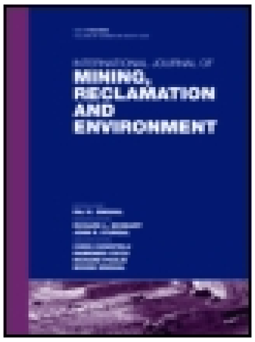

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Evaluating crusher system location in an open pit mine using Markov chains

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ABSTRACT
A methodology is presented for deciding the location of a third ore crusher to be installed at Chile's Chuquicamata copper mine. The proposed approach determines which location alternative exhibits the minimum capital and operating costs when equipment failure probability is considered. Two alternative location configurations are evaluated using the stationary probabilities of a Markov chain model, and the results are validated with a discrete-event simulation model. The Markov model generates insights into the relationships between the variables that a discrete-event simulation cannot provide, and does so without the latter's greater costs and complexities of modelling, solving and calibration.

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Open-pit crusher location; Markov chains; reliability; simulation

1. Introduction

The Chuquicamata copper mine, a division of Chile's state-owned CODELCO mining company, is an open-pit operation located in the far north of the country that has been worked continuously since 1915. The natural growth of extraction operations at the mine has driven the evolution of a mixed system of material transport that currently consists of 15 loading units (power shovels and front loaders), 96 high-payload haul trucks, 2 primary (gyratory) crushers and 4 conveyor belt systems.

The material extracted at the property is separated into waste rock, which has no economic value and is hauled away to waste dumps, and ore, which contains the valuable minerals and is sent to the first in a series of processing plants whose final output is copper metal. In the initial process, the ore is crushed and the output is transported to the concentrator plant on conveyor belts.

When the problem considered in this study arose, the operating configuration of the mine included two crusher units of equal capacity, one installed inside the pit and the other outside of it (see Figure 1). Both of them fed their output to the concentrator plant through two conveyor belts. Since crushers and belts, like any other equipment, are subject to failure, it was resolved that a third crusher should be installed to improve reliability.

For reasons beyond the scope of our enquiry, the new unit would have to be built alongside one of the existing crushers and the two units would then share the same two conveyor belts. This posed a major decision problem defined by a complex interplay of factors including the failure rates of the belts and crushers, the operating costs of truck haulage vs. conveyor belt transport, the capital costs of truck acquisition and the much greater expense involved in an in-pit (inside) crusher installation compared to an ex-pit (outside) one.

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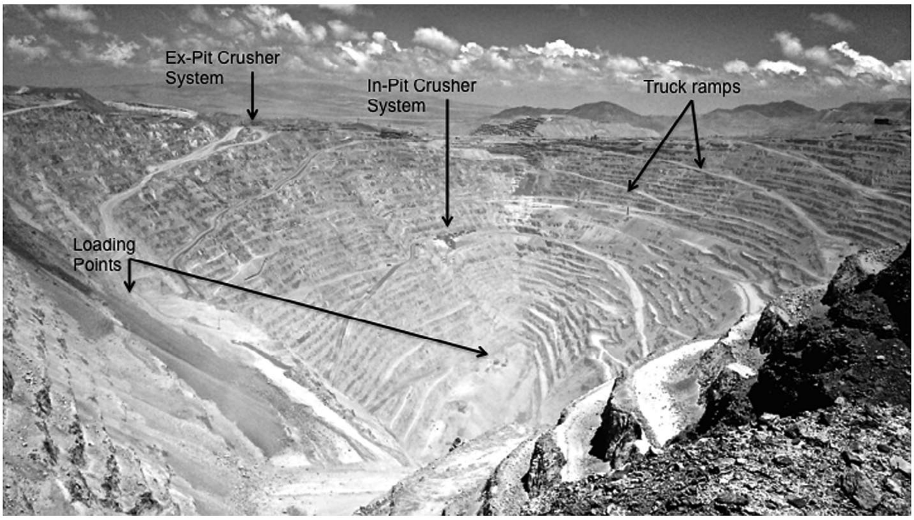

Figure 1. Current Chuquicamata operating configuration showing relative locations of equipment.

In broad terms, the decision problem faced by Chuquicamata management was fundamentally one of evaluating the trade-off between cost and reliability, a decidedly non-trivial exercise that required a methodology which could take proper account of the element of randomness in the belt and crusher failures. At other large-scale operations such as Bingham Canyon, decision makers have opted for an in-pit set-up, suggesting the balance was tipped by the lower transport costs [1].

The most commonly used method for studying the productivity of mine systems incorporating randomness is simulation. The first applications of the technique in this context date back to the 1960s in cases mainly involving underground mines (see [2–6]). In the 1980s, with the growing use of dispatching systems in open-pit operations, simulation was employed to investigate the benefits of different computational algorithms [7–10]. Subsequent advances in computer technology enabled the design of the first simulation model with graphical animations, which was used in the study of stockpiling strategies for minimising costs at an iron mine [11]. More recently, simulation has been applied to analyses of open-pit mine productivity under operating constraints [12], new truck dispatching algorithms in real time [13] and autonomous vehicle dispatch algorithms [14].

Yet despite its widespread application to stochastic mining problems, simulation is costly in time and resources spent on modelling, solving and calibration. Furthermore, it provides no insights into the problems it is used to study or the implications of the factors and parameters involved in them. Finally, conclusions based on simulation are in practice derived from the numerical results of multiple realisations. Solving mine operating problems analytically, on the other hand, affords a clearer understanding of the production system and how its configuration and operation can be optimised. The present article aims to demonstrate that Markov chains can contribute to the solution of complex problems, and in particular to the study of the productivity of a materials handling system as a function of its operating configuration. Thus, the proposed approach offers an alternative to the simulation of discrete events.

The theory behind Markov chains first appeared in a paper by Andréi Markov in 1907, but attracted wider attention only after 1923 when the article was cited by Bernstein in his work extending the central limit theorem of probability theory [15]. Since that time, Markov chains have been used to solve problems in a variety of fields. For example, the significant expansion of personal loans in the United States in the post-World War II era prompted Cyert and Thompson [16] to develop a methodology for determining customer credit risk in the retail trade. Their approach was based on customers' historical behaviour (ability to pay). The model they proposed divides the portfolio of a retail firm into multiple risk categories across which varying proportions of its customers are distributed. Another early application of Markov chains was a model proposed by Judge and Swanson [17] for analysing the evolution of the pig producer sector starting from a set of initial conditions. The study used data for 83 firms in Illinois over a period of 13 years. The strong simplifying assumptions of the model facilitated the paper's main purpose, which was to demonstrate the usefulness of the Markov chain concept for future studies in agricultural economics.

A paper by Burnham [18] furthered the development of Markov chain models in agriculture. The author's model analysed the effects of various policies in the southern Mississippi alluvial valley. In an application to manufacturing, given that no production system is perfectly reliable, Abboud [19] analysed production and inventory in a system consisting of one machine producing a single good that is subject to breakdown. Output and demand rates were assumed constant while failure and repair rates were considered to be random. The author compared the Markov chain model to a simulation-based formulation, demonstrating that the two are strongly correlated.

If we assume that the passage of time has little impact on equipment availability, Markov chains can be used to determine the reliability of mechanical systems [20]. This is valid where systems are always maintained in excellent state of repair [21]. But where that is not the case, that is where availability declines over time (i.e. non-renewable), more complex models must be resorted to. These generally involve the use of simulation to find the solution (see [22–26]).

More generally, other studies have been done that propose integrating the crusher haulage system with the mine design optimization problem [27].

This article develops two stochastic models for solving the above-described problem of determining the best location for a third crusher. The first model uses discrete-time Markov chains while the second one uses a continuous-time formulation. Both versions are validated with a discrete-event simulation model.

2. The problem

The production system at the Chuquicamata mine consists for present purposes mainly of the following equipment:

(1) Power shovels: Used to load haul trucks inside the pit, these shovels are highly specialised units with a capacity of 100 metric tonnes and can load 5000 metric tonnes per hour. Each shovel costs approximately US$20 million.
(2) Haul trucks: These vehicles are designed for use, exclusively, in mining applications and have a load capacity of 330 metric tonnes. They are built to order and sell for about US$4 million. The trucks are at their slowest when climbing fully loaded and can reach full speed when running empty on level ground.
(3) Fixed Crushers: These are specialised units used to reduce the size of the ore-bearing rocks extracted from the pit. They are built to order and designed to fit the characteristics of a specific mine. Each crusher at Chuquicamata is of the gyratory type, measures $60'' \times 109''$, consumes 800 kW of power and can process 5600 metric tonnes per hour at an $8''$ setting. Their basic selling price is US$15 million but to this must be added another US$35 million for installation, principally involving construction of the foundations. It is precisely this high installation cost that underlies the significance of this study.
(4) Conveyor belts: The output of the crushers is transported by conveyor belts. Although more efficient than truck haulage, this method can only be used with ore that has already been crushed. At Chuquicamata, the in-pit and ex-pit crushers are each connected to the concentrator plant by a pair of 72-inch-wide parallel belts. The capacity of each belt is enough to carry the output of a single crusher. A belt costs about US$2500 per metre, but the final amount for in-pit belts is higher due to the tunnel construction costs.

(5) Concentrator plant: This is a major installation and a strategic unit in the mining process, and is therefore also a limiting factor in the mine's installed capacity. Concentrators consume huge quantities of energy and water, which in some cases constrains their design capacity and operation. A plant such as the one at Chuquicamata can today cost some US$ 8 billion.

The problem we propose to solve covers the following mine processes:

- P1: Loading of truck inside the pit by a shovel. There are about 15 loading units located at four different points around the pit.
- P2: Transport of material with no economic value to the waste dump. This operation is carried out by about 80 trucks.
- P3a: Transport of material with economic value to the in-pit crusher. After crushing, it will be processed by the concentrator.
- P3b: Transport of material to the ex-pit crusher, analogously to process P3a above. Processes P3a and P3b together use about 15 trucks.
- P4: Transport of crushed material from the in-pit and ex-pit crushers to the concentrator plant by the conveyor belts.

As noted in the introduction, the original configuration includes 1 in-pit and 1 ex-pit crusher, each served by a pair of belts. The concentrator plant capacity is equal to the combined capacities of the two crushers and is therefore one-half that of the four belts combined. Since each belt has a capacity equal to that of a crusher, the crushers' individual capacities are the limiting factor on the capacity of each of the two crusher–conveyor systems. A flow chart depicting the operating configuration of the equipment and processes described above is shown in Figure 2.

The central elements in the third crusher decision problem are the randomness in the possibility of crusher failure and the ore material hauling cost. Since the two conveyor belts have sufficient capacity to transport the throughput of two crushers, having two crushers installed at a single location, either inside or outside the pit, means that a belt failure would idle one of them. Originally, with just one crusher at each location, the failure of one belt did not affect production since the other belt could handle the unit's entire output

If the crusher location alternative is evaluated using the equipment's average availability rates, the optimal solution would be to install the third crusher inside the pit. This is the case because the capacity of the two belts at the existing inside crusher would be sufficient to carry the output of the two crushers (recall the exogenous condition that the new unit has to be built alongside the existing one) and belt transport is much cheaper than truck haulage, the cost relationship being 1:5. Furthermore, with only 1 inside crusher, its failure means additional trucks have to be pressed into service to maintain production. With the added second unit, the number of trucks required could be reduced.

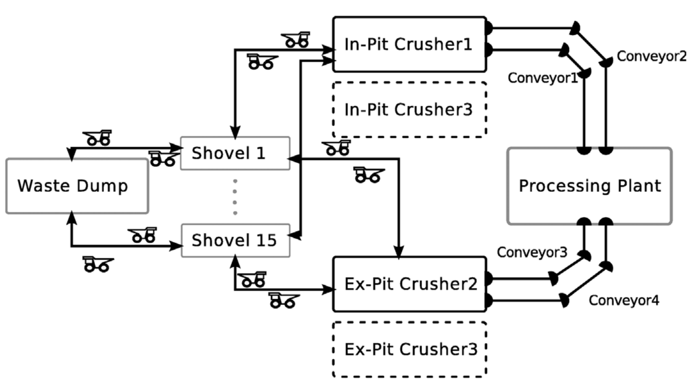

Figure 2. Operating configuration flow chart.

On the other hand, a logistic factor that must be taken into account seems to point to the opposite solution. The inside crusher location is 400 m down into the 1000-m-deep pit. For safety reasons, the trucks cannot descend when loaded so material extracted by shovels operating above the 400-m level can only be sent to the ex-pit crusher. This implies that a failure of a single outside crusher would cause a major interruption in global output. Thus, despite the greater operating expense, installing the third crusher outside the pit would ensure feed to the concentrator is more reliable.

In reality, however, there is significant variability in both crusher and belt failure rates, and this considerably complicates the third crusher location decision if average values are used. In what follows, therefore, we develop a methodology for addressing the problem of locating the third crusher that incorporates this randomness in the failure factors as well as the cost and logistic considerations.

3. The methodology and the model

Consider two production system configurations that include a third crusher. Consider two alternative configurations, each originally with two crushers, to which a third crusher has been added alongside one of the original ones. The first configuration, denoted $C1$, has two ex-pit crushers and one in-pit crusher while the second configuration, called $C2$, has two in-pit crushers and one ex-pit crusher. Each pit location already had two conveyor belts and continues to have two after the installation of the third crusher.

At the location now with two crushers, whether inside or outside the pit, each one will be uniquely associated with one of the two belts. There are thus two crushing systems at this location, each one composed of a belt and a crusher. A system is said to be operative if both belt and crusher are operating normally. If either component fails, the system it belongs is said to be inoperative.

At the location still with just one crusher, whether inside or outside the pit, the crusher will be associated with both belts. In this case, there is a single crushing system consisting of two belts and a crusher. This system is said to be operative if its crusher and at least one of its belts are operating normally. If either the crusher or both belts fail, the system is said to be inoperative.

From the foregoing, it is immediately evident that the mechanical reliability of the crushing systems depends on two factors: the mechanical reliability of the crushers and belts and the number of belts associated with each crusher.

An analysis of the mechanical performance of each crusher and belt at Chuquicamata over a period of a year revealed that the distributions of both the times between failures and the repair times follow a negative exponential function. Under these conditions, standard mechanical reliability theory [28] allows us to represent a set of equipment connected in series or in parallel as a single mechanical system. In our case, the crushing systems consist of one crusher and either one or two belts, depending on the configuration. We can thus estimate values for the failure rate λ and the repair rate μ for each crusher–belt system in configurations $C1$ and $C2$ as shown in Table 1, where $I1$ and $I2$ are the in-pit systems and $E1$ and $E2$ the ex-pit systems. Note that in the case of $C2$, the rates for $I1$ and $I2$ are not identical, unlike $E1$ and $E2$ in $C1$. This asymmetry reflects differences in the reliability of the various belts, particularly between the two installed inside the pit. Also, the reliability of the new crusher is assumed to be equal to that of the existing external crusher.

It should be emphasised here that even with just two alternatives, conducting an economic evaluation that takes into account the time between failures and repair time factors is far from trivial. The

Table 1. Parameter values for time between failures (λ) and repair time (μ).

Configuration	$C1$			$C2$		
Crusher–Conveyor system	$I1$	$E1$	$E2$	$I1$	$I2$	$E1$
No of conveyors per crusher	2	1	1	1	1	2
λ [1/h]	0.081	0.079	0.079	0.085	0.102	0.073
μ [1/h]	0.850	0.250	0.250	0.690	0.230	0.630

use of Markov chains is fundamental to the inclusion of these stochastic elements and for explaining the model to management at the mine in simple terms.

3.1. Discrete-time Markov chains

Based on the above considerations, we can model the two configurations using Markov chains. In each case, there will be eight states representing all possible combinations of the 3 crushing systems and their two possible states, operative and inoperative (see Table 2). Variable $I1(t)$ is defined as 1 if crusher $I1$ is operative at time t and 0 otherwise; the other variables $I2(t)$, $E1(t)$ and $E2(t)$ are defined analogously.

A schematic of the Markov chain for $C1$ and $C2$ with the possible transitions for a given period is shown in Figure 3. In the depicted case, the graph is complete because all of the transitions between states are possible, though as we shall see later when discussing the infinitesimal period case, some of the transitions are very unlikely.

We now want to determine the percentage of time in a steady state that a production system configuration is in each state. Let $\mathbf{P}$ be the transition matrix whose elements p_{ij} are the probabilities of passing from state S_i to state S_j at time instant δt. If the Markov chain is irreducible and aperiodic, then $\pi = \pi \cdot \mathbf{P}$ where $\sum_j \pi_j = 1$.

Now let $\mathbf{P}_1$ be the transition matrix for configuration $C1$ and $\mathbf{P}_2$ the transition matrix for configuration $C2$. To calculate the transition probabilities p_{ij}, we use the failure and repair rates of each crushing system. For example, the value of p_{12} for configuration $C1$ is calculated as follows:

Table 2. State definitions by operational configuration.

	$C1$			$C2$		
State	$I1$	$E1$	$E2$	$I1$	$I2$	$E1$
S_1	1	1	1	1	1	1
S_2	1	1	0	1	0	1
S_3	1	0	1	0	1	1
S_4	0	1	1	1	1	0
S_5	0	0	1	0	1	0
S_6	0	1	0	1	0	0
S_7	1	0	0	0	0	1
S_8	0	0	0	0	0	0

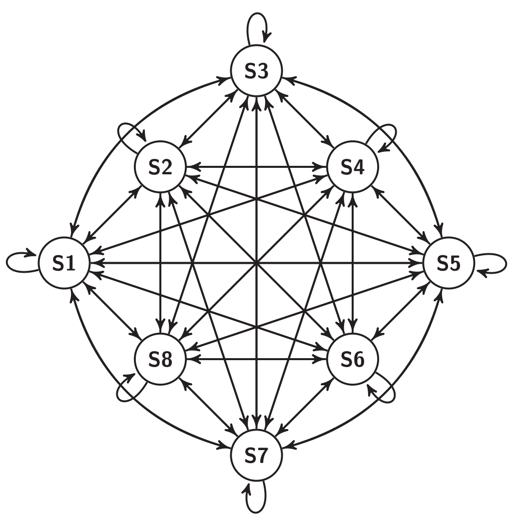

Figure 3. Discrete-time Markov chain graph.

$$p_{12} = P\left\{I1(t+\delta t) = 1, E1(t+\delta t) = 1, E2(t+\delta t) = 0 \middle| I1(t) = 1, E1(t) = 1, E2(t) = 1\right\} \quad (1)$$

Since the crushing systems' respective failure probabilities are independent of each other,

$$p_{12} = P\left\{I1(t+\delta t) = 1, E1(t+\delta t) = 1, E2(t+\delta t) = 0 \middle| I1(t) = 1, E1(t) = 1, E2(t) = 1\right\} \quad (2)$$

where

$$P\left\{I1(t+\delta t) = 1 \middle| I1(t) = 1\right\} = e^{-\lambda_{I1}\delta t} \quad (3)$$

$$P\left\{E1(t+\delta t) = 1 \middle| E1(t) = 1\right\} = e^{-\lambda_{E1}\delta t} \quad (4)$$

$$P\left\{E2(t+\delta t) = 0 \middle| E2(t) = 1\right\} = 1 - e^{-\lambda_{E2}\delta t} \quad (5)$$

Substituting (3), (4) and (5) into (2), we have

$$p_{12} = e^{-\lambda_{I1}\delta t} \cdot e^{-\lambda_{E1}\delta t} \cdot \left(1 - e^{-\lambda_{E2}\delta t}\right) \quad (6)$$

where the values for λ and μ are given in Table 1.

If we chose a small value for δt of 1/13 of an hour, $p_{12} = 0.60\%$ and the rest of the p_{ij} can be calculated in a similar manner for the two configurations. Transition matrix $\mathbf{P}_1$ is given in Table 3 and transition matrix $\mathbf{P}_2$ in Table 4. Note that the 0% entries in the matrices represent probabilities that are small but non-zero.

Since both Markov chains are clearly ergodic (aperiodic and connected), we can solve the equation $\pi = \pi \cdot \mathbf{P}$ for each configuration. The time percentages π^{C1} for configuration 1 and π^{C2} for configuration 2 are given in Table 5.

Table 3. Transition matrix for configuration *C*1.

$\mathbf{P}_1 =$	98.18%	0.60%	0.60%	0.61%	0.00%	0.00%	0.00%	0.00%
	1.86%	96.92%	0.01%	0.01%	0.00%	0.60%	0.59%	0.00%
	1.86%	0.01%	96.92%	0.01%	0.60%	0.00%	0.59%	0.00%
	6.29%	0.04%	0.04%	92.50%	0.56%	0.56%	0.00%	0.00%
	0.12%	0.00%	6.21%	1.75%	91.32%	0.01%	0.04%	0.56%
	0.12%	6.21%	0.00%	1.75%	0.01%	91.32%	0.04%	0.56%
	0.04%	1.83%	1.83%	0.00%	0.01%	0.01%	95.68%	0.60%
	0.00%	0.12%	0.12%	0.03%	1.73%	1.73%	6.13%	90.15%

Table 4. Transition matrix for configuration *C*2.

$\mathbf{P}_2 =$	98.02%	0.77%	0.64%	0.55%	0.00%	0.00%	0.01%	0.00%
	1.77%	97.03%	0.01%	0.01%	0.00%	0.54%	0.64%	0.00%
	5.14%	0.04%	93.53%	0.03%	0.52%	0.00%	0.74%	0.00%
	4.69%	0.04%	0.03%	93.88%	0.62%	0.74%	0.00%	0.00%
	0.25%	0.00%	4.47%	4.92%	89.58%	0.04%	0.04%	0.71%
	0.08%	4.64%	0.00%	1.69%	0.01%	92.93%	0.03%	0.61%
	0.09%	5.09%	1.68%	0.00%	0.01%	0.03%	92.58%	0.52%
	0.00%	0.24%	0.08%	0.09%	1.61%	4.87%	4.43%	88.67%

Table 5. Steady-state probabilities for discrete-time Markov chain, by configuration.

States	π^{C1}	π^{C2}
S_1	52.1%	55.3%
S_2	16.8%	24.3%
S_3	16.8%	6.9%
S_4	5.1%	6.5%
S_5	1.6%	0.8%
S_6	1.6%	2.8%
S_7	5.4%	3.0%
S_8	0.5%	0.4%

3.2. Continuous-time Markov chains

If we assume δt is an infinitesimal time interval, the probability that two or three crushing systems change state in any one interval is negligible. The problem can then be modelled as a continuous-time Markov chain.

In analogous fashion to the discrete-time model, we define the eight possible states of each production system configuration as a function of the three crushing systems' two operating states, operative (1) and inoperative (0). Simplifying the notation, the failure rate and the repair rate of crushing system i are denoted λ_i and μ_i, respectively. The values of these two parameters for each configuration are based on the historical data summarised in Table 1. The continuous-time Markov chain can then be graphed as in Figure 4. Each node in the graph must be in equilibrium, that is the inflow to any node must be equal to the outflow from it. The flow conservation equations for each node in the steady state are given in the following system:

$$\pi_{(1,1,1)} \cdot \left(\lambda_1 + \lambda_2 + \lambda_3\right) = \pi_{(0,1,1)} \cdot \mu_1 + \pi_{(1,0,1)} \cdot \mu_2 + \pi_{(1,1,0)} \cdot \mu_3$$

$$\pi_{(1,1,0)} \cdot \left(\lambda_1 + \lambda_2 + \mu_3\right) = \pi_{(0,1,0)} \cdot \mu_1 + \pi_{(1,0,0)} \cdot \mu_2 + \pi_{(1,1,1)} \cdot \lambda_3$$

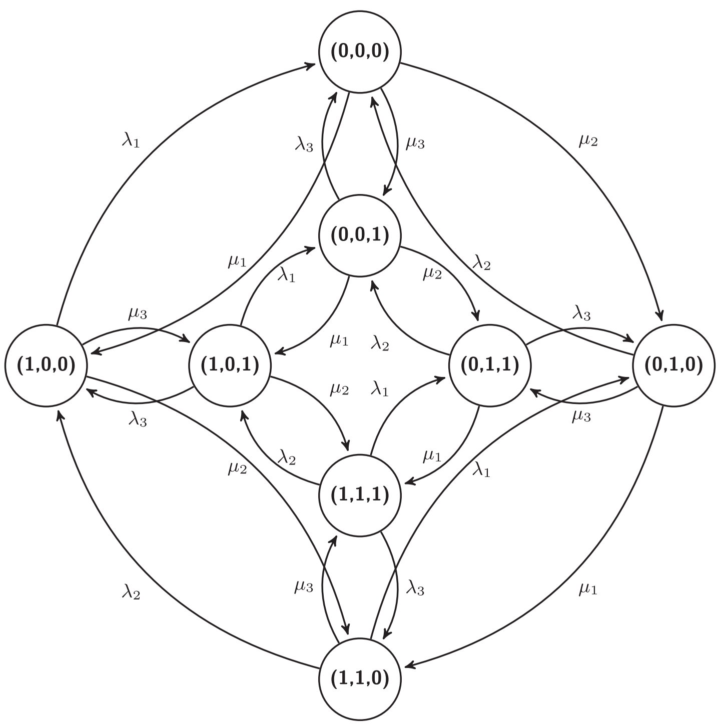

Figure 4. Continuous-time Markov chain graph.

$$\pi_{(1,0,1)} \cdot \left(\lambda_1 + \mu_2 + \lambda_3\right) = \pi_{(0,0,1)} \cdot \mu_1 + \pi_{(1,1,1)} \cdot \lambda_2 + \pi_{(1,0,0)} \cdot \mu_3$$

$$\pi_{(0,1,1)} \cdot \left(\mu_1 + \lambda_2 + \lambda_3\right) = \pi_{(1,1,1)} \cdot \lambda_1 + \pi_{(0,0,1)} \cdot \mu_2 + \pi_{(0,1,0)} \cdot \mu_3$$

$$\pi_{(0,0,1)} \cdot \left(\mu_1 + \mu_2 + \lambda_3\right) = \pi_{(1,0,1)} \cdot \lambda_1 + \pi_{(0,1,1)} \cdot \lambda_2 + \pi_{(0,0,0)} \cdot \mu_3$$

$$\pi_{(0,1,0)} \cdot \left(\mu_1 + \lambda_2 + \mu_3\right) = \pi_{(1,1,0)} \cdot \lambda_1 + \pi_{(0,0,0)} \cdot \mu_2 + \pi_{(0,1,1)} \cdot \lambda_3$$

$$\pi_{(1,0,0)} \cdot \left(\lambda_1 + \mu_2 + \mu_3\right) = \pi_{(0,0,0)} \cdot \mu_1 + \pi_{(1,1,0)} \cdot \lambda_2 + \pi_{(1,0,1)} \cdot \lambda_3$$

$$\pi_{(0,0,0)} \cdot \left(\mu_1 + \mu_2 + \mu_3\right) = \pi_{(1,0,0)} \cdot \lambda_1 + \pi_{(0,1,0)} \cdot \lambda_2 + \pi_{(0,0,1)} \cdot \lambda_3 \tag{7}$$

If we then add the condition that the π_j values are a probability distribution, so that $\sum_j \pi_j = 1$, we obtain a system of eight linearly independent equations in eight unknowns. The π value for each state represents the fraction of time the system is in the corresponding state over the long term. Solving the system yields the following π values:

$$\pi_{(1,1,1)} = \frac{\mu_1 \cdot \mu_2 \cdot \mu_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

$$\pi_{(1,1,0)} = \frac{\mu_1 \cdot \mu_2 \cdot \lambda_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

$$\pi_{(1,0,1)} = \frac{\mu_1 \cdot \lambda_2 \cdot \mu_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

$$\pi_{(0,1,1)} = \frac{\lambda_1 \cdot \mu_2 \cdot \mu_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

$$\pi_{(0,0,1)} = \frac{\lambda_1 \cdot \lambda_2 \cdot \mu_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

$$\pi_{(0,1,0)} = \frac{\lambda_1 \cdot \mu_2 \cdot \lambda_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)}$$

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$$\pi_{(0,0,0)} = \frac{\lambda_1 \cdot \lambda_2 \cdot \lambda_3}{\left(\lambda_1 + \mu_1\right)\left(\lambda_2 + \mu_2\right)\left(\lambda_3 + \mu_3\right)} \tag{8}$$

Table 6. Steady-state probabilities for continuous-time Markov Chain, by configuration.

States	π^{C1}	π^{C2}
S_1	52.4%	55.6%
S_2	16.8%	24.3%
S_3	16.8%	6.8%
S_4	4.9%	6.4%
S_5	1.6%	0.8%
S_6	1.6%	2.8%
S_7	5.4%	3.0%
S_8	0.5%	0.3%

From these results, we derive the π^{C1} values for configuration $C1$ and the π^{C2} values for configuration $C2$, set out in Table 6. Since the transition period defined for the discrete-time Markov chain is relatively short, the solutions for the steady-state probabilities in the two cases are similar.

More generally, for a system of n components where each component is subject to failure at a rate λ_i and can be repaired at a rate μ_i, the probability of being in state $(k_1, k_2, \ldots, k_n,)$ with $k_i \in \{0, 1\}$ is given by the following formula:

$$\pi_{(k_1,k_2,\ldots,k_n)} = \prod_{i=1}^{n} \frac{\lambda_i^{1-k_i} \cdot \mu_i^{k_i}}{(\lambda_i^{1-k_i} \cdot \mu_i^{k_i} + \lambda_i^{k_i} \cdot \mu_i^{1-k_i})} \tag{9}$$

4. Model validation

To validate our results, we modelled configurations $C1$ and $C2$ using the discrete-event simulation software Promodel. The simulation period was one year, consisting of 8610 h discretised into intervals of 1/13 of an hour. For each configuration, 100 realisations were simulated.

The results of the simulations are contrasted with those obtained by the Markov model in Figure 5 for $C1$ and in Figure 6 for $C2$. The bars in the figures represent the ranges of the realisations while the markers indicate their expected values.

The goodness of fit between the value predicted by our Markov model and the average simulated realisation value was calculated using the traditional coefficient of determination indicator R^2 (see [29]). For $C1$, the result was $R^2 = 0.990$ while for $C2$ it was $R^2 = 0.981$.

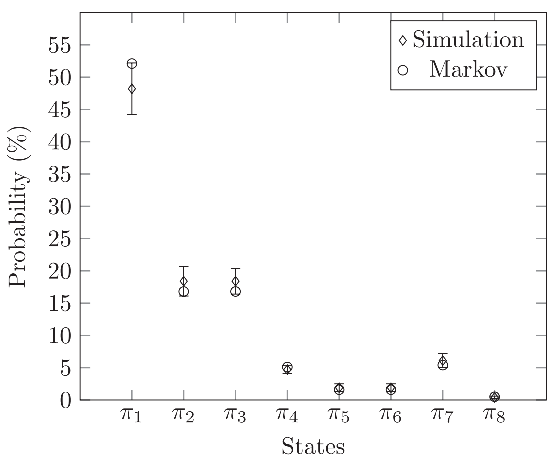

Figure 5. Markov vs. discrete-event simulation, configuration 1.

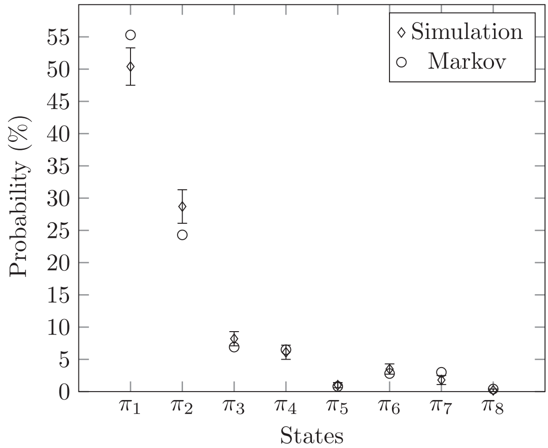

Figure 6. Markov vs. discrete-event simulation, configuration 2.

Table 7. Distance associated with states, by configuration and year.

		Distance (m)					Distance (m)			
States	π^{C1} (%)	d_S^{year1}	d_S^{year2}	d_S^{year3}	d_S^{year4}	π^{C2} (%)	d_S^{year1}	d_S^{year2}	d_S^{year3}	d_S^{year4}
S_1	52.1	4310	3207	3666	3366	55.3	4295	3173	3663	3364
S_2	16.8	4310	3207	3666	3366	24.3	4295	3173	3663	3364
S_3	16.8	4310	3207	3666	3366	6.9	4295	3173	3663	3364
S_4	5.1	6328	5203	6094	4395	6.5	4513	4579	3987	4910
S_5	1.6	6328	5203	6094	4395	0.8	4513	4579	3987	4910
S_6	1.6	6328	5203	6094	4395	2.8	4513	4579	3987	4910
S_7	5.4	4910	4677	3990	4912	3.0	6924	5306	6094	4395
S_8	0.5	6928	6673	6418	5941	0.4	7143	6713	6419	5941

5. Results

In our specification of the problem, the truck haulage distance (hereafter simply 'distance') depends on three factors: first, the location of the shovels; second, the location of the crushers; and third, the crushers' mechanical availability. The total truck haulage distance depends on the configuration: in the case of $C1$, since crusher capacity inside the pit is half the capacity outside of it, the distance covered by the trucks will, generally, be greater than that for $C2$ where inside crusher capacity is double that outside. Haulage distance is also affected by crusher system failures: if, for example, an inside system (regardless of configuration) is inoperative, the trucks are reassigned to the outside system(s). In the case where all systems have failed, the trucks are assigned to the stockpiling of uncrushed ore outside the pit so that mining operations are not interrupted.

Total haulage distance is calculated as the average of the distances between origins and destinations (crushers or stockpiles) as determined by the production plan. Separate calculations are made for each configuration and system state using the Minehaul software, specially designed for this type of application. Thus, we associated a transport distance d_S with each state S. Since the shovels must be relocated periodically as ore extraction advances, d_S will vary over time.

To evaluate the best location of the planned third crusher, we defined a time horizon of 4 years. For each year, the distances associated with each state were calculated. The values so computed for each configuration are summarised in Table 7.

The average distance d is a function of the time percentage π and the distances d_S. This relationship determines the productivity of the system as a function of the operating configuration, the failure rates and the crushing system repair rates. Therefore, using the continuous-time Markov chain solution π and the distance vector d_S, we calculated the average distance with the following formula:

Table 8. Truck requirement by operational configuration and year.

Year	Tonnage [kton]	Configuration $C1$ Distance [m]	Cycle time [min]	Productivity [ton/h]	Trucks [unit]	Configuration $C2$ Distance [m]	Cycle time [min]	Productivity [ton/h]	Trucks [unit]
Y_1	44,816	4524	36.24	497	19	4408	34.03	529	17
Y_2	44,941	3471	29.76	605	15	3393	28.71	627	15
Y_3	44,817	3900	30.57	589	15	3779	29.69	606	15
Y_4	44,569	3549	28.80	625	15	3561	28.81	625	15

Table 9. Economic evaluation by cost item and configuration.

Economic breakdown	Configuration $C1$ [US$ millions]	Configuration $C2$ [US$ millions]
Manpower	−12.199	−11.776
Fuel	−37.380	−35.011
Tyres	−11.678	−11.440
Maintenance	−40.686	−39.314
Truck CAPEX	−7.407	0.000
Crusher installation	−13.889	−32.407
Total NPV (8%)	−123.233	−129.949

$$d = \frac{\mu_1 \cdot \mu_2 \cdot \mu_3 \cdot d_{(1,1,1)} + \mu_1 \cdot \mu_2 \cdot \lambda_3 \cdot d_{(1,1,0)} + \mu_1 \cdot \lambda_2 \cdot \mu_3 \cdot d_{(1,0,1)} + \lambda_1 \cdot \mu_2 \cdot \mu_3 \cdot d_{(0,1,1)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)} + \frac{\lambda_1 \cdot \lambda_2 \cdot \mu_3 \cdot d_{(0,0,1)} + \lambda_1 \cdot \mu_2 \cdot \lambda_3 \cdot d_{(0,1,0)} + \mu_1 \cdot \lambda_2 \cdot \lambda_3 \cdot d_{(1,0,0)} + \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot d_{(0,0,0)}}{(\lambda_1 + \mu_1)(\lambda_2 + \mu_2)(\lambda_3 + \mu_3)}$$

The truck cycle time was then identified from the values for the distance and the average velocity, the latter computed for each of the 4 years. The productivity of the truck haulage system was defined as the average hourly tonnage per truck hauled to the crusher. The truck requirement (number of trucks) was determined by two factors: first, the tonnage to be transported; and second, the transport system productivity as just defined. For a given tonnage, the greater the productivity, the smaller the truck requirement, and vice versa. The truck requirement for each configuration and period along with the distance, cycle time and productivity are set forth in Table 8.

In carrying out the economic evaluation, we followed the accounting practices set by CODELCO management. Thus, the operating cost items included wages, maintenance, fuel and truck tyres, all of which are directly related to the truck requirement. Capital costs were also included even though they are not affected by randomness and are therefore not directly incorporated into our model. These items included crusher installation and truck acquisition. Note particularly that installing a crusher inside the pit costs US$20 million more than doing so outside it. Both operating and capital costs were discounted at a rate of 8% over a 4-year horizon (at the time of our study, shutdown of the Chuquicamata pit was slated for the end of this period but was later rescheduled for 2018). The resulting present values were approximately US$ −123 millions for $C1$ and US$ −130 millions for $C2$ and are shown in Table 9 broken down by individual cost item. On the basis of these results, configuration C1 was recommended.

6. Conclusions

This article presented a solution methodology for the decision problem faced by Chile's Chuquicamata open-pit copper mine regarding the location of a third ore crusher. The two available alternatives were to situate it next to either one of the two existing units, one installed inside the pit and the other outside it. These alternatives posed a complex trade-off between the acquisition and operating costs of the crushers and associated equipment and their reliability or probability of failure.

The proposed methodology was built around a Markov chain model that incorporated failure as a randomness factor in determining the productivity of the crusher systems. Two different configurations, one each for the two location alternatives just described, were modelled using discrete-time simulation and the results were compared with those derived from the Markov chains approach. The goodness-of-fit measurements found that the coefficient of determination for the inside and outside installation configurations were 0.990 and 0.981, respectively, demonstrating that the Markov chain model could replace the simulation model for productivity calculations.

A continuous-time Markov model was then employed to determine how each relevant variable affected the behaviour of the crusher systems in the two configurations. These results were used to conduct an economic evaluation establishing which of the configurations was more cost effective and would therefore be recommended for adoption. This model also showed how the different variables interacted and was thus able to deliver insights into the dynamics of the crusher systems that a simulation could not provide, despite the latter approach's greater cost due to the many complexities of modelling, solving and calibrating discrete-time formulations.

Finally, the methodology was successfully applied to the Chuquicamata case in 2010, and the more optimal of the two configurations was implemented at the mine. Some of the specific data presented in this study were changed for reasons of confidentiality.

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