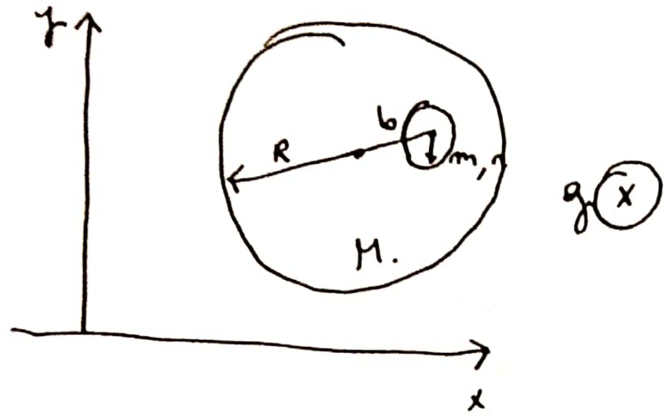
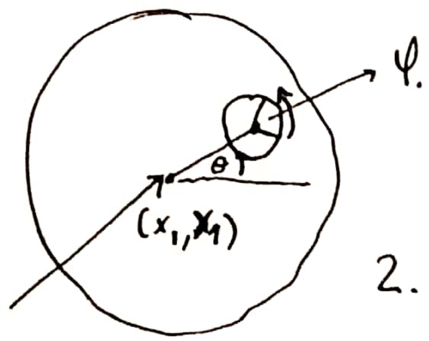
P3 | Semidiscos



Disco mayor := (x_1, y_1)
" menor := (x_2, y_2)
(al centro de masas)

1. coordenadas del centro de masas.

$$\vec{r}_1 = x_1 \hat{x} + y_1 \hat{y}$$
$$\vec{r}_2 = (x_1 + b\cos\theta)\hat{x} + (y_1 + b\sin\theta)$$



2. Derivar coordenadas.

$$\vec{v}_1 = \dot{x}_1 \hat{x} + \dot{y}_2 \hat{y}$$
$$v_2 = (\dot{x}_1 - b\dot{\theta}\sin\theta)\hat{x} + (\dot{y}_1 + b\cos\theta\,\dot{\theta})\hat{y}$$
$$K = \frac{1}{2} M \cdot \vec{v}_1 \cdot \vec{v}_1 + \frac{1}{2} m\, \vec{v}_2 \cdot \vec{v}_2 + E_{R_1} + E_{R_2}$$
$$E_{R_i} = \frac{1}{2} I_i \, w_i^2$$

3. Calculos energía $I_{R_1} = \frac{M}{2}$; $I_{R_2} = \frac{m}{2}$

$$K = \frac{1}{2} M (\dot{x}_1^2 + \dot{x}_1^2) + \frac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2 + b^2 \dot{\theta}^2 - 2b \dot{x} \dot{\theta} \sin\theta + 2b \dot{y} \dot{\theta} \cos\theta)$$

$$+ \frac{1}{4} M \underbrace{R^2 \dot{\theta}^2}_{W_{R_1}^2} + \frac{1}{4} m \underbrace{r^2 \dot{\varphi}^2}_{W_{R_2}^2}$$

$V = 0.$

4. Ocupar ecuaciones de Euler-Lagrange (E-L)

$\mathcal{L} = T - V$ ó $K - U$ (cinética - potencial)

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} = 0} \quad (E-L)$$

Si $\frac{\partial \mathcal{L}}{\partial x_i} = 0 \Rightarrow$ que hay una cantidad conservada.

$\cancel{x_1}$ | $\frac{\partial \mathcal{L}}{\partial x_1} = 0$; $\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = (M+m)\dot{x}_1 - mb\dot{\theta}\sin\theta$

$$(E-L) \quad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_1}\right) = 0 \quad / \int dt$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}_1} = cte.$$

$(M+m)\dot{x}_1 - mb\dot{\theta}\sin\theta = cte$

análogo para $\dot{y}_1$

$\varphi)$ $\frac{\partial L}{\partial \varphi} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\varphi}} = cte$

$$\frac{\partial L}{\partial \dot{\varphi}} = \frac{1}{2} m r^2 \dot{\varphi} = cte$$

$$\Rightarrow \dot{\varphi} = cte.$$

$\theta)$ $\frac{\partial L}{\partial \theta} = -mb\dot{x}_1\dot{\theta}\cos\theta - mb\dot{y}_1\dot{\theta}\sin\theta$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} M R^2 \dot{\theta} + mb^2\dot{\theta} - mb\dot{x}_1\sin\theta + mb\dot{y}_1\cos\theta$$

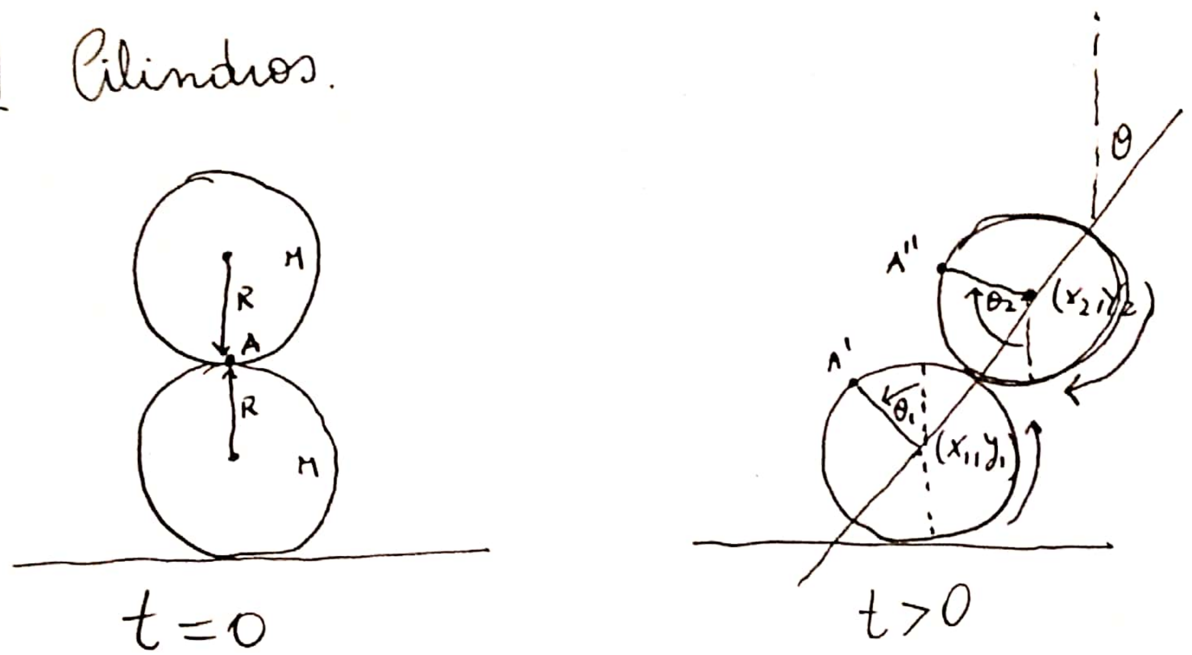
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{1}{2} M R^2 \ddot{\theta} + mb^2\ddot{\theta} - mb\ddot{x}_1\sin\theta - mb\dot{x}_1\dot{\theta}\cos\theta + mb\ddot{y}_1\cos\theta - mb\dot{y}_1\dot{\theta}\sin\theta$$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \Leftrightarrow$ Calcular la ecuación de movimiento para θ

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{1}{2} M R^2 \ddot\theta + m b \ddot\theta - m b \ddot x_1 \sin\theta + m b \ddot y \cos\theta = 0.$$

P1 Cilindros.



θ_1 y θ_2 se miden desde la vertical a sus centros de masa.

$$\theta_1 + \theta = \theta_2 - \theta \Rightarrow \theta_2 = \theta_1 + 2\theta$$

$x_1 = -R\theta_1$ (se mueve hacia la izquierda rodando)

$y_1 = R$ (Centro de masa estático.)

$X_2 = X_1 + 2R\sin\theta$; $Y_2 = \underset{\sim}{3R} - 2(R - R\cos\theta)$

$X_2 = -R\theta_1 + 2R\sin\theta$

3R es la altura maxima del centro de masas.

$y_2 = R + 2R\cos\theta$

$\vec{v}_1 = -R\dot{\theta}_1\hat{x} + 0\hat{y}$

$\vec{v}_2 = (-R\dot{\theta}_1 + 2R\dot{\theta}\cos\theta)\hat{x} - 2R\dot{\theta}\sin\theta\,\hat{y}$

$T_1 = \frac{1}{2}M(\vec{v}_1\cdot\vec{v}_1) + \frac{1}{2}I_1 w_1^2$; $I = \frac{1}{2}M$; $w = R\dot{\theta}_1$

$= \frac{1}{2}MR^2\dot{\theta}_1^2 + \frac{1}{4}MR^2\dot{\theta}_1^2 = \frac{3}{4}MR^2\dot{\theta}_1^2$

$T_2 = \frac{1}{2}M(\vec{v}_2\cdot\vec{v}_2) + \frac{1}{2}I_2 w_2$

$= \frac{1}{2}M(\vec{v}_2\cdot\vec{v}_2) + \frac{1}{2}\frac{M}{2}\cdot(\dot{\theta}_2 R)^2$, pero $\dot{\theta}_2 = \dot{\theta}_1 + 2\dot{\theta}$

$= \frac{1}{2}M(\vec{v}_2\cdot\vec{v}_2) + \frac{1}{4}MR^2(\dot{\theta}_1^2 + 4\dot{\theta}_1\dot{\theta} + 4\dot{\theta}^2)$

$= \frac{1}{2}M\left[(-R\dot{\theta}_1 + 2R\dot{\theta}\cos\theta)^2 + (-2R\dot{\theta}\sin\theta)^2\right) + \ldots$

$T_2 = \frac{1}{4}MR^2\left[3\dot{\theta}_1^2 + 4\dot{\theta}_1\dot{\theta}(1 - 2\cos\theta) + 12\dot{\theta}^2\right]$

$$V = Mg(y_1 + y_2) = 2MR(1+\cos\theta)\, g.$$

$$L = T_1 + T_2 - V$$

$$\boxed{L = \frac{1}{2} MR^2 [3\dot\theta_1^2 + 2\dot\theta_1\dot\theta(1-2\cos\theta) + 6\dot\theta^2] - 2MRg(1+\cos\theta)}$$

b) Como no hay fuerzas que se opongan al movimiento de manera conservativa, una cantidad conservada es la energía mecánica total

$$E = T + V \quad (1)$$

$$\frac{\partial L}{\partial \theta_1} = 0 \Rightarrow \frac{\partial L}{\partial \dot\theta_1} = cte$$

$$\frac{\partial L}{\partial \dot\theta_1} = MR^2 [3\dot\theta_1 + \dot\theta(1-2\cos\theta)] = cte. \quad (2)$$

c) de (2) tenemos que $\theta = 0$ y $\dot\theta_1 = \dot\theta = 0$

$$\Rightarrow MR^2 [3\ddot\theta_1 + \dot\theta(1-2\cos\theta)] = 0 \quad (3)$$

① $E_i = 4MRg$ (MRg por 1er centro de masas
$3MRg$ " segundo centro de masas)

$$E_f = \frac{1}{2} MR^2 [3\dot{\theta}_1^2 + 2\dot{\theta}_1\dot{\theta}(1-2\cos\theta) + 6\dot{\theta}^2] + 2MR(1+\cos\theta)g$$

$$E_i = E_f$$

de ③ como $ML^2 \neq 0$

$$3\dot{\theta}_1 + \dot{\theta}(1-2\cos\theta) = 0$$

$$\Rightarrow \boxed{\dot{\theta}_1 = \frac{\dot{\theta}(2\cos\theta - 1)}{3}} \quad ④$$

④ en $E_i = E_f$

$$MR^2 [\dot{\theta}_1^2(-1+2\cos\theta)^2 + 2\dot{\theta}^2(1-2\cos\theta)(2\cos\theta-1) + 18\dot{\theta}^2]$$
$$+ 12MgR(1+\cos\theta) = 24MgR.$$

$$MR^2 [18\dot{\theta}^2 - (1-2\cos\theta)^2\dot{\theta}^2] = 12MgR(1-\cos\theta)$$

Escaneado con CamScanner

$$\dot{\theta}^2\left[18-(1-2\cos\theta)^2\right]=\frac{12g}{R}(1-\cos\theta)$$

$$\boxed{\dot{\theta}^2=\frac{12(1-\cos\theta)g}{R(17+4\cos\theta-4\cos^2\theta)}}$$