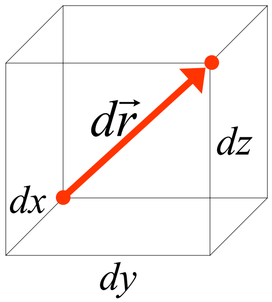
Elementos de línea, superficie y volumen

CARTESIANAS

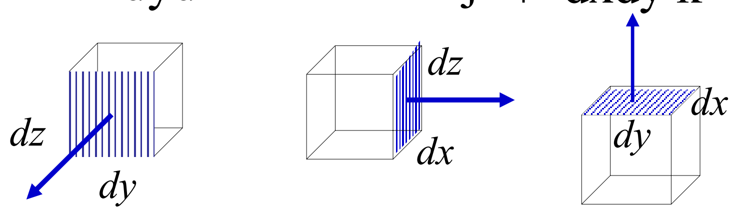


$$d\vec{r} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k}$$

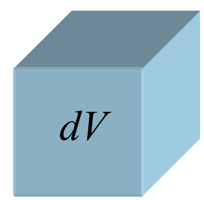
$$C = \int_C \vec{E}\cdot d\vec{r}$$

$$d\vec{S} = dS_x\,\vec{i} + dS_y\,\vec{j} + dS_z\,\vec{k}$$
$$= \mathrm{dydz}\,\vec{\mathrm{i}} + \mathrm{dxdz}\,\vec{\mathrm{j}} + \mathrm{dxdy}\,\vec{\mathrm{k}}$$

$$\Phi = \int_S \vec{E}\cdot d\vec{S}$$



$$dV = dx\cdot dy\cdot dz$$



$$M = \int_V \rho\cdot dV$$

$$\Phi = \int_V (\vec{\nabla}\cdot\vec{E})dV$$

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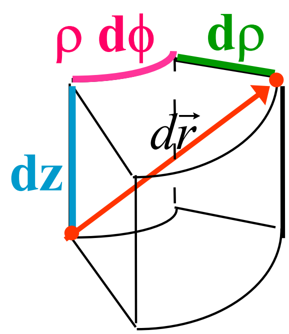
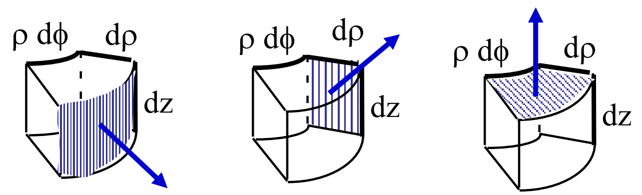
CILÍNDRICAS

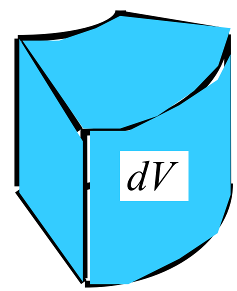


$$\mathbf{d}\vec{\mathbf{r}} = d\rho\,\vec{u}_\rho + \rho\, d\phi\,\vec{u}_\phi + dz\,\vec{u}_z$$

$$\mathbf{d}\vec{\mathbf{S}} = dS_\rho\,\vec{u}_\rho + dS_\phi\,\vec{u}_\phi + dS_z\,\vec{u}_z;$$

$$= \rho\, d\phi\, dz\,\vec{u}_\rho + d\rho\, dz\,\vec{u}_\phi + \rho\, d\phi\, d\rho\,\vec{u}_z;$$





$$dV = \rho\, d\rho\, d\phi\, dz$$

Ejemplo: calcular el flujo a través de una superfície circular de radio a del campo vectorial $\vec{A} = (x^2 + y^2)\vec{u}_z$

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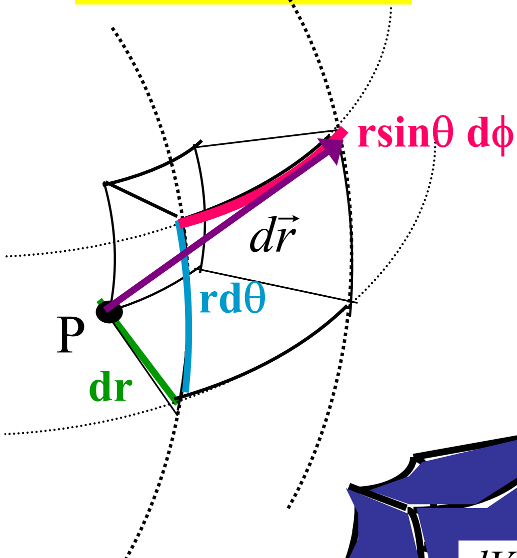
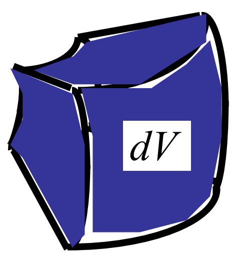
ESFÉRICAS

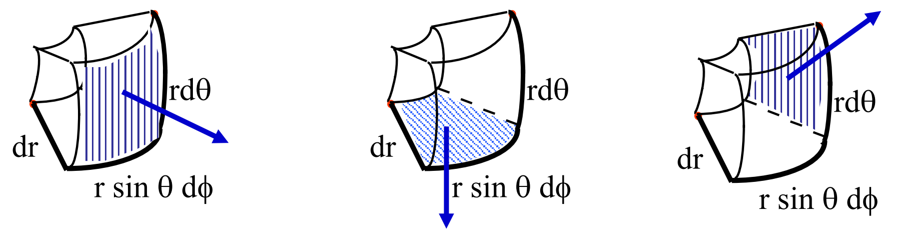




$$d\vec{r} = dr\,\vec{u}_r + r\,d\theta\vec{u}_\theta + r\sin\theta\,d\phi\,\vec{u}_\phi;$$

$$d\vec{S} = dS_r\,\vec{u}_r + dS_\theta\,\vec{u}_\theta + dS_\phi\,\vec{u}_\phi$$

$$= r^2\sin\theta d\theta\,d\phi\,\vec{u}_r + r\sin\theta dr\,d\phi\,\vec{u}_\theta + r\,dr\,d\theta\,\vec{u}_\phi$$



$$dV = r^2 dr\underbrace{\sin\theta\,d\theta\,d\phi}_{d\Omega}$$

Elemento de ángulo sólido

Ángulo sólido de una esfera: $\Omega = \int_0^\pi \sin\theta\,d\theta \int_0^{2\pi} d\phi = 4\pi$

Superficie de una esfera: $S = \int dS_r = a^2 \int d\Omega = 4\pi a^2$

Ejemplo : calcular la carga de una esfera de radio a centrada en el origen, cuya densidad volúmica es

$$\rho(\vec{r}) = x^2 + y^2 + z^2$$

Problema del Boletín: 6.6

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