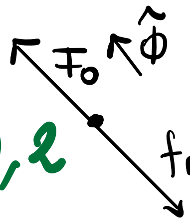
P3

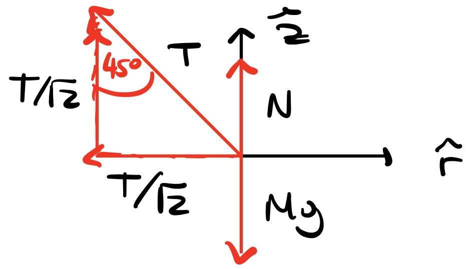
(a) En coordenadas cilíndricas la aceleración: (a): 2 ptos.

$$\vec{a} = (\ddot{r} - r\dot{\phi}^2)\hat{r} + (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z} \longrightarrow +0.2$$

Las fuerzas que actúan sobre la partícula son:

+0,2





+0,2

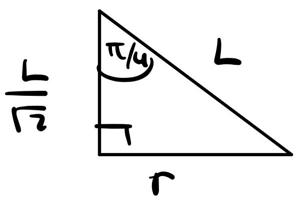
f_r; N y T son desconocidas. Se tiene; por componente:

$\hat{r}$) $\quad m(\ddot{r} - r\dot{\phi}^2) = -\dfrac{T}{\sqrt{2}} \quad +0.1$

$\hat{\phi}$) $\quad m(2\dot{r}\dot{\phi} + r\ddot{\phi}) = F_0 - f_r \quad +0.1$

$\hat{z}$) $\quad m\ddot{z} = N + \dfrac{T}{\sqrt{2}} - mg \quad +0.1$

Como el ángulo que define el radio de la circunf. es $\pi/4$; y la cuerda está fija; describiendo la partícula una circunf se tiene que:



$\Rightarrow r = L\sin\pi/4 = \dfrac{L}{\sqrt{2}} \quad +0.1$

$\Rightarrow \dot{r} = \ddot{r} = 0 \quad +0.2$

+0,2

Además $\ddot{z} = 0$ (mov. solo en el plano). Así; las ecuaciones son:

$\hat{r}$) $\quad \dfrac{mL}{\sqrt{2}}\dot{\phi}^2 = \dfrac{T}{\sqrt{2}} \Rightarrow \boxed{mL\dot{\phi}^2 = T}$ (1) $\quad +0.2$

$\hat{\phi}$) $\quad \boxed{\dfrac{mL}{\sqrt{2}}\ddot{\phi} = F_0 - f_r}$ (2) $\quad +0.2$

$\hat{z}$) $\quad \boxed{0 = N - mg + \dfrac{T}{\sqrt{2}}}$ (3) $\quad +0.2$

(b) Determinemos $\dot{\phi}(\phi)$: Reemplazando (1) en (3):

$$N = mg - \frac{mL\dot{\phi}^2}{\sqrt{2}}$$ + 0.3 (Normal)

+0.3 roce total

Dado que $f_r = \mu N \Rightarrow f_r = \mu m g - \frac{\mu m L \dot{\phi}^2}{\sqrt{2}}$. Reemplazando en (2):

+0.1 roce det

$$\frac{mL\ddot{\phi}}{\sqrt{2}} = F_0 - \mu m g + \frac{\mu m L \dot{\phi}^2}{\sqrt{2}} \qquad / \times \frac{\sqrt{2}}{mL}$$

$$\Rightarrow \quad \ddot{\phi} = \frac{\mu\sqrt{2}F_0}{\mu m L} - \frac{\sqrt{2}\mu g}{L} + \mu\dot{\phi}^2$$

$$\ddot{\phi} = \mu\left(\frac{\sqrt{2}F_0}{\mu m L} - \frac{\sqrt{2}g}{L} + \dot{\phi}^2\right)$$ +0.3 llegar a EDO

Definiendo $\frac{\sqrt{2}F_0}{\mu m L} - \frac{\sqrt{2}g}{L} = -A$

$$\Rightarrow \quad \ddot{\phi} = \mu(\dot{\phi}^2 - A) \quad ; \quad \text{Usando} \quad \ddot{\phi} = \dot{\phi}\frac{d\dot{\phi}}{d\phi}:$$

$$\frac{\dot{\phi}\, d\dot{\phi}}{(\dot{\phi}^2 - A)} = \mu\, d\phi \qquad / \int$$

$$\Rightarrow \int_0^{\dot{\phi}} \frac{\dot{\phi}'\, d\dot{\phi}'}{(\dot{\phi}'^2 - A)} = \mu \int_{\phi_0}^{\phi} d\phi$$

Introduciendo $\omega = \dot{\phi}^2 - A \rightarrow d\omega = 2\dot{\phi}\, d\dot{\phi}$

$$\rightarrow \dot{\phi}^2 = \omega + A \Rightarrow \dot{\phi} = \sqrt{\omega + A}$$

Si $\dot{\phi} \to 0 \Rightarrow \omega \to -A$.

+0.5 (integrar)

Así:

$$\int_{-A}^{\sqrt{\omega + A}} \frac{d\omega}{\omega} = 2\mu(\phi - \phi_0)$$

$$\Rightarrow \ln\left(\frac{\sqrt{\omega + A}}{-A}\right) = 2\mu(\phi - \phi_0)$$

$$\Rightarrow \quad \frac{\sqrt{\omega+A}}{-A} = \exp\left[2\mu(\phi-\phi_0)\right]$$

$$\frac{\omega+A}{A^2} = e^{4\mu(\phi-\phi_0)} \quad \Rightarrow \quad \dot{\phi}^2 = A^2 e^{4\mu(\phi-\phi_0)}$$

$$\Rightarrow \quad \dot{\phi} = A\,e^{2\mu(\phi-\phi_0)}$$

$$\Rightarrow \quad \boxed{\dot{\phi} = \left(\frac{\sqrt{2}\,g}{L} - \frac{\sqrt{2}\,F_0}{\mu m L}\right) e^{2\mu(\phi-\phi_0)}}$$

+0.5 (despeje)

(c) Integrando la última expresión:

$$\frac{d\phi}{dt} = A\,e^{2\mu\phi}\,e^{-2\mu\phi_0} \qquad / \int dt$$

$$\rightarrow \quad d\phi\, e^{-2\mu\phi} = A e^{-2\mu\phi_0}\, dt$$

$$\rightarrow \quad \int_{\phi_0}^{\phi} e^{-2\mu\phi}\, d\phi = A e^{-2\mu\phi_0}(t-t_0)$$

$$-\frac{1}{2\mu}\left(e^{-2\mu\phi} - e^{-2\mu\phi_0}\right) = A e^{-2\mu\phi_0}(t-t_0)$$

$$\Rightarrow \quad -e^{-2\mu\phi_0}\left(e^{-2\mu\phi+2\mu\phi_0} - 1\right) = 2\mu A e^{-2\mu\phi_0}(t-t_0)$$

$$\Rightarrow \quad e^{-2\mu\phi+2\mu\phi_0} - 1 = -2\mu A(t-t_0)$$

$$\Rightarrow \quad e^{-2\mu(\phi-\phi_0)} = 1 - 2\mu A(t-t_0)$$

$$\Rightarrow \quad -2\mu(\phi-\phi_0) = \ln\left(1 - 2\mu A(t-t_0)\right)$$

(innecesario)

$$\Rightarrow \quad \boxed{\phi - \phi_0 = -\frac{1}{2\mu}\ln\left(1 - 2\mu A(t-t_0)\right)}$$

Ahora; en las ecs; si $N=0 \rightarrow \dot{\phi}^2\Big|_{\phi=\phi_s} = \frac{T}{mL}$ (de (1))

+0.5

y además $T = \sqrt{2}\, mg$ (de (3))

$$\Rightarrow \quad \dot{\phi}^2\Big|_{\phi=\phi_s} = \frac{\sqrt{2}\,g}{L}.$$

+0.5

Evaluando lo obtenido en (b) en $\phi=\phi_s$ y elevando al cuadrado:

$$\dot{\phi}^2\Big|_{\phi=\phi_s} = \sqrt{2}\,\frac{g}{L} = \left(\sqrt{2}\frac{g}{L} - \frac{\sqrt{2}F_0}{\mu m L}\right)^2 e^{2\mu(\phi_s-\phi_0)}$$

+0.5 (igualar a $\dot{\phi}^2$ evaluado en ϕ_s)

$$\Rightarrow \quad e^{2\mu(\phi_s-\phi_0)} = \frac{\sqrt{2}\,g/L}{\left(\sqrt{2}\frac{g}{L} - \frac{\sqrt{2}F_0}{\mu m L}\right)^2} \qquad / \ \ln(\cdot)$$

$$\Rightarrow \quad \boxed{\phi_s - \phi_0 = \frac{1}{2\mu}\ln\left[\frac{\sqrt{2}\,g/L}{(\sqrt{2}g/L - \sqrt{2}F_0/\mu m L)^2}\right]}$$

+0.5 despejar

ángulo crítico

(Misma forma que $\phi_s(t)-\phi_0$).