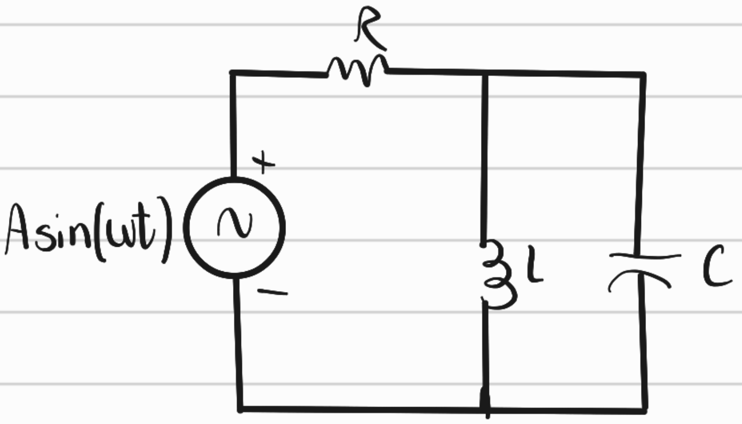
P1



$R = 4\,[\text{Ohm}]$

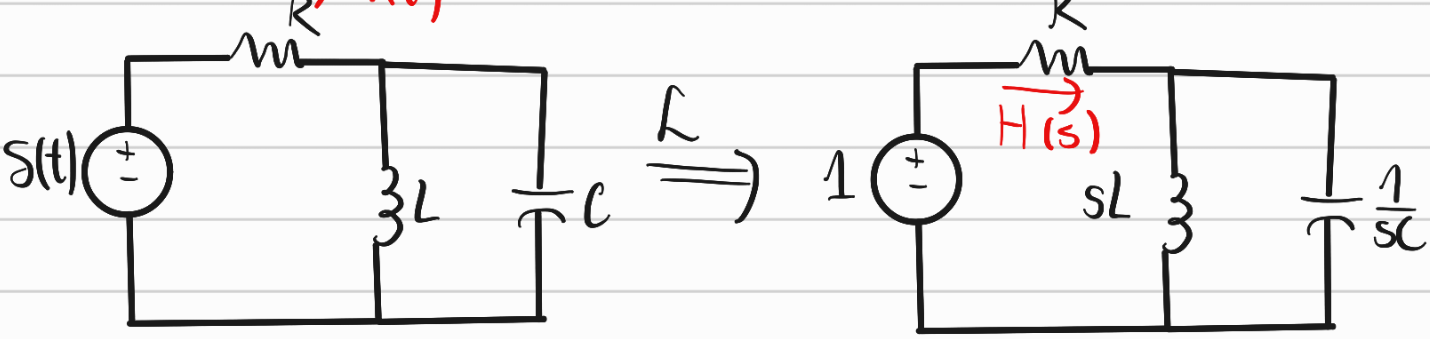
$L = 0.1\,[mH]$ $\quad 10^{-4}$

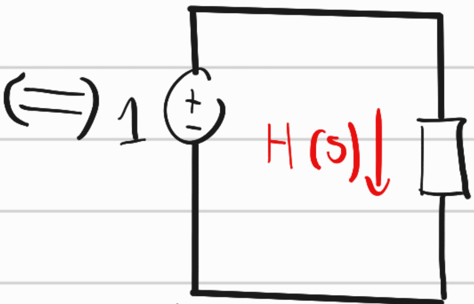
$C = 10\,[mF]$ $\quad 10^{-2}$

$A = 5\,[V]$

$f = 50\,Hz$

a) Obtener la función de red (o respuesta al impulso, es lo mismo)
Para ello consideramos la fuente como un impulso $\delta(t)$ y
CI's nulas: $\rightarrow h(t)$





$Z_{eq} = R + \left(sL \,//\, \frac{1}{sC}\right)$

Por lo tanto H(s) se calcula como:

$$H(s) = \frac{1}{Z_{eq}}$$

$$= \frac{1}{R + \left(sL \,//\, \frac{1}{sC}\right)}$$

$$= \frac{1}{R + \frac{L/C}{sL + 1/sC}}$$

$$= \frac{1}{R + \frac{sL}{s^2LC + 1}}$$

$$= \frac{1}{R + \frac{s\cancel{L}}{\cancel{L}C\left(s^2 + 1/LC\right)}}$$

$$= \frac{C\left(s^2 + 1/LC\right)}{RC\left(s^2 + 1/LC\right) + s}$$

$$\boxed{= \frac{\left(s^2 + 1/LC\right)}{R\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right)}}$$

b) Queremos calcular la componente transiente de la RESC(t). Para ello calculamos la TL de la entrada y multiplicamos por H(s):

$$I(s) = \mathcal{L}\left[5\sin(wt)\right] = 5\int_0^\infty \sin(wt)e^{-st}\,dt$$

$$= 5\int_0^\infty \frac{\left(e^{jwt} - e^{-jwt}\right)}{2j} e^{-st}\,dt$$

$$= \frac{5}{2j}\left[\int_0^\infty e^{-(s-jw)t}\,dt - \int_0^\infty e^{-(s+jw)t}\,dt\right]$$

$$\boxed{= \frac{5}{2j}\left[\frac{1}{s-jw} - \frac{1}{s+jw}\right]}$$

(acá en la pauta anterior puse mal los signos)

$$\frac{5}{2j}\,\frac{\cancel{s}+jw-\cancel{s}+jw}{s^2+w^2} = \frac{5}{\cancel{2j}}\,\frac{\cancel{2}jw}{s^2+w^2} = 5\,\frac{w}{s^2+w^2}$$

otra forma conocida que para este caso no usaremos

Y ahora si:

$$RESC(s) = H(s) \cdot I(s)$$

$$= \frac{\left(s^2 + 1/LC\right)}{R\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)} \cdot \frac{5}{2j}\left[\frac{1}{s-jw} - \frac{1}{s+jw}\right]$$

$$= \frac{5}{2jR}\underbrace{\left[\frac{1}{s-jw}\,\frac{s^2 + 1/LC}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} - \frac{1}{s+jw}\cdot\frac{s^2 + 1/LC}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right]}_{\text{fracciones parciales}}$$

Primero factoricemos $s^2 + \frac{1}{RC}s + \frac{1}{LC}$ encontrando sus ceros:

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$\Rightarrow s_{1,2} = -\frac{1}{2RC} \pm \frac{1}{2}\sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}$$

(999,92187...) ≈ 1000

$$= -\frac{1}{2\cdot 4\cdot 10^{-2}}\left[\frac{1}{s}\right] \pm \frac{1}{2}\overbrace{\sqrt{\left(\frac{1}{4\cdot 10^{-2}}\right)^2 - \frac{4}{10^{-4}\cdot 10^{-2}}}}\left[\frac{1}{s}\right]$$

$$s_1 = -12.5 + 1000j = 1000.078\angle 1.58^\circ$$
$$s_2 = -12.5 - 1000j = 1000.078\angle -1.58^\circ$$

Por lo que de forma general, la descomposición en fracciones parciales se hace de la sgte forma:

$$\frac{s^2 + 1/LC}{(s-j\omega)(s-s_1)(s-s_2)} = \frac{a_1}{s-s_1} + \frac{a_2}{s-s_2} + \frac{a_3}{s-j\omega}$$

$$\frac{s^2 + 1/LC}{(s+j\omega)(s-s_2)(s-s_3)} = \frac{b_1}{s-s_1} + \frac{b_2}{s-s_2} + \frac{b_3}{s+j\omega}$$

$$a_1 - b_1$$

respuesta transiente

RPS (regimen permanente sinusoidal)

Para encontrar estos coefs descomponemos en fracciones parciales o hallando el residuo asociado a su polo respectivo, esto es:

$$a_i = \lim_{s\to s_i} (s-s_i)\frac{s^2 + \frac{1}{LC}}{(s-j\omega)(s-s_1)(s-s_2)}$$

$$b_i = \lim_{s\to s_i} (s-s_i)\frac{1}{(s+j\omega)(s-s_1)(s-s_2)}$$

Explicitamente:

$$a_1 = \lim_{s\to s_1} \frac{s^2 + \frac{1}{LC}}{(s-j\omega)(s-s_2)} = \boxed{\frac{s_1^2 + \frac{1}{LC}}{(s_1-j\omega)(s_1-s_2)}}$$

$$\approx \frac{25000\angle -1.56^\circ}{1.37\cdot 10^6\angle -3.12^\circ}$$

$$b_1 = \lim_{s\to s_1} \frac{s^2 + \frac{1}{LC}}{(s+j\omega)(s-s_2)} = \boxed{\frac{s_1^2 + \frac{1}{LC}}{(s_1+j\omega)(s_1-s_2)}}$$

$$\approx \frac{25000\angle -1.56^\circ}{2.62\cdot 10^6\angle -3.13^\circ}$$

$$a_2 = \lim_{s\to s_2} \frac{s^2 + \frac{1}{LC}}{(s-j\omega)(s-s_1)} = \boxed{\frac{s_2^2 + \frac{1}{LC}}{(s_2-j\omega)(s_2-s_1)}}$$

$$= \frac{25000\angle 1.56^\circ}{2.62\cdot 10^6\angle 3.13^\circ}$$

$$b_2 = \lim_{s\to s_2} \frac{s^2 + \frac{1}{LC}}{(s+j\omega)(s-s_1)} = \boxed{\frac{s_2^2 + \frac{1}{LC}}{(s_2+j\omega)(s_2-s_1)}}$$

$$= \frac{25000\angle 1.56^\circ}{1.37\cdot 10^6\angle 3.12^\circ}$$

Quedando:

$$RESC^{transiente}(s) = \frac{5}{R\,2j}\left(\frac{(a_1-b_1)}{(s-s_1)} + \frac{(a_2-b_2)}{(s-s_2)}\right)$$

$\mathcal{L}^{-1}$

$$RESC^{transiente}(t) = \frac{5}{R\,2j}\left(e^{s_1 t}(a_1-b_1) + e^{s_2 t}(a_2\ b_2)\right)$$

$$= \frac{5}{R}\frac{e^{-12.5t}}{2j}\left[e^{1000jt}(a_1-b_1)+e^{-1000jt}(a_2-b_2)\right]$$

Definiendo $A=\frac{25000}{1.37\cdot 10^6} \Rightarrow$ $a_1 = A\angle 1.56°,\ b_1=\frac{A}{2}\angle 1.56°$, $a_2=\frac{A}{2}\angle -1.56°,\ b_2 = A\angle -1.56°$

$$RESC^{transiente}(t) = \frac{5}{R}\frac{e^{-12.5t}}{2j}\left[e^{10^3jt}\underbrace{(a_1-b_1)}_{=\frac{A}{2}\angle 1.56°}+e^{-10^3jt}\underbrace{(a_2-b_2)}_{=-\frac{A}{2}\angle -1.56°}\right]$$

$$= \frac{5}{R}\cdot\frac{A}{2}\,e^{-12.5t}\,\frac{1}{2j}\left[e^{(10^3t+1.56°)j}-e^{-(10^3t+1.56°)j}\right]$$

$$= \frac{5}{R}\cdot\frac{A}{2}\,e^{-12.5t}\,\sin(10^3t+1.56°)$$

$$\tau=\frac{1}{12.5}$$

c) La parte transitoria desaparece para $t \geq 5\cdot 2RC = 10.4\cdot 10^{-2}[s]$
$= 4\cdot 10^{-1}[s]$