Formulario de Mecánica

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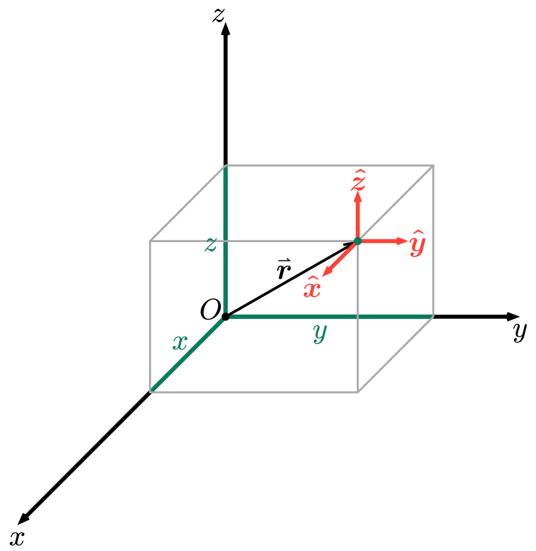
Cinemática y coordenadas

Sistemas de coordenadas

Coordenadas cartesianas



Los dominios de cada coordenada son:

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

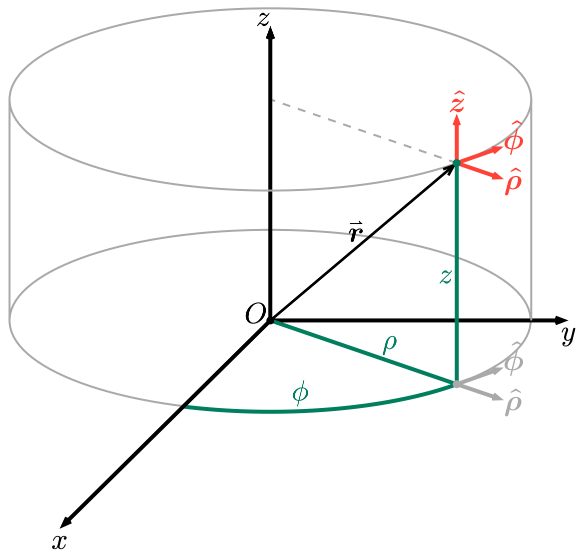
Vector posición, velocidad y aceleración:

$$\boldsymbol{r}(t) = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$$
$$\boldsymbol{v}(t) = \dot{x}\hat{\boldsymbol{x}} + \dot{y}\hat{\boldsymbol{y}} + \dot{z}\hat{\boldsymbol{z}}$$
$$\boldsymbol{a}(t) = \ddot{x}\hat{\boldsymbol{x}} + \ddot{y}\hat{\boldsymbol{y}} + \ddot{z}\hat{\boldsymbol{z}}$$

Rapidez $v = \|\boldsymbol{v}\|$:

$$v(t) = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

Coordenadas cilíndricas (polares en $z = 0$)



Los dominios de cada coordenada son:

$$0 \le \rho < \infty$$
$$0 \le \phi < 2\pi$$
$$-\infty < z < \infty$$

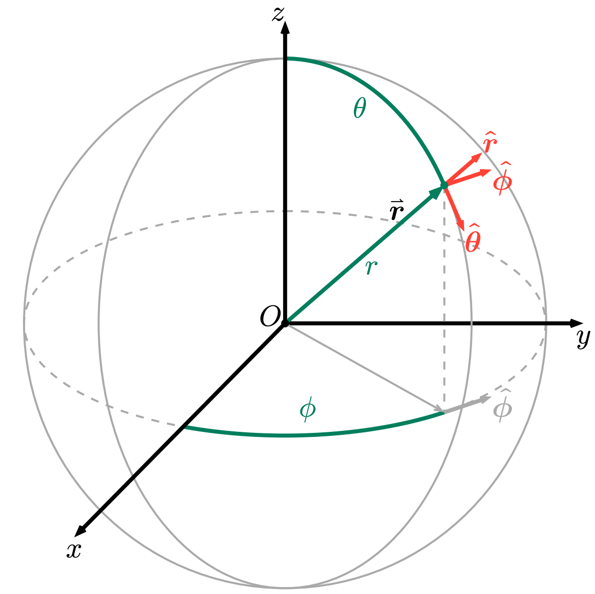
Vector posición, velocidad y aceleración:

$$\boldsymbol{r}(t) = \rho\hat{\boldsymbol{\rho}} + z\hat{\boldsymbol{z}}$$
$$\boldsymbol{v}(t) = \dot{\rho}\hat{\boldsymbol{\rho}} + \rho\dot{\phi}\hat{\boldsymbol{\phi}} + \dot{z}\hat{\boldsymbol{z}}$$
$$\boldsymbol{a}(t) = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{\boldsymbol{\rho}} + \underbrace{\left(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi}\right)}_{\frac{1}{\rho}\frac{\mathrm{d}}{\mathrm{d}t}\left(\rho^2\dot{\phi}\right)}\hat{\boldsymbol{\phi}} + \ddot{z}\hat{\boldsymbol{z}}$$

Rapidez $v = \|\boldsymbol{v}\|$:

$$v(t) = \sqrt{\dot{\rho}^2 + \rho^2\dot{\phi}^2 + \dot{z}^2}$$

Coordenadas esféricas



Los dominios de cada coordenada son:

$$0 \leq r < \infty$$
$$0 \leq \theta \leq \pi$$
$$0 \leq \phi < 2\pi$$

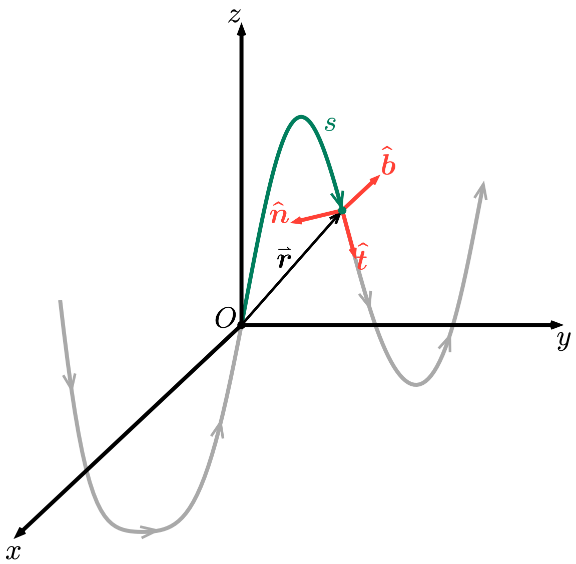
Vector posición, velocidad y aceleración:

$$\boldsymbol{r}(t) = r\hat{\boldsymbol{r}}$$
$$\boldsymbol{v}(t) = \dot{r}\hat{\boldsymbol{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}} + r\dot{\phi}\sin\theta\hat{\boldsymbol{\phi}}$$
$$\begin{aligned}\boldsymbol{a}(t) = {} & \left(\ddot{r} - r\dot{\phi}^2\sin^2\theta - r\dot{\theta}^2\right)\hat{\boldsymbol{r}} \\ & + \left(r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta + 2\dot{r}\dot{\theta}\right)\hat{\boldsymbol{\theta}} \\ & + \underbrace{\left(2\dot{r}\dot{\phi}\sin\theta + r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta\right)}_{\frac{1}{r\sin\theta}\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\dot{\phi}\sin^2\theta\right)}\hat{\boldsymbol{\phi}}\end{aligned}$$

Rapidez $v = \|\boldsymbol{v}\|$:

$$v(t) = \sqrt{\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta}$$

Coordenadas intrínsecas



Vector velocidad y aceleración:

$$\boldsymbol{v}(s) = \dot{s}\hat{\boldsymbol{t}}$$
$$\boldsymbol{a}(s) = \ddot{s}\hat{\boldsymbol{t}} + \frac{\dot{s}^2}{R_\mathrm{c}}\hat{\boldsymbol{n}}$$

Vectores unitarios tangente y normal:

$$\hat{\boldsymbol{t}} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s} \qquad \hat{\boldsymbol{n}} = \frac{\mathrm{d}\hat{\boldsymbol{t}}/\,\mathrm{d}s}{\left\|\mathrm{d}\hat{\boldsymbol{t}}/\,\mathrm{d}s\right\|}$$

Radio de curvatura y longitud de una curva:

$$R_\mathrm{c} = \frac{v^3}{\|\boldsymbol{v}\times\boldsymbol{a}\|} \qquad L = \int_0^t \left\|\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}\right\|\mathrm{d}t$$

Propiedades de los vectores base

Variación temporal de vectores unitarios

Base cilíndrica

$$\dot{\hat{\boldsymbol{\rho}}} = \dot{\phi}\hat{\boldsymbol{\phi}} \qquad \text{y} \qquad \dot{\hat{\boldsymbol{\phi}}} = -\dot{\phi}\hat{\boldsymbol{\rho}}$$

Base esférica

$$\dot{\hat{\boldsymbol{r}}} = \dot{\theta}\hat{\boldsymbol{\theta}} + \dot{\phi}\sin\theta\hat{\boldsymbol{\phi}} \qquad \dot{\hat{\boldsymbol{\theta}}} = -\dot{\theta}\hat{\boldsymbol{r}} + \dot{\phi}\cos\theta\hat{\boldsymbol{\phi}} \qquad \text{y} \qquad \dot{\hat{\boldsymbol{\phi}}} = -\dot{\phi}\sin\theta\hat{\boldsymbol{r}} - \dot{\phi}\cos\theta\hat{\boldsymbol{\theta}}$$

Ortonormalidad y regla de base derecha

Base cartesiana

$$\begin{aligned}
\hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{x}} &= 1 & \hat{\boldsymbol{x}}\cdot\hat{\boldsymbol{y}} &= 0 & \hat{\boldsymbol{x}}\times\hat{\boldsymbol{y}} &= \hat{\boldsymbol{z}} \\
\hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{y}} &= 1 & \hat{\boldsymbol{y}}\cdot\hat{\boldsymbol{z}} &= 0 & \hat{\boldsymbol{y}}\times\hat{\boldsymbol{z}} &= \hat{\boldsymbol{x}} \\
\hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{z}} &= 1 & \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{x}} &= 0 & \hat{\boldsymbol{z}}\times\hat{\boldsymbol{x}} &= \hat{\boldsymbol{y}}
\end{aligned}$$

Base cilíndrica

$$\begin{aligned}
\hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\rho}} &= 1 & \hat{\boldsymbol{\rho}}\cdot\hat{\boldsymbol{\phi}} &= 0 & \hat{\boldsymbol{\rho}}\times\hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{z}} \\
\hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} &= 1 & \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{z}} &= 0 & \hat{\boldsymbol{\phi}}\times\hat{\boldsymbol{z}} &= \hat{\boldsymbol{\rho}} \\
\hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{z}} &= 1 & \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{\rho}} &= 0 & \hat{\boldsymbol{z}}\times\hat{\boldsymbol{\rho}} &= \hat{\boldsymbol{\phi}}
\end{aligned}$$

Base esférica

$$\begin{aligned}
\hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{r}} &= 1 & \hat{\boldsymbol{r}}\cdot\hat{\boldsymbol{\theta}} &= 0 & \hat{\boldsymbol{r}}\times\hat{\boldsymbol{\theta}} &= \hat{\boldsymbol{\phi}} \\
\hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} &= 1 & \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} &= 0 & \hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{r}} \\
\hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} &= 1 & \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{r}} &= 0 & \hat{\boldsymbol{\phi}}\times\hat{\boldsymbol{r}} &= \hat{\boldsymbol{\theta}}
\end{aligned}$$

Base intrínseca

$$\begin{aligned}
\hat{\boldsymbol{t}}\cdot\hat{\boldsymbol{t}} &= 1 & \hat{\boldsymbol{t}}\cdot\hat{\boldsymbol{n}} &= 0 & \hat{\boldsymbol{t}}\times\hat{\boldsymbol{n}} &= \hat{\boldsymbol{b}} \\
\hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{n}} &= 1 & \hat{\boldsymbol{n}}\cdot\hat{\boldsymbol{b}} &= 0 & \hat{\boldsymbol{n}}\times\hat{\boldsymbol{b}} &= \hat{\boldsymbol{t}} \\
\hat{\boldsymbol{b}}\cdot\hat{\boldsymbol{b}} &= 1 & \hat{\boldsymbol{b}}\cdot\hat{\boldsymbol{t}} &= 0 & \hat{\boldsymbol{b}}\times\hat{\boldsymbol{t}} &= \hat{\boldsymbol{n}}
\end{aligned}$$

Relaciones entre coordenadas, bases, y distancias

Relaciones entre coordenadas y bases

Base cartesiana a base cilíndrica o esférica

La transformación de las coordenadas y de la base cartesiana a la cilíndrica es

$$\begin{aligned} x &= \rho\cos\phi & \hat{\boldsymbol{x}} &= \cos\phi\hat{\boldsymbol{\rho}} - \sin\phi\hat{\boldsymbol{\phi}} \\ y &= \rho\sin\phi & \hat{\boldsymbol{y}} &= \sin\phi\hat{\boldsymbol{\rho}} + \cos\phi\hat{\boldsymbol{\phi}} \\ z &= z & \hat{\boldsymbol{z}} &= \hat{\boldsymbol{z}} \end{aligned}$$

La transformación de las coordenadas y de la base cartesiana a la esférica es

$$\begin{aligned} x &= r\cos\phi\sin\theta & \hat{\boldsymbol{x}} &= \sin\theta\cos\phi\hat{\boldsymbol{r}} + \cos\theta\cos\phi\hat{\boldsymbol{\theta}} - \sin\phi\hat{\boldsymbol{\phi}} \\ y &= r\sin\phi\sin\theta & \hat{\boldsymbol{y}} &= \sin\theta\sin\phi\hat{\boldsymbol{r}} + \cos\theta\sin\phi\hat{\boldsymbol{\theta}} + \cos\phi\hat{\boldsymbol{\phi}} \\ z &= r\cos\theta & \hat{\boldsymbol{z}} &= \cos\theta\hat{\boldsymbol{r}} - \sin\theta\hat{\boldsymbol{\theta}} \end{aligned}$$

Base cilíndrica a base cartesiana o esférica

La transformación de las coordenadas y de la base cilíndrica a la cartesiana es

$$\begin{aligned} \rho &= \sqrt{x^2+y^2} & \hat{\boldsymbol{\rho}} &= \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{x}} + \frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{y}} \\ \phi &= \operatorname{atan2}(y,x) & \hat{\boldsymbol{\phi}} &= -\frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{x}} + \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{y}} \\ z &= z & \hat{\boldsymbol{z}} &= \hat{\boldsymbol{z}} \end{aligned}$$

La transformación de las coordenadas y de la base cilíndrica a la esférica es

$$\begin{aligned} \rho &= r\sin\theta & \hat{\boldsymbol{\rho}} &= \sin\theta\hat{\boldsymbol{r}} + \cos\theta\hat{\boldsymbol{\theta}} \\ \phi &= \phi & \hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{\phi}} \\ z &= r\cos\theta & \hat{\boldsymbol{z}} &= \cos\theta\hat{\boldsymbol{r}} - \sin\theta\hat{\boldsymbol{\theta}} \end{aligned}$$

Base esférica a base cartesiana o cilíndrica

La transformación de las coordenadas esféricas a las cartesianas es

$$\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \arccos\frac{z}{\sqrt{x^2+y^2+z^2}} \\ \phi &= \operatorname{atan2}(y,x) \end{aligned}$$

y de la base esférica a la cartesiana es

$$\begin{aligned} \hat{\boldsymbol{r}} &= \frac{x}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{x}} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{y}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{z}} \\ \hat{\boldsymbol{\theta}} &= \frac{x}{\sqrt{x^2+y^2}}\frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{x}} + \frac{y}{\sqrt{x^2+y^2}}\frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{y}} \\ &\quad -\frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\hat{\boldsymbol{z}} \\ \hat{\boldsymbol{\phi}} &= -\frac{y}{\sqrt{x^2+y^2}}\hat{\boldsymbol{x}} + \frac{x}{\sqrt{x^2+y^2}}\hat{\boldsymbol{y}} \end{aligned}$$

La transformación de las coordenadas y de la base esférica a la cilíndrica es

$$
\begin{aligned}
r &= \sqrt{\rho^2 + z^2} & \hat{\boldsymbol{r}} &= \frac{\rho}{\sqrt{\rho^2 + z^2}} \hat{\boldsymbol{\rho}} + \frac{z}{\sqrt{\rho^2 + z^2}} \hat{\boldsymbol{z}} \\
\theta &= \arccos \frac{z}{\sqrt{\rho^2 + z^2}} & \hat{\boldsymbol{\theta}} &= \frac{z}{\sqrt{\rho^2 + z^2}} \hat{\boldsymbol{\rho}} - \frac{\rho}{\sqrt{\rho^2 + z^2}} \hat{\boldsymbol{z}} \\
\phi &= \phi & \hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{\phi}}
\end{aligned}
$$

Distancia entre dos puntos

La distancia entre dos puntos $\boldsymbol{r}$ y $\boldsymbol{r}'$ usando norma euclidiana es

$$d = \|\boldsymbol{r} - \boldsymbol{r}'\|$$

Base cartesiana

La distancia entre los puntos $\boldsymbol{r} = (x, y, z)$ y $\boldsymbol{r}' = (x', y', z')$ es

$$d = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$

Base cilíndrica

La distancia entre los puntos $\boldsymbol{r} = (\rho, \phi, z)$ y $\boldsymbol{r}' = (\rho', \phi', z')$ es

$$d = \sqrt{\rho^2 + {\rho'}^2 - 2\rho\rho' \cos(\phi - \phi') + (z - z')^2}$$

Base esférica

La distancia entre los puntos $\boldsymbol{r} = (r, \theta, \phi)$ y $\boldsymbol{r}' = (r', \theta', \phi')$ es

$$d = \sqrt{r^2 + {r'}^2 - 2rr'[\sin\theta \sin\theta' \cos(\phi - \phi') + \cos\theta \cos\theta']}$$

Elementos y operadores diferenciales

Elementos diferenciales

Base cartesiana

El *elemento de línea* es

$$\mathrm{d}\boldsymbol{r} = \mathrm{d}x\hat{\boldsymbol{x}} + \mathrm{d}y\hat{\boldsymbol{y}} + \mathrm{d}z\hat{\boldsymbol{z}}$$

Los *elementos de área* a x, y o z constantes son, respectivamente,

$$\begin{aligned} \mathrm{d}S_x &= \mathrm{d}y\,\mathrm{d}z \\ \mathrm{d}S_y &= \mathrm{d}z\,\mathrm{d}x \\ \mathrm{d}S_z &= \mathrm{d}x\,\mathrm{d}y \end{aligned}$$

El *elemento de volumen* es

$$\mathrm{d}V = \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

Base cilíndrica

El *elemento de línea* es

$$\mathrm{d}\boldsymbol{r} = \mathrm{d}\rho\hat{\boldsymbol{\rho}} + \rho\,\mathrm{d}\phi\hat{\boldsymbol{\phi}} + \mathrm{d}z\hat{\boldsymbol{z}}$$

Los *elementos de área* a ρ, ϕ o z constantes son, respectivamente,

$$\begin{aligned} \mathrm{d}S_\rho &= \rho\,\mathrm{d}\phi\,\mathrm{d}z \\ \mathrm{d}S_\phi &= \mathrm{d}z\,\mathrm{d}\rho \\ \mathrm{d}S_z &= \rho\,\mathrm{d}\rho\,\mathrm{d}\phi \end{aligned}$$

El *elemento de volumen* es

$$\mathrm{d}V = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z$$

Base esférica

El *elemento de línea* es

$$\mathrm{d}\boldsymbol{r} = \mathrm{d}r\hat{\boldsymbol{r}} + r\,\mathrm{d}\theta\hat{\boldsymbol{\theta}} + r\sin\theta\,\mathrm{d}\phi\hat{\boldsymbol{\phi}}$$

Los *elementos de área* a r, θ o ϕ constantes son, respectivamente,

$$\begin{aligned} \mathrm{d}S_r &= r^2\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi \\ \mathrm{d}S_\theta &= r\sin\theta\,\mathrm{d}\phi\,\mathrm{d}r \\ \mathrm{d}S_\phi &= r\,\mathrm{d}r\,\mathrm{d}\theta \end{aligned}$$

El *elemento de volumen* es

$$\mathrm{d}V = r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi = r^2\,\mathrm{d}r\,\mathrm{d}\Omega$$

donde se define el *elemento de ángulo sólido* $\mathrm{d}\Omega = \sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$.

Operadores diferenciales

Base cartesiana

El *gradiente* y el *laplaciano* de un campo escalar f son, respectivamente,

$$\nabla f = \frac{\partial f}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial f}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

La *divergencia* y el *rotor* de un campo vectorial $\boldsymbol{A}$ son, respectivamente,

$$\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\boldsymbol{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\boldsymbol{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\boldsymbol{z}}$$

Base cilíndrica

El *gradiente* y el *laplaciano* de un campo escalar f son, respectivamente,

$$\nabla f = \frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$$

$$\nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

La *divergencia* y el *rotor* de un campo vectorial $\boldsymbol{A}$ son, respectivamente,

$$\nabla \cdot \boldsymbol{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \boldsymbol{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\boldsymbol{\rho}} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\phi}} + \frac{1}{\rho}\left(\frac{\partial(\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol{z}}$$

Base esférica

El *gradiente* y el *laplaciano* de un campo escalar f son, respectivamente,

$$\nabla f = \frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

La *divergencia* y el *rotor* de un campo vectorial $\boldsymbol{A}$ son, respectivamente,

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \boldsymbol{A} = \frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r}\right)\hat{\boldsymbol{\theta}}$$
$$+\frac{1}{r}\left(\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$$