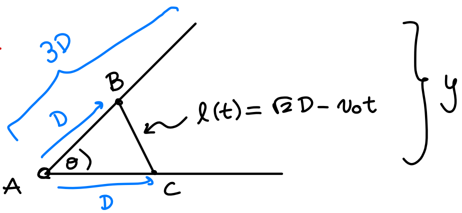
Pauta Aux 4

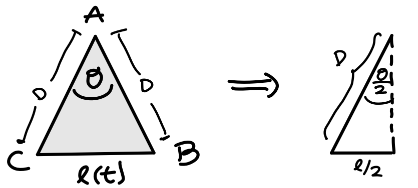
P1



a) Para esta parte nos piden información sobre $y(t)$ (en qué tiempo es igual a $\frac{3D}{2}$)

Pero $y = 3D \sin\theta$, por lo que , es más cómodo trabajar con $\theta(t)$

Como $\overline{AB} = \overline{AC}$ tenemos un triángulo isósceles



$\Rightarrow$ tenemos $D\sin(\theta/2) = \ell/2$

reemplazo $\ell(t)$

$$2D\sin\left(\frac{\theta(t)}{2}\right) = (\sqrt{2}D - v_0 t) \qquad (*)$$

Por otra parte , $y = \frac{3D}{2} \Rightarrow \cancel{3D}\sin\theta = \frac{\cancel{3D}}{2} \Rightarrow \sin\theta = \frac{1}{2}$ y esto se cumple para $\theta = \pi/6$

Luego en el tiempo t^* , $\theta = \pi/6$, reemplazamos en $(*)$

$$\rightarrow 2D\underbrace{\sin\left(\frac{\pi}{6}\frac{1}{2}\right)}_{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \sqrt{2}D - v_0 t^*$$

$$t^* = \frac{1}{v_0}\sqrt{2}D\left(1 - \frac{\cancel{2}(\sqrt{3}-1)}{2\cancel{\sqrt{2}}}\right)$$

$$= \frac{1}{v_0}\sqrt{2}D\left(1 + \frac{1}{2} - \frac{\sqrt{3}}{2}\right)$$

$$\boxed{t^* = \frac{D}{v_0\sqrt{2}}(3 - \sqrt{3})}$$

b) Para esto ocupamos nuevamente $(*)$ y para obtener $\dot{\theta}$ hacemos $\frac{d}{dt}(*)$

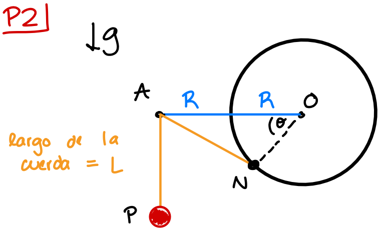
$$\frac{d}{dt}2D\sin\left(\frac{\theta(t)}{2}\right) = \frac{d}{dt}(\sqrt{2}D - v_0 t)$$

$$\cancel{2}D\cos\left(\frac{\theta(t)}{2}\right)\frac{\dot{\theta}}{\cancel{2}} = -v_0$$

y nos piden la vel. cuando $\theta = \pi/6$

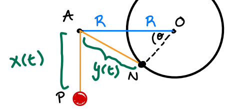
$$D\cos\left(\frac{\pi}{4}\frac{1}{2}\right)\dot{\theta} = -v_0 \Rightarrow \boxed{\dot{\theta} = \frac{-v_0}{D\underbrace{\cos(\pi/12)}_{\frac{1+\sqrt{3}}{2\sqrt{2}}}} = \frac{-v_0\, 2\sqrt{2}}{D(1+\sqrt{3})}}$$

P2|



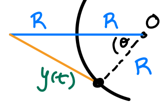
a) $\dot{\theta} = \omega_0$ constante

nos piden encontrar la rapidez máxima de P (se mueve sólo verticalmente) por lo que, tenemos que parametrizar la posición de P



Sabemos que $x(t) + y(t) = L \quad / \frac{d}{dt}$

$\dot{x}(t) + \dot{y}(t) = 0 \implies$ nos basta encontrar $\dot{y}(t)$, lo hacemos geométricamente, tenemos un triángulo



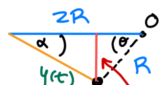
teo coseno

(★) $y^2(t) = (2R)^2 + R^2 - 2(2R)R\cos\theta \quad / \frac{d}{dt}$

$2y\dot{y} = +4R^2 \sin\theta\, \dot{\theta}$

$2y\dot{y} = 4R^2 \sin\theta\, \omega_0$

$\dot{y} = 2R^2 \omega_0 \frac{\sin\theta}{y} \longrightarrow$ para maximizar $|\dot{y}|$ hay que max. $\frac{\sin\theta}{y}$ pero notar

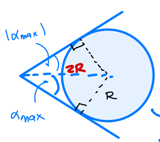


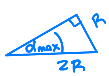
$$\frac{\boxed{R\sin\theta}}{y} = \frac{\cancel{y}\sin\alpha}{\cancel{y}}$$

Por lo que $\max\limits_{\theta} \frac{\sin\theta}{y} = \frac{1}{R} \max\limits_{\alpha} \sin\alpha$

importante, para maximizar $\sin\alpha$, tenemos que ver cuáles son los α's permitidos

y α es máximo cuando la cuerda es tangente al aro





$\alpha_{max} = \sin^{-1}\frac{R}{2R} = \pi/6$

$\implies \alpha \in [-\pi/6, \pi/6]$

$\implies \dot{y} = 2R\omega_0 \sin\alpha \implies \max|\dot{x}| = \max|\dot{y}| = 2R\omega_0 \max\limits_{\alpha \in [-\frac{\pi}{6}, \frac{\pi}{6}]} \sin\alpha$

$= 2R\omega_0 \sin\pi/6$

$= R\omega_0$

Otra forma: maximicemos $\frac{\sin\theta}{y}$ directamente

$$\frac{\sin\theta}{y} \overset{\text{usando (★)}}{=} \frac{\sin\theta}{R\sqrt{5-4\cos\theta}} = f(\theta)$$

para encontrar el máximo

$$\frac{d}{d\theta} f(\theta) = \frac{\cos\theta \, R\sqrt{5-4\cos\theta} - \sin\theta \, R \, \frac{+4\sin\theta}{2\sqrt{5-4\cos\theta}}}{R^2 (5-4\cos\theta)} \overset{!}{=} 0 \qquad /\cdot\, R(5-4\cos\theta)$$

$$\cos\theta \, \sqrt{5-4\cos\theta} - 2\sin^2\theta \, \frac{1}{\sqrt{5-4\cos\theta}} = 0 \qquad \cdot / \sqrt{5-4\cos\theta}$$

$$\cos\theta\,(5-4\cos\theta) - 2(1-\cos^2\theta) = 0$$

$$-2\cos^2\theta + 5\cos\theta - 2 = 0$$

$$\cos\theta = \frac{-5 \pm \sqrt{25-4\cdot 2\cdot 2}}{4}$$

$$\cos\theta = -\frac{5}{4} \pm \frac{\sqrt{9}}{4} = \frac{-5\pm 3}{4}$$

me quedo con el + porque el − me da $\cos\theta < -1$

$$\Rightarrow \cos\theta = -\frac{2}{4} = -\frac{1}{2} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

$$\max f(\theta) = \frac{\cancel{\sqrt{3}}/2}{R\underbrace{\sqrt{5-4\cdot 1/2}}_{=\cancel{\sqrt{3}}}} = \frac{1}{2R}$$

$$|\dot{x}|_{max} = 2R^2\omega_0 \underbrace{\max \frac{\sin\theta}{y}}_{\max f(\theta) = \frac{1}{2R}} = R\omega_0 \quad //$$

b) ahora queremos $\dot{x} = v_0$ cte y encontrar $\dot{\theta}(\theta = \pi/3)$

$\to \dot{y} = -v_0$

teníamos la ecuación:

$$2y\dot{y} = +4R^2 \sin\theta\,\dot{\theta}$$

$$-y v_0 = 2R^2 \sin\theta\,\dot{\theta}$$

$$\dot{\theta} = -\frac{v_0 \cancel{R}\sqrt{5-4\cos\theta}}{2R^{\cancel{2}}\sin\theta}$$

usando $y^2(t) = (2R)^2 + R^2 - 2(2R)R\cos\theta$

$$y = R\sqrt{5-4\cos\theta}$$

$\cos \pi/3 = \frac{1}{2}$

$\sin \pi/3 = \sqrt{3}/2$

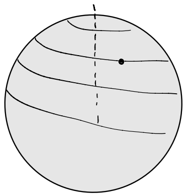
$$\dot{\theta}(\theta = \pi/3) = -\frac{v_0}{\cancel{2}R} \frac{\cancel{\sqrt{5-4/2}}}{\cancel{\sqrt{3}}/\cancel{2}}$$

$$= -\frac{v_0}{R}$$

y la aceleración centrípeta es $a_c = \frac{v^2}{R} = R\dot{\theta}^2$

$$= \frac{v_0^2}{R}$$

P3



$r = R_0$ $\qquad \underbrace{\phi = N\theta \qquad \dot{\theta} = \omega_0}_{\dot{\phi} = N\dot{\theta} = N\omega_0}$ $\qquad$ y $\quad \ddot{\phi} = \ddot{\theta} = 0$

1. Ocupamos las fórmulas en coords. esféricas

r cte

$r = R_0 \quad \dot{\phi} = N\omega_0 \quad \dot{\theta} = \omega_0$

$$\vec{v} = \cancel{\dot{r}}^{\,0}\,\hat{r} + r\dot{\theta}\,\hat{\theta} + r\dot{\phi}\sin\theta\,\hat{\phi} = R_0\omega_0\,\hat{\theta} + R_0 N\omega_0 \sin\theta\,\hat{\phi}$$

$\ddot{\theta} = 0$, $\ddot{\phi} = 0$

$$\begin{aligned}\vec{a} = &\left(\cancel{\ddot{r}}^{\,0} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} &= &-\left(R_0\omega_0^2 + R_0(N\omega_0)^2\sin^2\theta\right)\hat{r}\\ &+\left(\cancel{r\ddot{\theta}}^{\,0} + \cancel{2\dot{r}\dot{\theta}}^{\,0} - r\dot{\phi}^2\sin\theta\cos\theta\right)\hat{\theta} & &- R_0(N\omega_0)^2\sin\theta\cos\theta\,\hat{\theta}\\ &+\left(\cancel{r\ddot{\phi}\sin\theta}^{\,0} + \cancel{2\dot{r}\dot{\phi}\sin\theta}^{\,0} + 2r\dot{\phi}\dot{\theta}\cos\theta\right)\hat{\phi} & &+ 2R_0 N\omega_0^2\cos\theta\,\hat{\phi}\end{aligned}$$

2. $\theta = \pi/2 \implies$

$\cos\theta = 0$
$\sin\theta = 1$

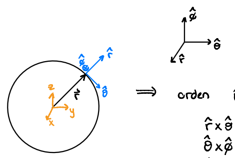
$$\vec{v} = R_0\omega_0\,\hat{\theta} + R_0 N\omega_0\,\hat{\phi} \Longrightarrow \overset{|\vec{v}|^2}{\overset{\parallel}{\vec{v}\cdot\vec{v}}} = (R_0\omega_0)^2\,\underbrace{\hat{\theta}\cdot\hat{\theta}}_{1} + (R_0 N\omega_0)^2\,\underbrace{\hat{\phi}\cdot\hat{\phi}}_{1} = R_0^2\omega_0^2(N^2+1)$$

$\hat{\theta}\cdot\hat{\phi} = 0$

$$\vec{a} = -\underbrace{\left(R_0\omega_0^2 + R_0(N\omega_0)^2\right)}_{R_0\omega_0^2(N^2+1)}\hat{r}$$

$$\Longrightarrow |\vec{v}|^3 = \left[R_0^2\omega_0^2(N^2+1)\right]^{3/2}$$

$$\begin{aligned}\vec{v}\times\vec{a} &= \left[R_0\omega_0\,\hat{\theta} + R_0 N\omega_0\,\hat{\phi}\right]\times\left[-\left(R_0\omega_0^2(N^2+1)\right)\hat{r}\right]\\ &= -R_0^2\omega_0^3(N^2+1)\,\underbrace{\hat{\theta}\times\hat{r}}_{-\hat{\phi}} - R_0^2 N\omega_0^3(N^2+1)\,\underbrace{\hat{\phi}\times\hat{r}}_{\hat{\theta}}\\ &= +R_0^2\omega_0^3(N^2+1)\,\hat{\phi} - R_0^2 N\omega_0^3(N^2+1)\,\hat{\theta}\end{aligned}$$



$\Longrightarrow$ orden $\hat{r}, \hat{\theta}, \hat{\phi}$

$\hat{r}\times\hat{\theta} = \hat{\phi}$ $\qquad \hat{\theta}\times\hat{r} = -\hat{\phi}$

$\hat{\theta}\times\hat{\phi} = \hat{r}$ $\quad$ y $\quad \hat{\phi}\times\hat{\theta} = -\hat{r}$

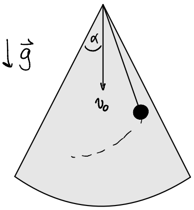
$\hat{\phi}\times\hat{r} = \hat{\theta}$ $\qquad \hat{r}\times\hat{\phi} = -\hat{\theta}$

Luego

$$\begin{aligned}|\vec{v}\times\vec{a}| &= \sqrt{R_0^4\omega_0^6(N^2+1)^2 + R_0^4 N^2\omega_0^6(N^2+1)^2}\\ &= \sqrt{R_0^4\omega_0^6(N^2+1)^2(1+N^2)}\\ &= \left(R_0^4\omega_0^6(N^2+1)^3\right)^{1/2} = R_0^2\omega_0^3(N^2+1)^{3/2}\end{aligned}$$

$$r_v = \frac{\left[R_0^2\omega_0^2(N^2+1)\right]^{3/2}}{R_0^2\omega_0^3(N^2+1)^{3/2}} = \frac{R_0^{\cancel{3}}\,\cancel{\omega_0^3}\,\cancel{(N^2+1)^{3/2}}}{\cancel{R_0^2}\,\cancel{\omega_0^3}\,\cancel{(N^2+1)^{3/2}}} = R_0$$

P4



Datos: $\theta = \alpha$ (usamos coord. esféricas) ojo: son coord. esféricas pero dadas vuelta, para que $\theta = \alpha$ cte y no $\theta = \pi - \alpha$

$\dot{\phi}(t=0) = \omega_0$

$r(t=0) = r_0$

Además se recoge la cuerda con velocidad v_0 $\Longrightarrow$ $\dot{r} = -v_0$

$\ddot{r} = 0$

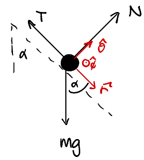
DCL

$$a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 \operatorname{sen}^2\theta = -r\dot{\phi}^2 \operatorname{sen}^2\alpha$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2 \operatorname{sen}\theta\cos\theta = -r\dot{\phi}^2 \operatorname{sen}\alpha\cos\alpha$$

$$a_\phi = \frac{1}{r\operatorname{sen}\theta}\frac{d}{dt}\left(r^2\dot{\phi}\sin^2\theta\right) = \frac{\sin^{\cancel{2}}\alpha}{r\cancel{\sin\alpha}}\frac{d}{dt}\left(r^2\dot{\phi}\right)$$

otra forma general de la aceleración en $\hat{\phi}$



$$\Longrightarrow \vec{F} = N\hat{\theta} + mg(\cos\alpha\,\hat{r} - \sin\alpha\,\hat{\theta})$$

ec. de mov

$$\hat{r}: \quad -mr\dot{\phi}^2 \operatorname{sen}^2\alpha = mg\cos\alpha - T$$

$$\hat{\theta}: \quad -mr\dot{\phi}^2 \operatorname{sen}\alpha\cos\alpha = N - mg\sin\alpha \longrightarrow -mr\frac{\ell^2}{r^4}\operatorname{sen}\alpha\cos\alpha = N - mg\sin\alpha$$

condición de despegue $N=0$

$$\hat{\phi}: \quad \frac{m\sin\alpha}{r}\frac{d}{dt}\left(r^2\dot{\phi}\right) = 0$$

$$r^2\dot{\phi} = cte = \ell$$

lo calculo con los valores en $t=0$

$$\ell = r(t=0)^2\,\dot{\phi}(t=0) = r_0^2\,\omega_0$$

$$+\cancel{m}\frac{\ell^2}{r^3}\cancel{\operatorname{sen}\alpha}\cos\alpha = +\cancel{m}g\,\cancel{\sin\alpha}$$

$$r^3 = \frac{\ell^2}{g}\cos\alpha$$

$$\boxed{r^3_{despegue} = \frac{r_0^4\omega_0^2}{g}\cos\alpha}$$

Para calcular T reemplazamos todo en la ec. de $\hat{r}$

$$-m\,r_{despegue}\frac{\ell^2}{r^4_{despegue}}\sin^2\alpha = mg\cos\alpha - T$$

$$-m\frac{1}{r_{despegue}^3}r_0^4\omega_0^2\sin^2\alpha = mg\cos\alpha - T$$

$$-m\frac{g}{\cancel{r_0^4\omega_0^2}\cos\alpha}\cancel{r_0^4\omega_0^2}\sin^2\alpha = mg\cos\alpha - T$$

$$T = mg\Big(\underbrace{\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}}_{\frac{\cos^2\alpha + \sin^2\alpha}{\cos\alpha} = \frac{1}{\cos\alpha}}\Big)$$

$$\boxed{T = \frac{mg}{\cos\alpha}}$$