbb{Z}\}$

2) $\cos(x) + \operatorname{sen}(x) = 0 \quad / \cdot \frac{\sqrt{2}}{2}$

1) $\frac{\sqrt{2}}{2}\cos(x) + \operatorname{sen}(x)\frac{\sqrt{2}}{2} = 0$ $\quad\Big|\quad$ $\boxed{\begin{array}{l}\operatorname{sen}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \\ \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\end{array}}$

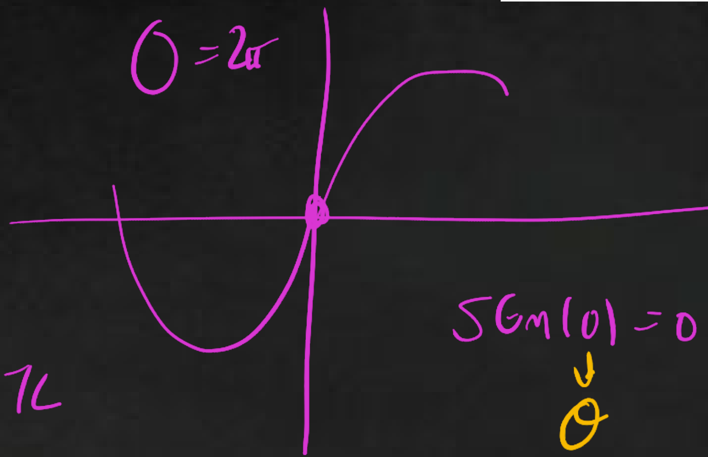
$\Leftrightarrow \operatorname{sen}\left(\frac{\pi}{4}\right)\cos(x) + \operatorname{sen}(x)\cos\left(\frac{\pi}{4}\right) = 0$

$\Leftrightarrow \operatorname{sen}\left(\frac{\pi}{4} + x\right) = 0$

$$x+\frac{\pi}{4} = k\pi + (-1)^k \underbrace{\arcsin(0)}_{0}$$

$$x+\frac{\pi}{4} = k\pi$$

$$0 = 2\pi$$



$\text{sen}(0) = 0$

$\downarrow$

0

$$x = k\pi - \frac{\pi}{4},\ k \in \mathbb{Z} \quad //$$

$$\text{sen}(x) + \cos(x) = 0$$

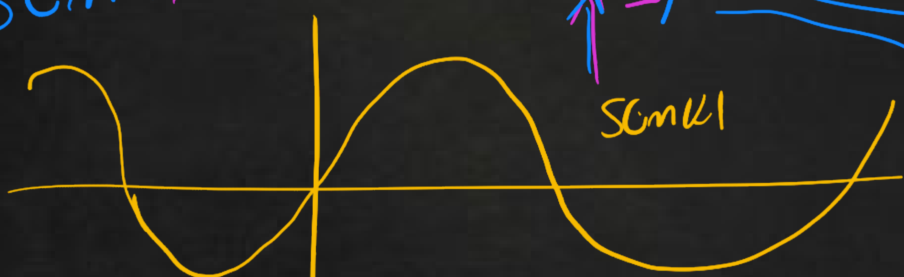
Propiedad (**)

$$s^2 + c^2 = 1$$

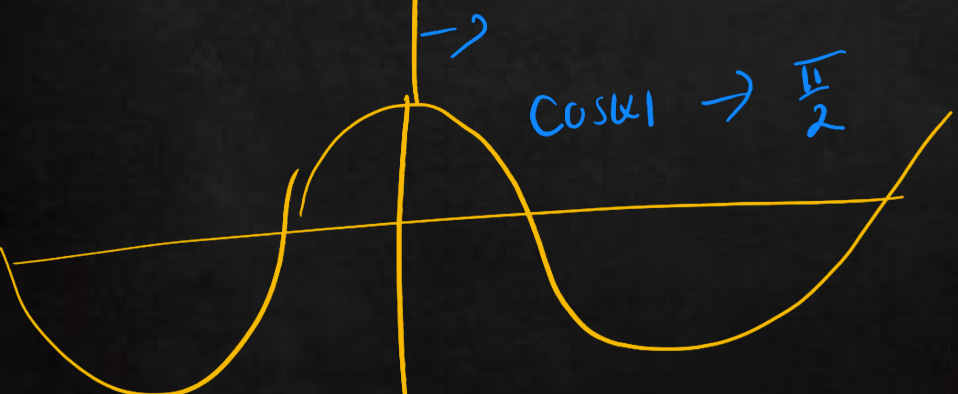
$$C(x) = \sqrt{1 - s^2(x)}$$

$$\text{sen}(x) = \cos\left(\frac{\pi}{2} - \boxed{x}\right)$$

$\uparrow$ sen(x)



$\cos(x) \rightarrow \frac{\pi}{2}$



$$\cos\left(\frac{\pi}{2} + -x\right)$$

$$= \underbrace{\cos\left(\frac{\pi}{2}\right)}_{0}\cos(-x) - \underbrace{\text{sen}\left(\frac{\pi}{2}\right)}_{1}\text{sen}(-x)$$

$$= -1 \cdot \sin(-x)$$

$$= \text{sen}(x) \quad //$$

$$\underbrace{\cos\left(x + -\frac{\pi}{2}\right)} = \underbrace{\operatorname{sen}(x)} \quad (*)$$

Resolvamos

$$\operatorname{sen}(x) + \cos(x) = 0$$

$$\overset{(*)}{\downarrow}$$

$$\Longleftrightarrow \cos\Big(\underbrace{x + -\frac{\pi}{2}}_{\alpha}\Big) + \cos(\underbrace{x}_{\beta}) = 0 \quad (PP)$$

$$\cos(\alpha) + \cos(\beta) = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) \rightarrow$$

$$\Longleftrightarrow 2\cos\left(\frac{x - \frac{\pi}{2} + x}{2}\right)\cos\left(\frac{\cancel{x} - \frac{\pi}{2} - \cancel{x}}{2}\right) = 0 \quad \Big/ \cdot \frac{1}{2}$$

$$\cos\left(\frac{2x - \frac{\pi}{2}}{2}\right)\underbrace{\cos\left(\frac{-\frac{\pi}{4}}{}\right)} = 0$$

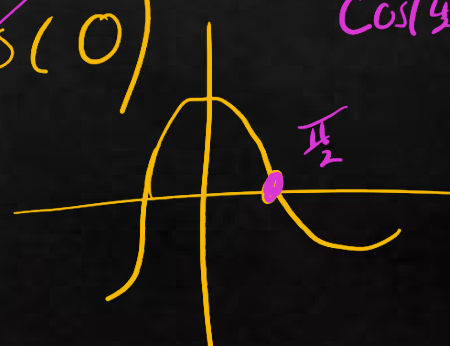
$$\cos\left(\frac{2x - \frac{\pi}{2}}{2}\right)\cos\left(\frac{\pi}{4}\right) = 0 \quad \Big/ \cdot \frac{1}{\frac{\sqrt{2}}{2}} \qquad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \neq 0$$

$$\cos\left(\frac{2x\frac{2}{2} - \frac{\pi}{2}}{2}\right) = 0$$

$$\cos\left(\frac{4x - \pi}{4}\right) = 0$$

$$\frac{4x - \pi}{4} = 2k\pi \pm \arccos(0) \rightarrow \frac{\pi}{2}$$

$$\cos\left(\frac{\pi}{2}\right) = 0$$



$$\frac{4x - \pi}{4} = 2k\pi \pm \frac{\pi}{2} \quad \Big/ \cdot 4$$

$$4x - \pi = 8k\pi \boxed{\pm} 2\pi$$

(1) $4x - \pi = 8k\pi + 2\pi \ \vee \ 4x - \pi = 8k\pi - 2\pi$

$4x = 8k\pi + 3\pi \ \vee \ 4x = 8k\pi - \pi$

$$\boxed{x = 2k\pi + \frac{3\pi}{4} \ \vee \ x = 2k\pi - \frac{\pi}{4}}$$

, $k \in \mathbb{Z}$ arbitrario

$K \cdot f(x) = 0$

$f(x) = \frac{0}{k} = 0$

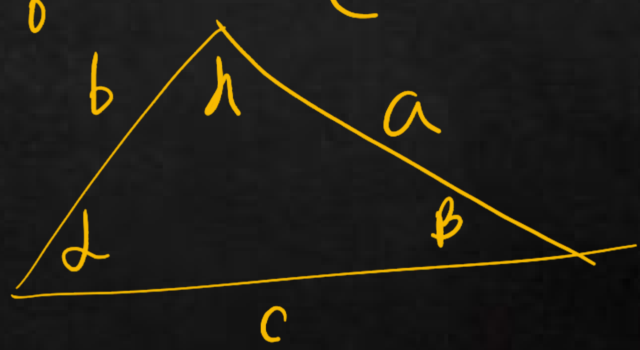
$K \neq 0$

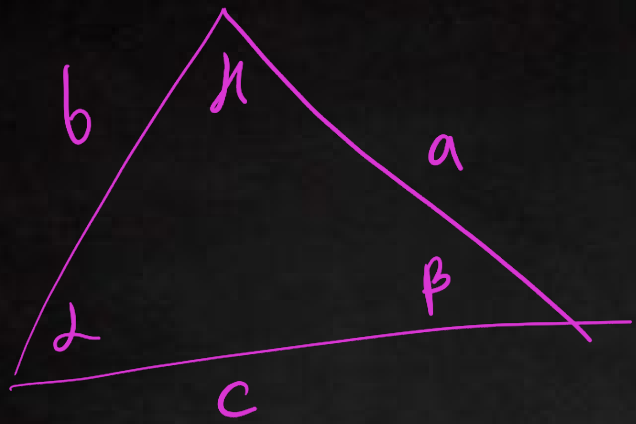
Geometria

Teo coseno

Teo seno → relación entre lados y angulos.

$$\frac{\text{sen}(\alpha)}{a} = \frac{\text{sen}(\beta)}{b} = \frac{\text{sen}(\gamma)}{c}$$



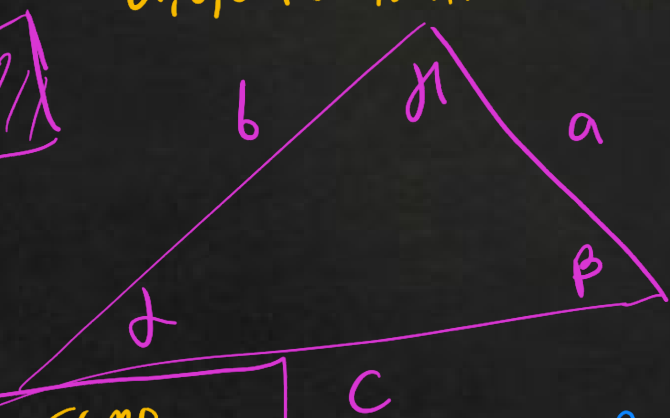


1) $a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

2) $b^2 = a^2 + c^2 - 2ac\cos(\beta)$

3) $c^2 = a^2 + b^2 - 2ab\cos(\gamma)$

P2) $a, b, c \neq 0$, $\alpha, \beta, \gamma \neq 0$



PDQ:

* $\alpha = 2\beta$

$\Rightarrow a^2 = b(c+b)$

TEO SENO

$$\frac{\sin(\alpha)}{a} = \frac{\sin(\beta)}{b}$$

$$\Rightarrow \frac{\sin(\alpha)}{\sin(\beta)} = \frac{a}{b}$$

$\alpha = 2\beta$ *

$$/\ \sin(\alpha) = \sin(2\beta) = 2\sin(\beta)\cos(\beta)$$

$$\frac{a}{b} = \frac{\sin(\alpha)}{\sin(\beta)} = 2\cos(\beta)$$

$$\frac{a}{b} = 2\cos(\beta)$$

$$\boxed{\frac{a}{2b} = \cos(\beta)}$$ **

¡Teo coseno!

Me piden $a^2 = b(c+b)$

$$\cancel{a^2 = b^2 + c^2 - 2bc\cos(\alpha)}$$

$$\rightarrow b^2 = a^2 + c^2 - 2ac\cos(\beta) \downarrow$$ **

$$b^2 = a^2 + c^2 - \cancel{2}a \cdot c \cdot \frac{a}{\cancel{2}b}$$

$$b^2 = a^2 + c^2 - a^2 \cdot \frac{c}{b}$$

$$b^2 - c^2 = a^2\left[1 - \frac{c}{b}\right]$$

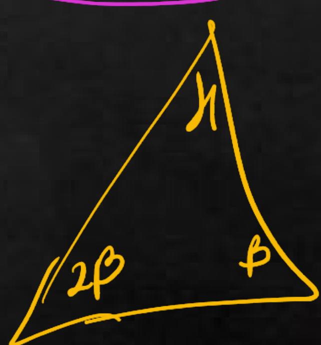
$$b^2 - c^2 = a^2\left[\frac{b-c}{b}\right]$$

$$\frac{\cancel{(b-c)}(b+c)b}{\cancel{(b-c)}} = a^2$$

$$\boxed{(b+c)b = a^2}$$ □

$$\boxed{b+c}$$

$\alpha = 2\beta$



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$$\sin(2x)\boxed{\cot(x)} - \sin^2(x) = \frac{1}{2}$$

$$\boxed{\frac{C}{S}}$$

$$\Leftrightarrow 2\cancel{\sin(x)}\cos(x)\cdot\frac{\cos(x)}{\cancel{\sin(x)}} - \sin^2(x) = \frac{1}{2}$$

$$\boxed{\sin(x) \neq 0}$$

$$2\cos^2(x) - \sin^2(x) = \frac{1}{2}$$

$$s^2 + c^2 = 1$$
$$s^2 = 1 - c^2$$

$$2\cos^2(x) - (1 - \cos^2(x)) = \frac{1}{2}$$

$$2\cos^2(x) - 1 + \cos^2(x) = \frac{1}{2}$$

$$3\cos^2(x) = \frac{1}{2} + 1 = \frac{3}{2}$$

$$3\cos^2(x) = \frac{3}{2}$$

$$\cos^2(x) = \frac{1}{2} \quad \sqrt{}$$

$$\sqrt{\alpha^2} = |\alpha|$$

$$\sqrt{\cos^2(x)} = \frac{1}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

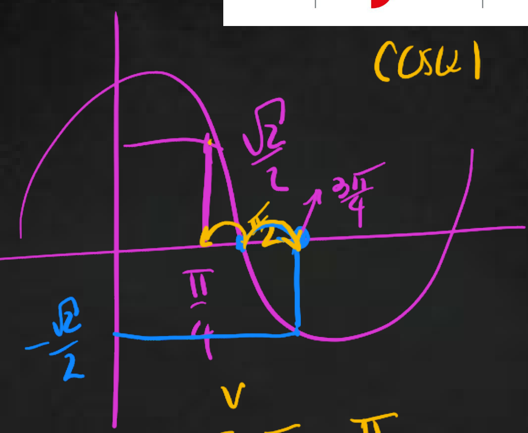
$$|\cos(x)| = \frac{\sqrt{2}}{2}$$

1) $\cos(x) = \frac{\sqrt{2}}{2}$ $\vee$ 2) $\cos(x) = -\frac{\sqrt{2}}{2}$

1) $\cos(x) = \frac{\sqrt{2}}{2}$

$x = 2k\pi \pm \underbrace{\arccos\left(\frac{\sqrt{2}}{2}\right)}_{\frac{\pi}{4}}$

$x = \left\{2k\pi \pm \frac{\pi}{4} \;,\; k \in \mathbb{Z}\right\}$



v

$\frac{2}{2}\cdot\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

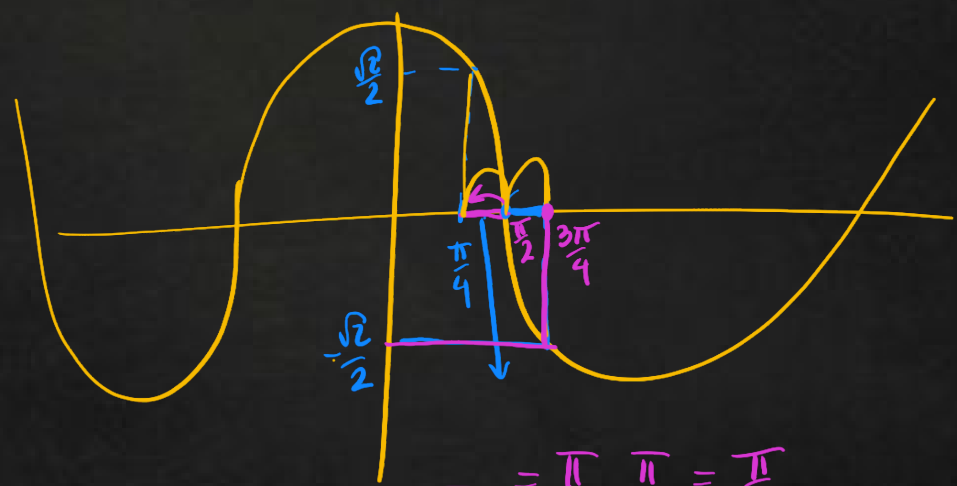
$\frac{2}{2}\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

2) $\cos(x) = -\frac{\sqrt{2}}{2}$

$x = 2k\pi \pm \underbrace{\arccos\left(-\frac{\sqrt{2}}{2}\right)}_{\frac{3\pi}{4}}$

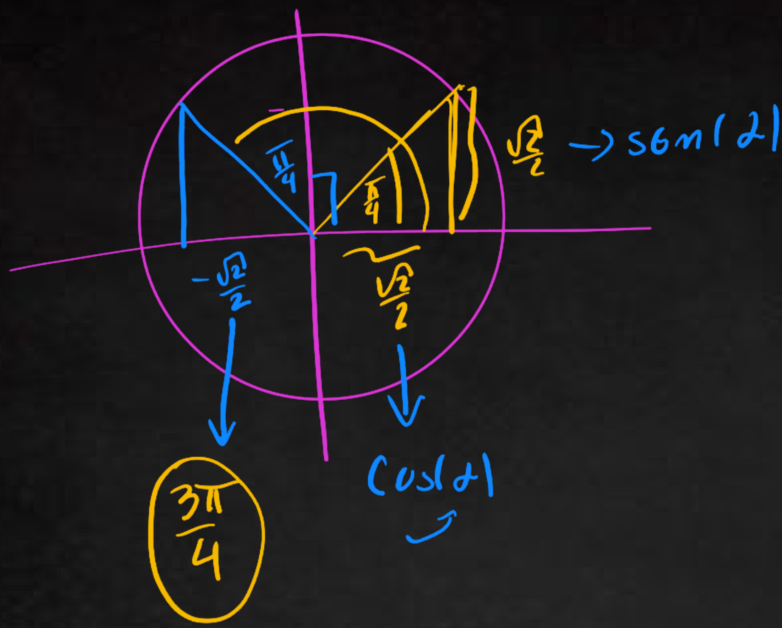
$x = \left\{2k\pi \pm \frac{3\pi}{4}\right\}$

$\arccos\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$ $\quad \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$



$— = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$



tarea: $f(x) = \dfrac{1 + \text{sen}(x)}{1 - \cos(x)}$

tarea 2: a) $1 + \cos(x) + \cos(2x) + \cos(3x) = 0$

$S_F = S_1 \cup S_2$ //

b) $\text{sen}(2x) = \cos(x/2)$ Miércoles

c) todos prop aux

Jueves

PDG

P1]

$$\frac{1}{2}\sin(x)\sec^2\left(\frac{x}{2}\right)+\cos(x)\,tg\left(\frac{x}{2}\right)-\sin(x)=0$$

$x = 2\alpha$ Cambio variable

$$=\frac{1}{2}\sin(2\alpha)\sec^2\left(\frac{\cancel{2}\alpha}{\cancel{2}}\right)+\cos(2\alpha)\,tg\left(\frac{\cancel{2}\alpha}{\cancel{2}}\right)-\sin(2\alpha)$$

α $\cos(\alpha)\neq 0$

$$=\frac{1}{\cancel{2}}\cancel{2}\sin(\alpha)\cancel{\cos(\alpha)}\frac{1}{\cos^{\cancel{2}}(\alpha)}+\left[\cos^2(\alpha)-\sin^2(\alpha)\right]\cdot\frac{\sin(\alpha)}{\cos(\alpha)}-\sin(2\alpha)$$

$$=\frac{\sin(\alpha)}{\cos(\alpha)}\left[1+\cos^2(\alpha)-\sin^2(\alpha)\right]-\sin(2\alpha)$$

$$=\frac{\sin(\alpha)}{\cos(\alpha)}\left[\cancel{\sin^2(\alpha)}+\cos^2(\alpha)+\cos^2(\alpha)-\cancel{\sin^2(\alpha)}\right]-\sin(2\alpha)$$

$$=\frac{\sin(\alpha)}{\cancel{\cos(\alpha)}}\cdot 2\cos^{\cancel{2}}(\alpha)-\sin(2\alpha)$$

$$=2\sin(\alpha)\cos(\alpha)-\sin(2\alpha)$$

$$=\sin(2\alpha)-\sin(2\alpha)=0 \;//$$

P1]

$$\tan(4x)=\frac{4\tan(x)-4\tan^3(x)}{1-6\tan^2(x)+\tan^4(x)}$$

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$$tg(4x) = tg(2x+2x)$$
$$= \frac{tg(2x) + tg(2x)}{1 - tg(2x)\,tg(2x)}$$

Propiedad:
$$tg(x+y) = \frac{tg(x) + tg(y)}{1 - tg(x)\,tg(y)}$$

$$= \frac{2\,tg(2x)}{1 - [tg(2x)]^2}$$
$$= \frac{2\,tg(x+x)}{1 - [tg(x+x)]^2}$$

$$tg(x+x) = \frac{tg(x) + tg(x)}{1 - tg(x)\,tg(x)}$$
$$\parallel \qquad \parallel$$
$$tg(2x) = \frac{2\,tg(x)}{1 - tg^2(x)}$$

$$1 \neq tg^2(x)$$

$$= \frac{2 \cdot \frac{2\,tg(x)}{1 - tg^2(x)}}{1 - \left[\frac{2\,tg(x)}{1 - tg^2(x)}\right]^2}$$

$$= \frac{\frac{4\,tg(x)}{1 - tg^2(x)}}{\frac{1 - 2tg^2(x) + tg^4(x)}{[1 - tg^2(x)]^2} - \frac{4\,tg^2(x)}{(1 - tg^2(x))^2}}$$

$$= \frac{4\,tg(x)}{\cancel{1 - tg^2(x)}} \cdot \frac{(1 - tg^2(x))^{\cancel{2}}}{(1 - 6\,tg^2(x) + tg^4(x))} \quad , \quad tg^2(x) \neq 1$$

$$= \frac{4\,tg(\alpha) - 4\,tg^3(\alpha)}{1 - 6\,tg^2 + tg^4(\alpha)} \;//$$

P3)

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$= 2\sin\left(\frac{x}{2}+\frac{y}{2}\right)\cos\left(\frac{x}{2}+\frac{-y}{2}\right)$$

$$= 2\left[\sin\left(\frac{x}{2}\right)\cos\left(\frac{y}{2}\right)+\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)\right]\left[\cos\left(\frac{x}{2}\right)\cos\left(\frac{-y}{2}\right)-\sin\left(\frac{x}{2}\right)\sin\left(\frac{-y}{2}\right)\right]$$

\# $\cos(\alpha) = \cos(-\alpha)$ \#

$$= 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{-y}{2}\right) + 2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\cos\left(\frac{-y}{2}\right)$$

$$- 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{y}{2}\right)\sin\left(\frac{x}{2}\right)\sin\left(\frac{-y}{2}\right) - 2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\sin\left(-\frac{y}{2}\right)$$

$$= \sin(x)\cos^2\left(\frac{y}{2}\right) + \sin(y)\cos^2\left(\frac{x}{2}\right) + \sin(y)\sin^2\left(\frac{x}{2}\right) + \sin(x)\sin^2\left(\frac{y}{2}\right)$$

$$= \sin(x)\Big[\underbrace{\cos^2\left(\tfrac{y}{2}\right)+\sin^2\left(\tfrac{y}{2}\right)}_{1}\Big] + \sin(y)\Big[\underbrace{\cos^2\left(\tfrac{x}{2}\right)+\sin^2\left(\tfrac{x}{2}\right)}_{1}\Big]$$

$$= \sin(x) + \sin(y) \quad \boxed{V/}$$