the corresponding grade at which this occurs is the treatment plant/market balancing cut-off grade, g_{kh}.

The determination of the balancing cut-off grades in a particular case is best achieved in practice by tabulating the grade distribution data for the relevant mine planning increment and extending it with columns of ratios and cumulative values. The balancing points can then frequently be found by inspection. The detailed calculations are illustrated by the example included in Case Study 3 (p.116).

Mines operating with more than one component of the system at capacity are the rule rather than the exception. Most mine operators appreciate that idle capacity usually involves some cost so, as far as possible, plans are moulded to exploit fully the capacities available. Should this prove difficult in the course of time, capital proposals to expand the restricting component are frequently prepared and commonly implemented because of the advantages which arise from eliminating a bottleneck at a marginal capital cost.

The role of cut-off as a balancing parameter is, in effect, understood but not usually admitted. The reason seems to be that cut-off is regarded, rightly, as a strategic parameter and, as a consequence, managements are reluctant to use it for tactical purposes. Instead, plans are reworked, including or excluding doubtful ore as necessary, until a satisfactory balance has been obtained. This amounts to changing the cut-off grade even though it is not acknowledged.

Balancing cut-off grades actually have both strategic and tactical elements. The strategy is the average level which achieves a balance in the longer term. The tactics are the week-to-week or the month-to-month changes which are found necessary to accommodate the shorter term variations in the mineralised material which is available for mining. One function of mine planning is to develop plans which iron out such short-term variations in the interest of smooth efficient running of the mine, but mineralised bodies are not always amenable. In other words, balancing cut-off grades are dynamic parameters which are dependent upon the grade distribution of the mineralised material that is available for mining at any point in time.

CHAPTER SEVEN

Effective Optimum Cut-Off Grades

The quest for an optimum has led to a plethora of cut-off grades — six, in fact. There are three limiting economic cut-off grades and three balancing cut-off grades corresponding to the three possible pairings of the limiting components of the mining system.

The multiplicity of contenders for the title of effective optimum is, of course, a consequence of the form of the economic model. It gives rise to six different possibilities, but only one of them is feasible under any given set of operating conditions. Frequently, the feasible possibility is apparent from the structure of the application, but this is not always the case and a logical procedure for identifying the right one is required.

The best way to examine the interrelations of the six cut-offs is to calculate the variable v which was introduced in Chapter 5. This variable v is actually the rate of change of V^*, the optimum present value, with respect to resource usage (dV^*/dR). In other words, it is the increment in present value per unit of resource utilised. It has to be maximised in order to determine the optimum cut-off grade. The formula is

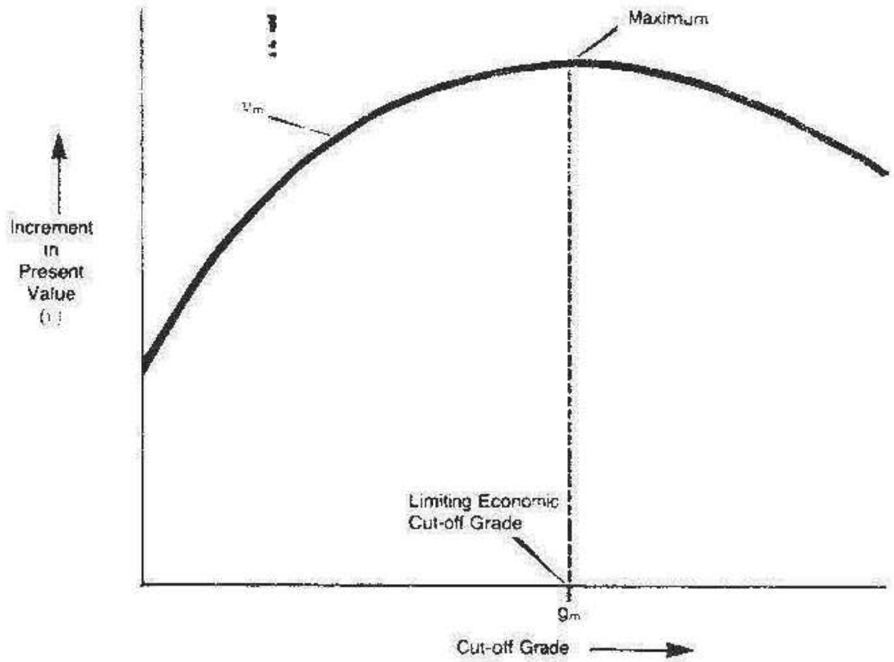
$$v = (p-k)xy\bar{g}-xh-m-(f+F)\tau$$

v takes three forms according to the determinant of τ

$$v_m = (p-k)xy\bar{g}-xh-m-(f+F)/M$$
$$v_h = (p-k)xy\bar{g}-x\{h+(f+F)/H\}-m$$
$$v_k = \{p-k-(f+F)/K\}xy\bar{g}-xh-m$$

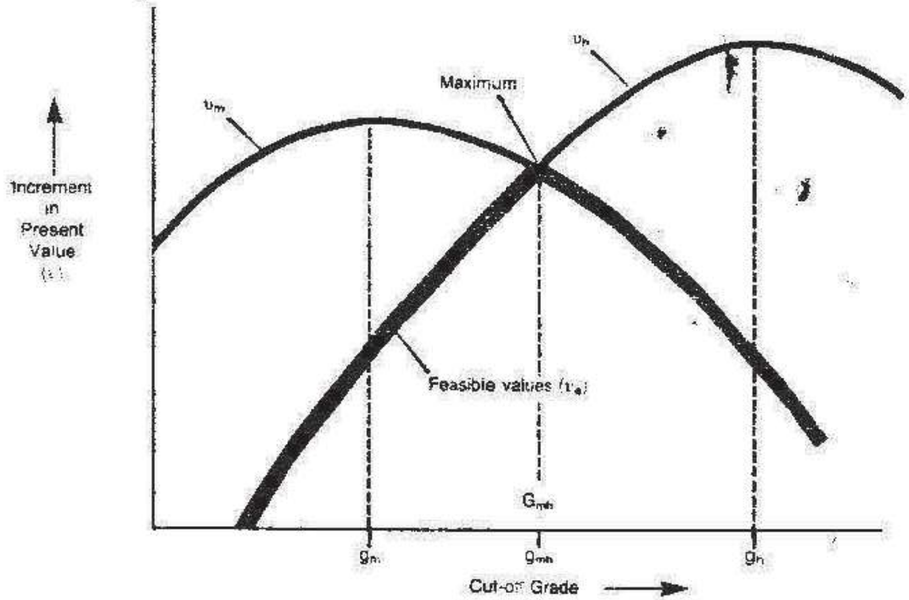
The graphs of all three forms of v as a function of the cut-off grade, g,

Figure 7.1



Increment in Present Value versus Cut-off Grade
Single Component (M); Limiting Economic Optimum

Figure 7.2



Increment in Present Value versus Cut-off Grade
Two Components (M & H); Balancing Optimum

are simila convex upwards with a single maximum. This maximum corresponds to the limiting economic cut-off grade for the component concerned. This is illustrated in figure 7.1. If the graphs for two forms of v are superimposed, then the picture is as shown in figure 7.2. The two forms in this case correspond to the mine and the treatment plant. The intersection of these two graphs is given by

$$v_m = v_h$$

which reduces to

$$x = H/M$$

This means that the point of intersection corresponds to the balancing cut-off grade, g_{mh}. Further, it can easily be shown from the formulae that for values of cut-off less than g_{mh} the treatment component is limiting and that for values above g_{mh} the mining component is limiting. In other words, the feasible form of v, at any cut-off grade, is always the lower of the two curves. It is shown as a bold line in figure 7.2. The maximum feasible value of v in the figure occurs at the point of intersection, g_{mh}. However, this is not always so and two other cases must also be examined. These, too, are best illustrated graphically and are shown in figures 7.3 and 7.4 presented overleaf.

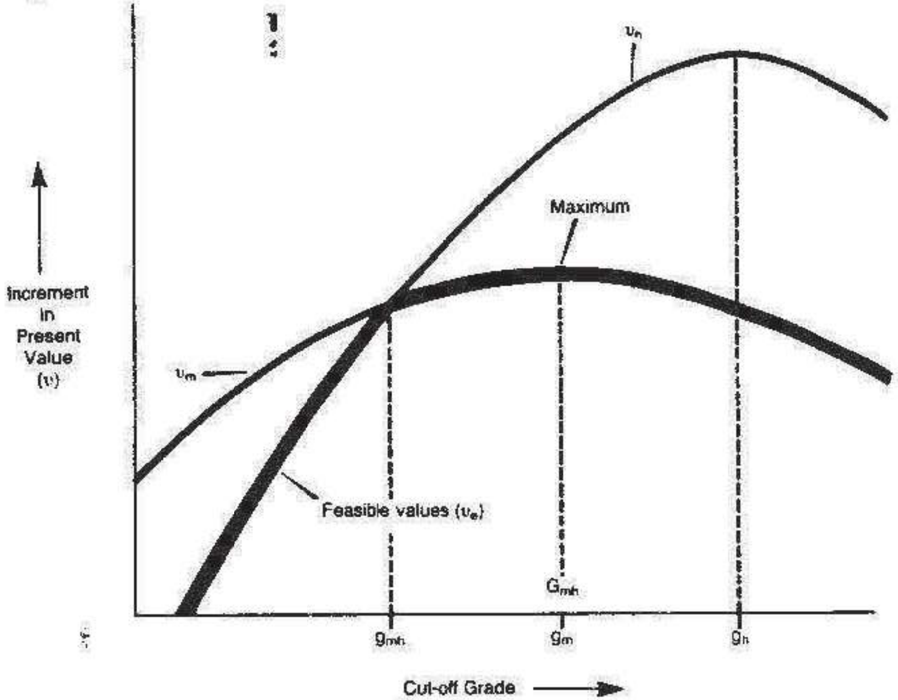
Figure 7.3 shows that when the balancing cut-off grade g_{mh} is less than g_m, the mine is really the bottleneck in the operation and g_m is the optimum cut-off grade. On the other hand, as figure 7.4 shows, when g_{mh} is greater than g_h, the treatment plant is the bottleneck and g_h is the optimum cut-off grade.

Thus, the following rule may be formulated for an operation limited by the mine and treatment plant. The effective optimum cut-off grade is called G_{mh}.
Mine and Treatment Plant

$$\begin{aligned} G_{mh} &= g_m \quad \text{if } g_{mh} < g_m \\ &= g_h \quad \text{if } g_{mh} > g_h \\ &= g_{mh} \quad \text{otherwise.} \end{aligned}$$

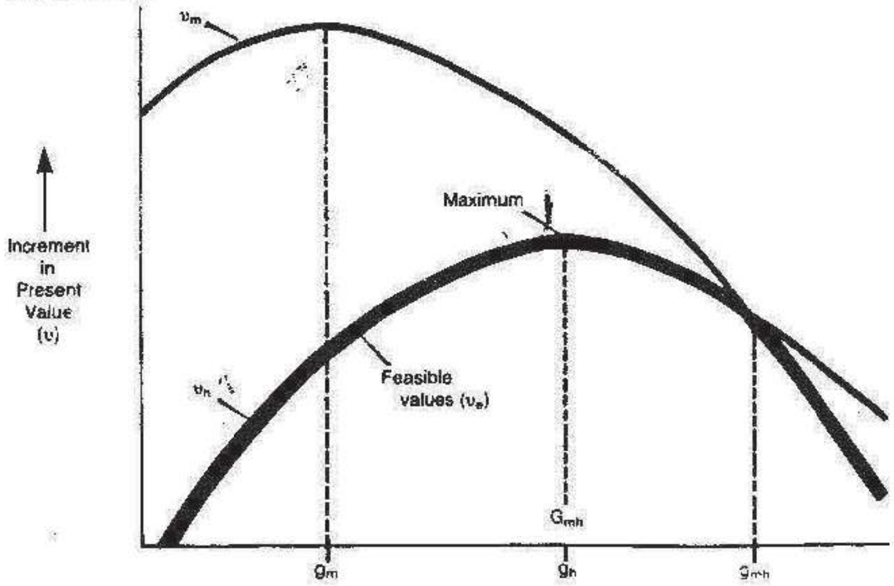
In a similar way, by considering the other pairs of stages:

Figure 7.3



Increment in Present Value versus Cut-off Grade
Two Components (M & H); M Limiting Optimum

Figure 7.4



Increment in Present Value versus Cut-off Grade
Two Components (M & H); H Limiting Optimum

Mine and Market

$$\begin{aligned} G_{mk} &= g_m \quad \text{if } g_{mk} < g_m \\ &= g_k \quad \text{if } g_{mk} > g_k \\ &= g_{mk} \quad \text{otherwise.} \end{aligned}$$

Treatment Plant and Market

$$\begin{aligned} G_{hk} &= g_k \quad \text{if } g_{hk} < g_k \\ &= g_h \quad \text{if } g_{hk} > g_h \\ &= g_{hk} \quad \text{otherwise.} \end{aligned}$$

The overall effective optimum cut-off grade is now one of the three, G_{mh}, G_{mk} and G_{hk}. The position can again be illustrated graphically, as in figure 7.5 overleaf.

The largest increase in present value that can be achieved at any cut-off grade, allowing for the capacity restrictions, is actually the least of v_m, v_k and v_h. This is represented by the three curved segments shown by the bold line in figure 7.5. The optimum cut-off grade obviously corresponds to the highest point on these segments and it can be shown that it always occurs at the middle value of G_{mk}, G_{mh} and G_{hk}.

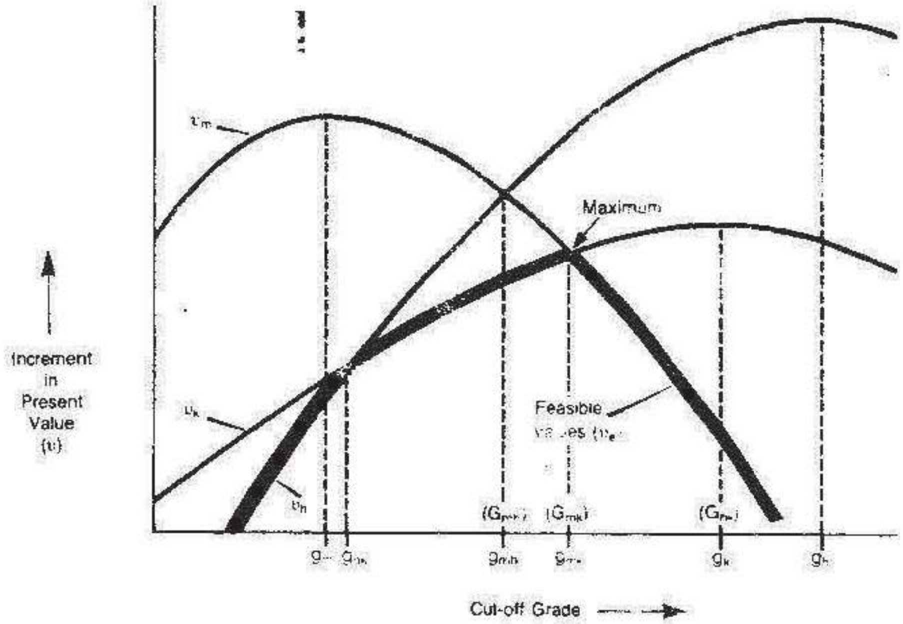
i.e. Effective Optimum Cut-Off Grade G

$$= \text{Middle Value } (G_{mk}, G_{mh}, G_{hk})$$

In the case illustrated in figure 7.5, it is G_{mk}. Further, the associated increase in present value v is always the least of the three increases which are possible considering the stages in pairs. The latter result is obvious; it cannot be possible that the imposition of a third restriction on capacity improves the position. The former result becomes apparent from enumerating the possibilities.

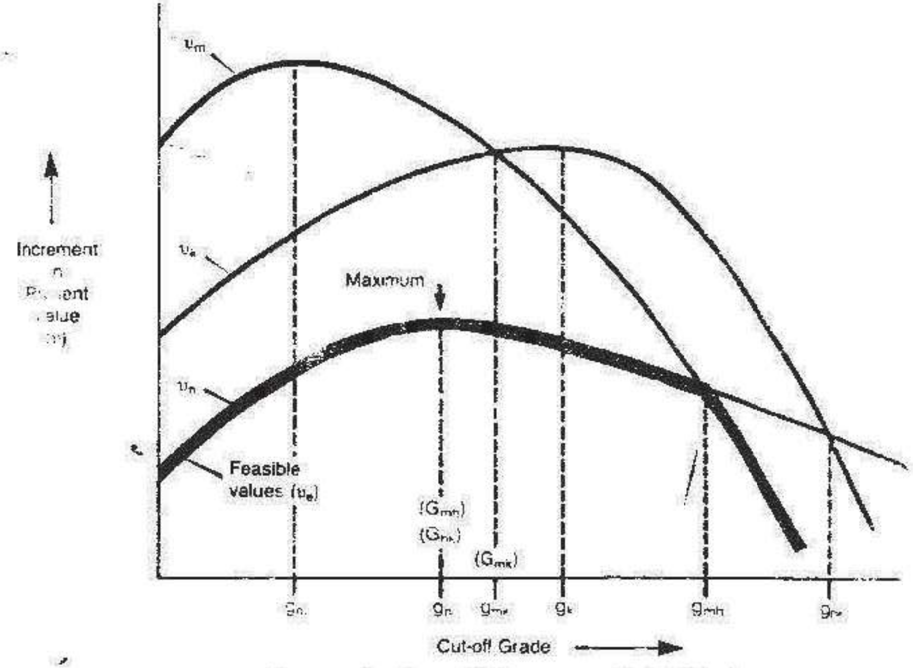
The highest point on the three segments need not be at one of the vertices as in the diagram. Depending on the relative positions of the three incremental present value curves, it can be at one of the maximum points at g_m, g_k or g_h. Under these circumstances, total capacity is limited by the one stage only and two of the associated values of G_{mh}, G_{mk} and G_{hk} coincide. Figure 7.6 illustrates this position. Here G_{mh} and G_{hk} coincide at g_h. This is the effective

Figure 7.5



Increment in Present Value versus Cut-off Grade
Three Components; Separate Paired Maxima

Figure 7.6



Increment in Present Value versus Cut-off Grade
Three Components; Two Paired Maxima Coincident

optimum and the treatment plant alone limits capacity. Case Study 3 illustrates the application of these ideas.

The method of first determining limiting economic cut-off grades and then balancing cut-off grades in order to arrive at an effective optimum cut-off has been described in detail because of the insight it gives into the factors which influence the optimum. Other more direct methods can be employed and are sometimes necessary, as when the maxima of the economic model cannot be located by simple means. One alternative method, a search technique, has already been mentioned. It is described in Chapter 11.