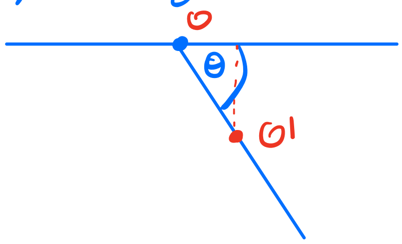
P1 La energía total del sistema será:

$$E = \varepsilon_1 + \varepsilon_2$$

Con ε_i la energía mecánica de la barra i. Para una barra de largo L; su energía será:



• Como $O' = G \Rightarrow \vec{R} = \vec{R}_{CM}$; Así:

$$\vec{R}_{CM} = \frac{L}{2}\cos\theta\,\hat{x} - \frac{L}{2}\sin\theta\,\hat{y}$$

$$\dot{\vec{R}}_{CM} = -\frac{L}{2}\sin\theta\,\dot{\theta}\,\hat{x} - \frac{L}{2}\cos\theta\,\dot{\theta}\,\hat{y} \Rightarrow \|\dot{\vec{R}}_{CM}\|^2 = \frac{L^2\dot{\theta}^2}{4}$$

$$I = \frac{1}{12}ML^2$$

De manera que:

$$\varepsilon_i = \frac{1}{2}M\|\dot{\vec{R}}_{CM}\|^2 + \frac{1}{2}I\dot{\theta}^2 + Mg\,Y_{CM}$$

$$= \frac{1}{8}ML^2\dot{\theta}^2 + \frac{1}{24}ML^2\dot{\theta}^2 - \frac{Mg}{2}L\sin\theta$$

$$\Rightarrow \varepsilon_i = \frac{1}{6}ML^2\dot{\theta}^2 - \frac{Mg}{2}L\sin\theta \quad ; \quad i = \{1,2\}.$$

De esta manera:

$$\boxed{E = \varepsilon_1 + \varepsilon_2 = \frac{1}{3}ML^2\dot{\theta}^2 - MgL\sin\theta}$$

(b) Utilizando cons. de la energía: $\boxed{\dot{E} = 0}$

$$\boxed{\dot{E} = \frac{2}{3} ML^2 \dot{\theta}\ddot{\theta} - MgL\cos\theta\,\dot{\theta} = 0}$$

$$\Rightarrow \boxed{\ddot{\theta} - \frac{3}{2}\frac{g}{L}\cos\theta = 0}$$

(c) Usando que en $t=0$ $\boxed{\theta = 0\,;\ \dot{\theta} = 0}$; y que $\boxed{E(t=0) = E(t)}$:

$$\boxed{0 = \frac{1}{3} ML^2\dot{\theta}^2 - MgL\sin\theta}$$

$$\Rightarrow \boxed{\dot{\theta} = \left(\frac{3g}{L}\sin\theta\right)^{1/2}}$$

Usando que en el instante en que chocan $\boxed{\theta = \pi/2}$:

$$\boxed{\dot{\theta}_c = \left(\frac{3g}{L}\right)^{1/2}}$$

De manera que las velocidades son: $\boxed{\begin{array}{l} \vec{v}^{\,CM}_1(\theta = \pi/2, \dot{\theta} = \dot{\theta}_c) \\ \vec{v}^{\,CM}_2(\theta = \pi/2, \dot{\theta} = \dot{\theta}_c) \end{array}}$

$$\Rightarrow \vec{v}^{\,CM}_{1,2} = \mp\frac{L}{2}\sin\theta_c\dot{\theta}_c\hat{x} - \cancel{\frac{L}{2}\cos\theta_c}^{\,0}\,\dot{\theta}_c\hat{y} = \mp\frac{L}{2}\left(\frac{3g}{L}\right)^{1/2}$$

$$= \mp\left(\frac{3}{4}Lg\right)^{1/2}$$

$$\therefore \boxed{\vec{v}^{\,CM}_{1,2} = \mp\left(\frac{3}{4}gL\right)^{1/2}}$$