Beam Propagation Method

Física
FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS UNIVERSIDAD DE CHILE

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Unos cuantos enjuages…

$$\nabla \times (\nabla \times \vec{E}) + \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = -\mu_0 \frac{\partial^2}{\partial t^2} \vec{P}$$

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Guiaje débil

Unos cuantos enjuages...

$$n^2(x,y) = 1 + \chi(x,y)$$

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Guiaje débil

$$\vec{E}(x,y,z;t) = \vec{f}(x,y)\Psi(x,y,z)e^{-i\omega t}$$

No hay acoplamiento entre modos TE y TM

Aproximación paraxial:

$$\left(\nabla^2 + k_0^2 n^2\right) \Psi(x, y, z) = 0$$

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$$\Psi(x, y, z) = \psi(x, y, z) e^{i\beta z}$$

Campo "lento"

Oscilación rápida

$$\beta \approx n_0 / \lambdabar$$

Aproximación paraxial:

$$\Psi(x, y, z) = \psi(x, y, z)e^{i\beta z}$$

$$\left|\frac{\partial^2 \psi}{\partial z^2}\right| \ll \left|\beta \frac{\partial \psi}{\partial z}\right|$$

$$\beta \approx n_0/\lambdabar$$

$$\nabla_\perp^2 \psi + \left(2i\beta \frac{\partial \psi}{\partial z} - \beta^2 \psi\right) + k_0^2 n^2 \psi = 0$$

Aproximación paraxial:

$$k_0 = \omega / c = 1/\lambdabar$$

$$\beta \approx n_0/\lambdabar$$

$$n = n_0 + \Delta n$$

$$\frac{n^2 - n_0^2}{2n_0} \approx \Delta n$$

$$\nabla_\perp^2 \psi + \left(2i\beta \frac{\partial \psi}{\partial z} - \beta^2 \psi \right) + k_0^2 n^2 \psi = 0$$

$$i\lambdabar \frac{\partial}{\partial z} \psi(x,y,z) = \left[-\frac{\lambdabar^2}{2n_0} \nabla_\perp^2 - \Delta n(x,y) \right] \psi(x,y,z)$$

Aproximación paraxial:

$$i\lambda\!\!\!^{-}\frac{\partial}{\partial z}\psi(x,y,z) = \underbrace{\left[-\frac{\lambda\!\!\!^{-2}}{2n_0}\nabla_\perp^2 - \Delta n(x,y)\right]}_{\hat{H}}\psi(x,y,z)$$

Fast Fourier Transform-BPM

$$i\hbar\frac{\partial}{\partial z}\psi(x,y,z) = \hat{H}\psi(x,y,z) \qquad \psi(x,y,z) = e^{\gamma z}A(x,y)$$

Fast Fourier Transform-BPM

$$i\lambda\!\!\!^{-}\frac{\partial}{\partial z}\psi(x,y,z) = \hat{H}\psi(x,y,z) \qquad \psi(x,y,z) = e^{\gamma z}A(x,y)$$

$$\implies \gamma = \frac{\hat{H}}{i\lambda\!\!\!^{-}}$$

Fast Fourier Transform-BPM

$$\psi(x, y, z) = e^{\gamma z} A(x, y)$$

$$\gamma = \frac{\hat{H}}{i\lambdabar}$$

$$\psi(x, y, z) = e^{-i\hat{H}z/\lambdabar} A(x, y)$$

Fast Fourier Transform-BPM

$$\psi(x, y, z) = e^{-i\hat{H}z/\lambdabar} A(x, y)$$

$$\Longrightarrow \; \psi(x, y, z + \Delta z) = e^{-i\hat{H}(z+\Delta z)/\lambdabar} A(x, y) = e^{-i\hat{H}\Delta z/\lambdabar} e^{-i\hat{H}z/\lambdabar} A(x, y)$$
$$= e^{-i\hat{H}\Delta z/\lambdabar} \psi(x, y, z)$$

Fast Fourier Transform-BPM

$$\psi(x, y, z + \Delta z) = \exp\left\{ i\Delta z \left(\frac{\lambda\!\!\!^{-2} \nabla_\perp^2}{2n_0} + \Delta n \right) / \lambda\!\!\!^{-} \right\} \psi(x, y, z)$$

En general, $\exp\left\{\hat{A} + \hat{B}\right\} \neq \exp\left\{\hat{A}\right\} \exp\left\{\hat{B}\right\}$ (La igualdad introduce un error $O(\Delta z^2)$)

Fast Fourier Transform-BPM

$$\psi(x,y,z+\Delta z) = \exp\left\{\frac{i\lambda\!\!\!^{-}\Delta z\nabla_\perp^2}{4n_0}\right\}\exp\left\{\frac{i\Delta z\Delta n(x,y)}{\lambda\!\!\!^{-}}\right\}\exp\left\{\frac{i\lambda\!\!\!^{-}\Delta z\nabla_\perp^2}{4n_0}\right\}\psi(x,y,z) + O(\Delta z^3)$$

Fast Fourier Transform-BPM

$$\psi(x,y,z+\Delta z) = \exp\left\{\frac{i\lambdabar\Delta z\nabla_\perp^2}{4n_0}\right\}\exp\left\{\frac{i\Delta z\Delta n(x,y)}{\lambdabar}\right\}\exp\left\{\frac{i\lambdabar\Delta z\nabla_\perp^2}{4n_0}\right\}\psi(x,y,z) + O(\Delta z^3)$$

Reservemos esta expresión de momento...

Transformada de Fourier discreta (DFT)

$$\tilde{\psi}_k = \sum_{n=0}^{N-1} \psi_n \exp\{-2\pi i k n/N\} = \sum_{n=0}^{N-1} \psi_n \exp\{-2\pi i k x/X\} = \sum_{n=0}^{N-1} \psi_n \exp\{-2\pi i \xi_k x\} \quad k = -N/2, \ldots, N/2$$

$$x = n\Delta x, X = N\Delta x, \xi_k = k/X$$

Transformada de Fourier discreta (DFT)

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$$x = n\Delta x, X = N\Delta x, \xi_k = k/X$$

Transformada de Fourier inversa

$$\psi_n = \frac{1}{N} \sum_{k=-N/2}^{N/2} \tilde{\psi}_k \exp\{2\pi i k n/N\} \quad n = 0, \ldots, N-1$$

$$\exp\left\{\frac{i\lambda\!\!\!^{-}\Delta z\frac{\partial^2}{\partial x^2}}{4n_0}\right\}\psi(x,z)=\sum_{k=-N/2}^{N/2}\exp\left\{\frac{i\lambda\!\!\!^{-}\Delta z\frac{\partial^2}{\partial x^2}}{4n_0}\right\}\exp\left\{\frac{2\pi ikx}{X}\right\}\tilde{\psi}_k(z)$$

$$
\begin{aligned}
\exp\left\{\frac{i\lambda\!\!\bar{}\,\Delta z\frac{\partial^2}{\partial x^2}}{4n_0}\right\}\psi(x,z) &= \sum_{k=-N/2}^{N/2}\exp\left\{\frac{i\lambda\!\!\bar{}\,\Delta z\frac{\partial^2}{\partial x^2}}{4n_0}\right\}\exp\left\{\frac{2\pi ikx}{X}\right\}\tilde{\psi}_k(z) \\
&= \sum_{k=-N/2}^{N/2}\exp\left\{\frac{i\lambda\!\!\bar{}\,\Delta z\left(\frac{2i\pi k}{X}\right)^2}{4n_0}\right\}\exp\left\{\frac{2\pi ikx}{X}\right\}\tilde{\psi}_k(z)
\end{aligned}
$$

$$= \sum_{k=-N/2}^{N/2} \exp\left\{ \frac{i\lambdabar \Delta z \left(\frac{2i\pi k}{X}\right)^2}{4n_0} \right\} \exp\left\{ \frac{2\pi i k x}{X} \right\} \tilde{\psi}_k(z)$$

$$= \frac{1}{N} \sum_{k=-N/2}^{N/2} \exp\left\{ \frac{-i\lambdabar \Delta z k_x^2}{4n_0} \right\} \exp\left\{ \frac{2\pi i k x}{X} \right\} \sum_{n=0}^{N-1} \psi_n(z) \exp\left\{ -\frac{2\pi i k x}{X} \right\}$$

$$= \frac{1}{N} \sum_{k=-N/2}^{N/2} \exp\left\{ \frac{-i\lambda\!\!\!^{-} \Delta z k_x^2}{4n_0} \right\} \exp\left\{ \frac{2\pi i k x}{X} \right\} \sum_{n=0}^{N-1} \psi_n(z) \exp\left\{ -\frac{2\pi i k x}{X} \right\}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{k=-N/2}^{N/2} \exp\left\{ \frac{-i\lambda\!\!\!^{-} \Delta z k_x^2}{4n_0} \right\} \exp\left\{ \frac{2\pi i k x}{X} \right\} \psi_n(z) \exp\left\{ -\frac{2\pi i k x}{X} \right\} \right)$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left(\sum_{k=-N/2}^{N/2}\exp\left\{\frac{-i\lambda\!\!\!^{-}\Delta z k_x^2}{4n_0}\right\}\exp\left\{\frac{2\pi i k x}{X}\right\}\psi_n(z)\exp\left\{-\frac{2\pi i k x}{X}\right\}\right)$$

$$= \mathcal{F}^{-1}\left(\exp\left\{\frac{-i\lambda\!\!\!^{-}\Delta z k_x^2}{4n_0}\right\}\mathcal{F}(\psi(x,z))\right)$$

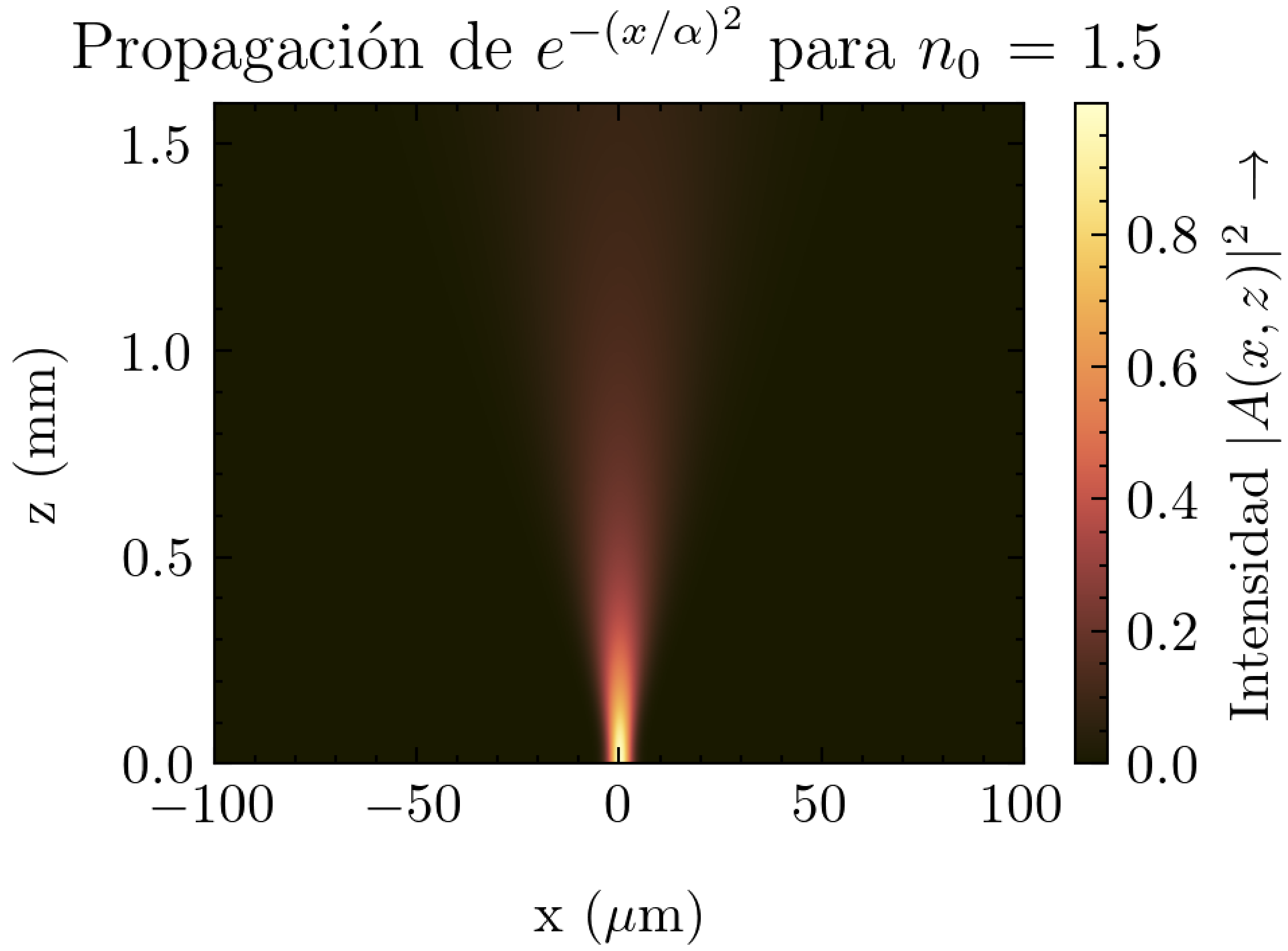
$$\exp\left\{\frac{i\lambda\!\!\!^{-}\Delta z\frac{\partial^2}{\partial x^2}}{4n_0}\right\}\psi(x,z) = \mathcal{F}^{-1}\left(\exp\left\{\frac{-i\lambda\!\!\!^{-}\Delta z k_x^2}{4n_0}\right\}\mathcal{F}(\psi(x,z))\right)$$

Ya podemos iterar!

$$\psi(x,y,z+\Delta z) = \exp\left\{\frac{i\lambda\Delta z\nabla_\perp^2}{4n_0}\right\}\exp\left\{\frac{i\Delta z\Delta n(x,y)}{\lambda}\right\}\exp\left\{\frac{i\lambda\Delta z\nabla_\perp^2}{4n_0}\right\}\psi(x,y,z) + O(\Delta z^3)$$

1. Aplicamos Fourier a la condición inicial
2. Multiplicamos lo obtenido por $\exp\left\{\frac{-i\lambda\Delta z k_x^2}{4n_0}\right\}$
3. Aplicamos Fourier inverso
4. Multiplicamos lo obtenido por $\exp\left\{\frac{i\Delta z\Delta n(x,y)}{\lambda}\right\}$
5. Aplicamos Fourier a lo obtenido
6. Multiplicamos lo obtenido por $\exp\left\{\frac{-i\lambda\Delta z k_x^2}{4n_0}\right\}$
7. Fourier inverso
8. Logramos propagar la condición inicial un step dz. Repetimos pasos 1-7 hasta llegar a la distancia final

Ejemplo de propagación libre



Largo de Rayleigh