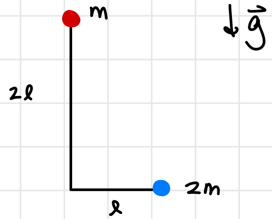
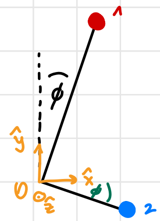
Pauta Aux 17

P1|





$$\vec{r}_1 = 2\ell\,(\sin\phi\,,\cos\phi)$$
$$\vec{r}_2 = \ell\,(\cos\phi\,,-\sin\phi)$$
$$\vec{r}_{CM} = \frac{1}{3m}\left(m\vec{r}_1 + 2m\vec{r}_2\right)$$
$$= \frac{1}{3}\Big(2\ell\,(\sin\phi\,,\cos\phi) + 2\ell\,(\cos\phi\,,-\sin\phi)\Big)$$
$$= \frac{2\ell}{3}\left(\sin\phi+\cos\phi\,,\ \cos\phi-\sin\phi\right)$$

para no tener que usar la normal

Sabemos que $\dfrac{d\vec{L}_O^{CM}}{dt} = \vec{\tau}_{ext}$

veamos $\vec{L}_O^{CM} = \vec{r}_{CM} \times M\,\vec{v}_{CM}$

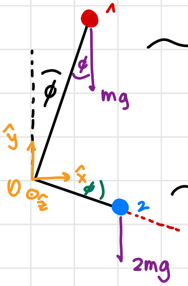
$$= \frac{2\ell}{3}\left(\sin\phi+\cos\phi\,,\ \cos\phi-\sin\phi\right) \times 3m\,\frac{2\ell}{3}\dot{\phi}\left(\cos\phi-\sin\phi\,,\ -\sin\phi-\cos\phi\right)$$

$$= \frac{4\ell^2}{3}m\dot{\phi}\,\Big[(\sin\phi+\cos\phi)(-\sin\phi-\cos\phi) - (\cos\phi-\sin\phi)(\cos\phi-\sin\phi)\Big]\hat{z}$$

$$-\sin^2\phi - \cancel{\sin\phi\cos\phi} - \cancel{\cos\phi\sin\phi} - \cos^2\phi - \cos^2\phi + \cancel{\cos\phi\sin\phi} + \cancel{\sin\phi\cos\phi} - \sin^2\phi\ \Big]\hat{z}$$

$$= \frac{4\ell^2 m\dot{\phi}}{3}\,[-2]\,\hat{z} \xrightarrow{\frac{d}{dt}} -\frac{8}{3}m\ell^2\ddot{\phi}$$

y veamos los $\vec{\tau}^{\,ext}$



$$\vec{\tau}_1 = \vec{r}_1 \times mg(-\hat{y}) = 2\ell\,(\sin\phi\,\hat{x} + \cos\phi\,\hat{y}) \times mg\,(-\hat{y})$$
$$= -2mg\ell\sin\phi\,\hat{z}$$

$$\vec{\tau}_2 = \vec{r}_2 \times 2mg(-\hat{y}) = \ell\,(\cos\phi\,\hat{x} - \sin\phi\,\hat{y}) \times 2mg(-\hat{y})$$
$$= -2mg\ell\cos\phi\,\hat{z}$$

$\therefore$ tenemos $\displaystyle -\frac{8}{3}m\ell^2\ddot{\phi}\,\hat{z} = -2mg\ell\,(\sin\phi+\cos\phi)\,\hat{z}$

$$\boxed{4\,\ddot{\phi} = 3\,g/\ell\,\left(\sin\phi+\cos\phi\right)}$$

Luego

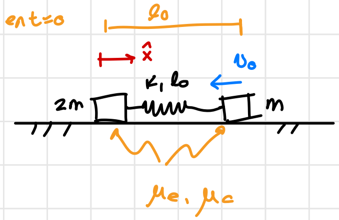
$$4\,\dot{\phi}\,\frac{d\dot{\phi}}{d\phi} = \frac{3g}{\ell}\,(\cos\phi + \sin\phi) \qquad \Big/ \int_0^{\phi} d\phi$$

$$4\int_0^{\dot{\phi}} \dot{\phi}'\,d\dot{\phi}' = \frac{3g}{\ell}\int_0^{\phi}(\cos\phi' + \sin\phi')\,d\phi'$$

$$4\,\frac{\dot{\phi}^2}{2} = \frac{3g}{\ell}\left(\sin\phi'\Big|_0^{\phi} - \cos\phi'\Big|_0^{\phi}\right)$$

$$\dot{\phi}^2 = \frac{3}{2}\,\frac{g}{\ell}\,(\sin\phi + 1 - \cos\phi)$$

P2|



a) si yo tiro mi masa m con vel. v_0 hacia la masa 2m se acercará a una distancia mínima Δ de 2m, en este punto, la F_e será máxima. Para que 2m no se mueva

$$|\mu_e N| \geq |k(\ell_0 - \Delta)|$$

en el límite $\mu_e 2mg = k(\ell_0 - \Delta)$

Calculemos Δ en función de v_0

ecuación de m (con 2m fijo) : $m\ddot{x} = -k(x-\ell_0) + \mu_c N$

$$\cancel{m\ddot{y}}\ 0 = N - mg \rightarrow = mg$$

ojo el signo

se tiene

$$m\ddot{x} = -k(x-\ell_0) + \mu_c mg$$

$$m\dot{x}\frac{d\dot{x}}{dx} = -k(x-\ell_0) + \mu_c mg \quad \Big/ \int_{\ell_0}^{\Delta} dx$$

boscamos el punto más cercano (llega con vel = 0)

parte con vel. v_0

$$m\int_{-v_0}^{0} \dot{x}\, d\dot{x} = -k\int_{\ell_0}^{\Delta} dx\,(x-\ell_0) + \mu_c mg \int_{\ell_0}^{\Delta} dx$$

$$m\frac{\dot{x}^2}{2}\Big|_{v_0}^{0} = -k\left(\frac{x^2}{2}\Big|_{\ell_0}^{\Delta} - \ell_0(\Delta-\ell_0)\right) + \mu_c mg(\Delta - \ell_0)$$

$$-\frac{m}{2}v_0^2 = -k\left(\frac{\Delta^2}{2} - \frac{\ell_0^2}{2} - \ell_0\Delta + \ell_0^2\right) + \mu_c mg\Delta - \mu_c mg\ell_0$$

$$0 = -\frac{k}{2}\Delta^2 + \Delta(k\ell_0 + \mu_c mg) - k\frac{\ell_0^2}{2} - \mu_c mg\ell_0 + \frac{m}{2}v_0^2$$

$$\Longrightarrow \Delta = \frac{-\mu_c mg - k\ell_0 \pm \sqrt{(k\ell_0+\mu_c mg)^2 - 4\frac{k}{2}\left(\frac{k\ell_0^2}{2} + \mu_c mg\ell_0 - \frac{m}{2}v_0^2\right)}}{-k}$$

$$= \ell_0 + \frac{\mu_c mg}{k} \mp \frac{1}{k}\sqrt{\cancel{k^2\ell_0^2} + \mu_c^2 m^2 g^2 + \cancel{2k\ell_0\mu_c mg} - \cancel{k^2\ell_0^2} - \cancel{2k\mu_c mg\ell_0} + km v_0^2}$$

$$= \ell_0 + \frac{\mu_c mg}{k} \mp \frac{1}{k}\sqrt{km v_0^2 + \mu_c^2 m^2 g^2}$$

Luego, tenemos

$$\mu_e 2mg = k(\ell_0 - \Delta)$$

$$= \cancel{k}\left(-\frac{\mu_c mg}{\cancel{k}} + \frac{1}{\cancel{k}}\sqrt{km v_0^2 + \mu_c^2 m^2 g^2}\right)$$

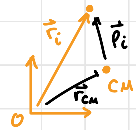
$$(2\mu_e + \mu_c)mg = \sqrt{km v_0^2 + \mu_c^2 m^2 g^2} \quad \Big/ (\,)^2$$

$$(4\mu_e^2 + \cancel{\mu_c^2} + 4\mu_e\mu_c)m^2g^2 = km v_0^2 + \cancel{\mu_c^2 m^2} g^2$$

$$(4\mu_e^2 + 4\mu_e\mu_c)\, m^2 g^2 = k m\, v_0^2$$

$$\boxed{v_0^2 = \frac{4mg^2}{k}\mu_e(\mu_e+\mu_c)}$$

b) $\mu_e = \mu_c = 0$



Usaremos $\quad K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2}\sum_i m_i \dot{\rho}_i$

viene de

$K = \frac{1}{2}\sum_i m_i v_i^2$, pero $\quad \vec{r}_i = \vec{r}_{cm} + \vec{\rho}_i \quad / \frac{d}{dt}$

$\vec{v}_i = \vec{v}_{cm} + \dot{\vec{\rho}}_i$

$$= \frac{1}{2}\sum_i m_i \underbrace{(\vec{v}_{cm} + \dot{\vec{\rho}}_i)^2}_{v_{cm}^2 + \dot{\rho}_i^2 + 2\vec{v}_{cm}\cdot\dot{\vec{\rho}}_i}$$

$$= \frac{1}{2}\left[\underbrace{\sum_i m_i}_{M} v_{cm}^2 + \sum_i m_i \dot{\rho}_i^2 + \sum_i m_i\, 2\vec{v}_{cm}\cdot\dot{\vec{\rho}}_i\right]$$

$$= \frac{1}{2}M\vec{v}_{cm}^2 + \frac{1}{2}\sum_i m_i\dot{\rho}_i^2 + \vec{v}_{cm}\cdot\underbrace{\sum_i m_i\dot{\vec{\rho}}_i}_{=0}$$

es la velocidad del CM c/r al CM, tiene que ser cero

Veamos $\vec{v}_{cm}$, sabemos que

$$M\vec{a}_{cm} = \vec{F}^{ext} = -2mg\hat{z} + \underbrace{N_{2m}}_{2mg}\hat{z} - mg\hat{z} + \underbrace{N_m}_{mg}\hat{z}$$
$$= 0$$

$$\Longrightarrow \vec{v}_{cm} = cte = \vec{v}_{cm}(t=0) = \frac{1}{M}\sum_i m_i v_i$$
$$= \frac{1}{3m}(2m\cdot 0 - m v_0\hat{x})$$
$$= -\frac{v_0}{3}\hat{x}$$

Por lo que

$$K = \frac{1}{2}3m\frac{v_0^2}{9} + \frac{1}{2}\sum_i m_i\dot{\rho}_i^2$$

$$\hookrightarrow E = \frac{1}{2}3m\frac{v_0^2}{9} + \frac{1}{2}\sum_i m_i\dot{\rho}_i^2 + \frac{1}{2}k\underbrace{(\ell-\ell_0)}_{\delta}^2$$

en el tiempo inicial

$$K_i = \frac{1}{2}m v_0^2 \quad y \quad U_i = 0 \Longrightarrow E_i = \frac{1}{2}m v_0^2$$

y cuando ℓ es mínimo (máxima compresión) $\dot{\rho}_i = 0$, si no se seguiría comprimiendo

$$E_f = \cancel{\tfrac{1}{2}}3m\frac{v_0^2}{9} + \cancel{\tfrac{1}{2}}k\,\delta_{min}^2 = \cancel{\tfrac{1}{2}}m v_0^2$$

E_i

$\delta^2 = (\ell-\ell_0)^2 \,/\sqrt{\ }$

$\sqrt{\delta^2} = |\ell-\ell_0|$

$$k\,\delta_{min}^2 = m v_0^2\left(1-\frac{1}{3}\right)$$

$$\underbrace{|\ell_{min}-\ell_0|}_{\ell_0-\ell_{min}} = \sqrt{\frac{2}{3}\frac{m v_0^2}{k}} \quad // \Longrightarrow \boxed{\ell_{min} = \ell_0 - \sqrt{\frac{2mv_0^2}{3k}}}$$